Shell Model Calculation Of Spectroscopic Properties Of Some Nuclei Participating In Positron Double Beta Decay

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***Abstract*:** Double beta decay (DBD) is clear sign of new physics beyond the standard model. Depending on the relative number of protons and neutrons in a nucleus, the three additional possible processes are double electron capture (ECEC), double positron decay ($\beta^+\beta^+$) and positron emitting electron capture (ECβ^+). These three processes are energetically competing and are referred as positron double beta decay (e^+DBD) modes. While DBD has been observed in about a dozen nuclei, the e^+DBD modes continue to be an elusive. The study of these modes require the knowledge of nuclear transition matrix elements (NTMEs), therefore, in this work, we have considered the nuclei namely ^{58}Ni, ^{74}Se, ^{78}Kr and ^{112}Sn, and studied their spectroscopic properties such as yrast energies, quadrupole moment (Q), magnetic moment (μ) and reduced transition probabilities B(E2). The obtained values are then compared with available experimental data to check the reliability of the wave functions. These wave functions may then be used for calculation of NTMEs for these positron double beta decay modes.

***Index Terms* -** Neutrinoless double beta decay, double positron decay, spectroscopic properties, nuclear transition matrix elements.

I. INTRODUCTION

The studies on neutrino oscillations provide information about differences of squared neutrino masses, and show that the neutrino mass matrix of weak neutrino eigenstates is non-diagonal. In this theoretical framework, processes which can occur in the standard model (SM) are modified and other processes due to non-vanishing neutrino masses can occur. Double beta decay with neutrino emission conserves the lepton number, providing a confirmation of the SM of weak interaction. On the contrary, the neutrino-less double beta decay ($0\nu\beta\beta$), which violates the lepton number by two units, can be a signature of new physics beyond the SM. Thus, investigations on neutrino-less double beta decay can provide information about neutrino properties, weak interaction, lepton number violation, and outline new theoretical frameworks [1]. However, in very recent years, interest in the double positron decay ($\beta^+\beta^+$), positron emitting electron capture, (ECβ^+), and double electron capture (ECEC) has been renewed. This is due to the fact that positron emitting processes have interesting signatures that could be detected experimentally [2-3]. ECEC is preferred by the available phase-space, but the rate is typically reduced by several orders of magnitude because an extra radiative process is required to satisfy energy-momentum conservation [4]. Furthermore, the ECβ^+ mode shows an enhanced sensitivity to right-handed weak currents [5] and could play an important role in the comprehension of the underlying mechanism in the event of a $0\nu\beta\beta$ discovery. These three processes are energetically competing and are referred as positron double beta decay (e^+DBD) modes.

The first direct observation of two-neutrino ECEC decay was made in ^{124}Xe with the XENON1T detector [6]. Half-life estimates for 0νECβ^+ in the most promising nuclei are of the order of 10^{29}-10^{33} years [2], while experimental limits for the isotopes, ^{64}Zn [7], ^{112}Sn [8] and ^{120}Te [9–11] investigated are in the range of $10^{18} – 10^{21}$ years.

If the $0\nu\beta\beta$ decay is observed, the e^+DBD processes would play a crucial role in discriminating the finer issues like dominance of Majorana neutrino mass or the right handed current. Once the neutrinoless $\beta\beta$ decay is experimentally observed, the values of different theoretical gauge parameters along with neutrino mass can be obtained through the calculation of NTMEs but these NTMEs can be calculated only when one has a set of reliable wave functions. Therefore, in the present work, we have calculated the spectroscopic properties of some nuclei namely ^{58}Ni, ^{74}Se, ^{78}Kr and ^{112}Sn to judge the validity of the wave functions. These wave functions can then be used to calculate the required NTMEs. The present work is organized as follows. In Sect. 2, we outline the theoretical framework to calculate the spectroscopic properties namely yrast energies, quadrupole moment [$Q(2^+)$], magnetic moment (μ) and reduced transition probabilities B(E2) for above nuclei. In Sect. 3, we have given the numerical results and discussed them. Finally, Sect. 4 is devoted to conclusions.

II. THEORETICAL FRAMEWORK

The calculations are performed with the help of ANTOINE shell model code [12]. To provide an effective interaction for nuclei in the upper part of the *pf* shell namely ^{58}Ni, ^{74}Se and ^{78}Kr, an effective interaction JUN45 [13] is constructed in the model space consisting of four spherical orbits, namely the $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, and $g_{9/2}$ single-particle orbits. The model space is called *f5pg9* shell. The single particle energies used are -9.8280, -8.7087, -7.8388 and -6.2617 MeV for $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, and $g_{9/2}$ orbits respectively and ^{58}Ni nucleus is assumed as inert core for this space. For ^{112}Sn, a *sdgh* shell nuclei, the BONN interaction [12] is used that provides us a systematic description of considered nucleus in this region. The model space consists of five single particle orbits namely $g_{7/2}$, $d_{5/2}$, $d_{3/2}$, $s_{1/2}$ and $h_{11/2}$ with single particle energies taken as 0.10, 0.0, 1.64, 1.55 and 2.50 MeV respectively. The ^{50}Sn nucleus is assumed to be an inert core for this model space. Using above model spaces and single particle energies, we have calculated the energies of 2^+, 4^+, 6^+ and 8^+ states of above nuclei and compared them with the available experimental data [14]. The calculated values are shown in Table 1 and energy states are plotted in Fig. 1. Similarly, we have calculated reduced transition probabilities BE(2) for $0^+ \rightarrow 2^+$ transition and quadrupole moments $Q(2^+)$ for 2^+ state. The calculated values are given in Table 2 along with the available experimental data [15].

The magnetic moment operator used in the present calculation is given by Eq. 1 as

$$\boldsymbol{\mu} = g_s \boldsymbol{s} + g_l \boldsymbol{l} \tag{1}$$

where g_s and g_l are the spin and the orbital g factors, respectively. By using the free-nucleon g factors $g_s = 5.586$, $g_l = 1$, for protons and $g_s = -3.826$, $g_l = 0$ for neutrons [16], the agreement between calculated value (μ_{fth}) and experiment (μ_{exp}) appears to be reasonable. However, there are small but systematic deviations from the experimental values. Such deviations are reduced when we introduce effective spin g factors, $g(\text{eff.}) = 0.7g(\text{free})$. Here, the "quenching" factor $q_s = 0.7$ is determined via a least squares fit to the experimental data [16]. The magnetic moment values taking both set of g factors are calculated and shown in Table 3.

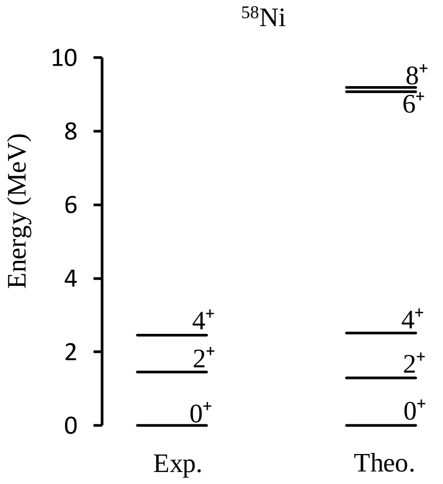
III. RESULTS AND DISCUSSION

In Table 1, we have calculated the energy eigen values of 2^+, 4^+, 6^+ and 8^+ states of nuclei and compared them with the available experimental data.

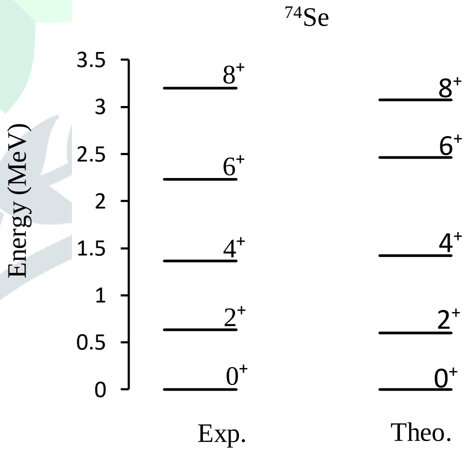
Table 1: Experimental and theoretical energy levels of ^{58}Ni, ^{74}Se, ^{78}Kr and ^{112}Sn nuclei

Energy levels	Nuclei							
	^{58}Ni		^{74}Se		^{78}Kr		^{112}Sn	
	Energy (MeV)		Energy (MeV)		Energy (MeV)		Energy (MeV)	
	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.
0^+	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2^+	1.454	1.298	0.635	0.601	0.455	0.492	1.257	1.267
4^+	2.459	2.508	1.363	1.423	1.119	1.434	2.247	2.121
6^+	-	9.074	2.231	2.463	1.978	2.635	2.549	2.628
8^+	-	9.192	3.198	3.072	2.993	2.801	-	2.934

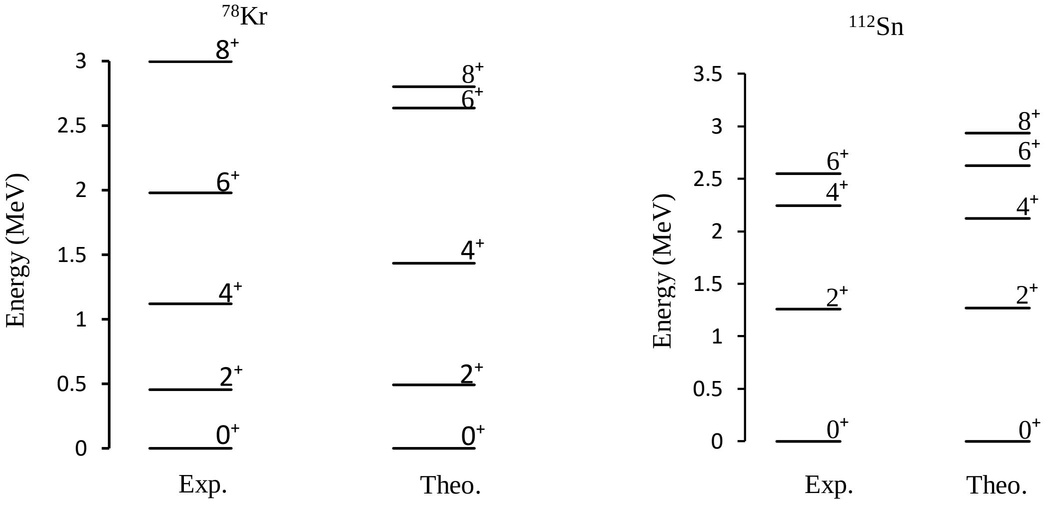
From the above table it is observed that calculated values for all the four nuclei are in good agreement with the experimental data. The experimental values of 6^+ and 8^+ states in case of ^{58}Ni and 8^+ state for ^{112}Sn nuclei are not available. We have also sketched the energy levels in Fig. 1.



(a)



(b)



(c) (d)

Fig. 1. Experimental and theoretical Energy levels of ^{58}Ni, ^{74}Se, ^{78}Kr and ^{112}Sn nuclei

In Table 2, we have shown the results for reduced transition probabilities for $0^+ \rightarrow 2^+$ transition in units of e^2b^2 for two sets of proton and neutron effective charges i.e. (0.5,1.5) and (1.1,1.5) and compared them with available experimental values [15].

Table 2: Reduced transition probabilities B(E2) for ^{58}Ni, ^{74}Se, ^{78}Kr and ^{112}Sn nuclei

Nuclei	B(E2) (e^2b^2) ($0^+ \rightarrow 2^+$)		
	Exp.	Theo.	
		(e_p,e_n)= (0.5,1.5)	(e_p,e_n)= (1.1, 1.5)
^{58}Ni	0.069	0.005	0.026
^{74}Se	0.387	0.094	0.190
^{78}Kr	0.600	0.094	0.150
^{112}Sn	0.240	0.038	0.187

It is seen that calculated values in this model show some deviations with the experimental data for both set of effective charges i.e. (0.5,1.5) and (1.1,1.5) for proton and neutron respectively although results come close to experimental one for later set of effective charges. In Table 3, we have presented the results for quadrupole moments Q (2^+) for 2^+ state and magnetic moments (μ) for considered nuclei and results are compared with available experimental data [17].

Table 3. Experimental and theoretical values of Quadrupole moments of 2^+ state and magnetic moments for ^{58}Ni, ^{74}Se, ^{78}Kr and ^{112}Sn nuclei

Nuclei	J^π	Quadrupole Moment (eb)			Magnetic Moment (μ_N)		
		Q_{exp}	Q_{th1}	Q_{th2}	μ_{exp}	μ_{free}^{th}	μ_{eff}^{th}
^{58}Ni	2^+	-0.10	-0.34	-0.74	-0.10	-0.70	-0.49
^{74}Se	2^+	-0.36	0.21	0.32	NA	0.54	0.58
^{78}Kr	2^+	NA	0.28	0.34	1.08	0.23	0.46
^{112}Sn	2^+	-0.03	-0.05	-0.12	0.70	0.29	0.21

The calculated quadrupole moments in units of eb are shown in column 4 and 5, written as Q_{th1} and Q_{th2} for two sets of effective charges (0.5, 1.5) and (1.1, 1.5) for proton and neutron respectively while column 3 shows experimental values. The values of Q are calculated for 2^+ states. The Q_{th1} values are in close agreement with the experimental values for ^{58}Ni and ^{112}Sn while for ^{74}Se nuclei, results are more close for Q_{th2} value with a change in sign. The experimental value Q_{exp} is not available for ^{78}Kr nuclei. The magnetic moments are calculated in units of nuclear magneton μ_N and are shown in column 7 and 8 as μ_{free}^{th} and μ_{eff}^{th} respectively. Experimental values of magnetic moments are shown in Column 6. The μ_{free}^{th} values are calculated taking free nucleon factors as $g_s = 5.586$, $g_l = 1$ for protons and $g_s = -3.826$, $g_l = 0$ for neutrons while μ_{eff}^{th} value is calculated by taking $g(\text{eff.})_s=0.7g(\text{free})_s$. No experimental values are available for ^{74}Se nuclei. In case of ^{58}Ni and ^{78}Kr nuclei, the theoretical value is close to experimental one when effective g is taken into account while for ^{112}Sn, the calculated values show more agreement with experimental one if we consider the free nucleon factors.

IV. CONCLUSION

We have calculated the spectroscopic properties of ^{58}Ni, ^{74}Se, ^{78}Kr and ^{112}Sn nuclei which participate in positron emitting mode of double beta decay process. The ANTOINE shell model code was used for calculating these properties and results are shown in Table 1,2 and 3. The near agreement between obtained values and experimentally available data ensures the reliability of wave functions to use them in further calculation of nuclear transition matrix elements for double beta decay transitions and calculation of half-lives of these nuclei.

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