MATHEMATICAL MODELLING OF DRYING OF *TOR-PUTITORA*

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Abstract: The aim of the presents study was to establish a suitable mathematical modelling of drying of Mahsheer fish (Tor-Putitora). In this perspective drying of the selected fish were performed by cutting it in butterfly shape at different temperature and at constant air velocity. A suitable model has been proposed on the basis of different statistical parameters viz. Correlation Coefficient (R), Root Mean Square Error (RMSE), Reduced Chi-Square (χ^{2}), Efficiency (EF) and Mean Bias Error (MBE). These parameters were evaluated on the basis of twelve well-established models available in the literature. Compared study, on the basis of these parameters shows that Midilli could satisfactorily describe the drying behavior of *Tor-Putitora*.
Index Terms: **Drying, Mathematical modelling, Tor-Putitora**

I. Introduction

Drying is a process of removal of moisture or other dissolvable by evaporation from the drying product before storage. Drying is one of the techniques used to protect nourishment by reducing the level of moisture or water action to levels where microorganisms cannot develop and the rate of response back off. The essential goal of food drying is to safeguard the raw material with a huge variety of items depending on the drying components (Chou et al, 2001). In addition to this, Drying has been used worldwide for a considerable length of time to save various food and agrarian items. The drying process is currently one of the key food conservation strategies (Saguy et al. 1980).The dried item achieved will minimize the cost and time of transportation, storage, bundling. Despite the fact that there are many ways in which drying can be done, however, the decision of the technique is based on the material and the clean dimension required (Kabiru et al, 2013, Mazza et al, 1980).

. Fish is one of the real protein foods that can be accessed in the tropics. This has made fisheries an essential part of the study. Fish accounts for 40 percent of creature protein intake; not at all like some other creature protein source with one issue of religious unthinkable or health hazard, has fish eaten the nation over (Olatunde et al, 1989). Unfortunately, as it may be, fish is one of the most short-lived of any stable product, and it will become unfit for human use within about one day of catch in the tropical atmosphere of most creating nations, except if it is exposed to some type of preparation (Ames et al, 1999). Tor-Putitora is an economically critical diversion fish just as highly regarded as food fish and occupies the two waterways and lakes, rising to fast streams with rough breeding ground (Menon et al, 1992).

Mathematical modelling of drying of any drying products plays an essential role to understand the phenomena of drying conditions and in the forecast execution of the drying procedure. It requires the establishment of an accurate model to recreate the drying curve at different drying environments (Fudholi et al, 2011).

In the present study, a suitable mathematical model of drying behavior of *Tor-Putitora* has been proposed on the basis of experimentally predicted drying curves.

II. Material and Methods

2.1 Sample Preparation and Drying Conditions

Drying of *Tor-Putitora* was performed in the incubator, which provides constant air temperature. To examine the drying behavior of locally available fish- *Tor-Putitora,* experiments were performed at three temperature *viz.* 50° C, 60° C, and 70° C and at constant air velocity of 3.6 m/s^2. In the experiments the unnecessary materials were removed from the sample with the help of water and dried by absorbent paper. The pre-treatment of the samples were undertaken in each experiment by putting some salt on the sample. The setup of the experiment is shown in Figure 1.

To ensure uniform drying, the fish was spread in the tray of wire mesh inside the incubator. The amounts of moisture removed from the product were monitored in the interval of one hour by weighing. For this purpose an electronic balance having least count of 0.1gm was taken. For the measurement of air velocity, anemometer having least count of 0.1m/s was taken.

2.2 Mathematical Modelling of drying curves of *Tor-Putitora*

Different Mathematical models which have been considered in the present study are summarized in Table 1. On the basis of experimentally observed data, fitted with different available models, different constants and statistical parameters were estimated. Model of drying of *Tor-Putitora* has been purposed, the evaluation of some statistical parameters viz. Correlation Coefficient (R), Root Mean Square Error (RMSE), Reduced Chi-Square (χ^{2}), Efficiency (EF) and Mean Bias Error (MBE) with the help of experimentally measured data. On the basis of highest values of EF and correlation coefficient (R) and the lowest values of $\chi 2$ and RMSE values, the best model has been purposed (Gunhan et al, 2005).

Figure 1: Drying Incubator, weighing machine and anemometer.

Table 1: Mathematical equations used to elaborate the drying curves of drying of *Tor-Putitora*.

Model no.	Model name	Model equation	References
1	Lewis	$M_R = \exp(-kt)$	(Lui et al,1997, Callaghen et al,1971)
2	Page	$M_R = \exp(-kt^n)$	(Agarwal et al,1997, Zhanget al,1991)
3	Modified page	$M_R = \exp[(-kt)^n]$	(Overhults et al,1973, White et al,1981)
4	Henderson and Pabis	$M_R = a\exp(-kt)$	(Henderson and Pabis,1961)
5	Yagcioglu et al.	$M_R = a\exp(-kt) + c$	(Yagcioglu et al,1999)
6	Two-term	$M_R = a\exp(-k_0 t) + b\exp(-k_1 t)$	(Henderson,1974, Rahman et al,1998)
7	Two-term exponential	$M_R = a\exp(-kt) + (1-a)\exp(-kat)$	(Sharaf-Elden et al,1980))
8	Wang and Singh	$M_R = 1 + at + bt^2$	(Wang and Singh,1978)
9	Diffusion approach	$M_R = a\exp(-kt) + (1-a)\exp(-kbt)$	(Kassem,1998)
10	Verma et al.	$M_R = a\exp(-kt) + (1-a)\exp(-gt)$	(Verma et al,1985)
11	Modified Henderson and Pabis	$M_R = a\exp(-kt) + b\exp(-gt) + c\exp(-ht)$	(Karathanos,1999)
12	Midili and Kucuk	$M_R = a\exp(-kt^n) + bt$	(Midili et al,2002)

2.2.1 Correlation coefficient (R):

$$R^2 = \frac{\sum_{i=1}^{N}(Mr_i - Mr_{pre,i})*(Mr_i - Mr_{exp,i})}{[\sum_{i=1}^{N}(Mr_i - Mr_{pre,i})^2]*[\sum_{i=1}^{N}(Mr_i - Mr_{exp,i})^2]} \quad (1)$$

2.2.2 Mean Bias Error (MBE):

$$MBE = \frac{1}{N}\sum_{i=1}^{N}(M r_{pre,i} - Mr_{exp,i}) \quad (2)$$

2.2. 3 Root Mean square error:

$$RMSE = \sqrt{\left[\frac{1}{N}\sum_{i=1}^{N}(M r_{pre,i} - Mr_{exp,i})^2\right]} \quad (3)$$

2.2.4 Reduced chi-square (χ^2):

$$\chi^2 = \frac{\sum_{i=1}^{N} M r_{exp,i} - Mr_{exp,i})^2}{N-n} \quad (4)$$

2.2.5 Efficiency (EF):

$$EF = \frac{\sum_{i=1}^{N}(MR_{i,exp} - MR_{i,exp_{mean}})^2 - \sum_{i=1}^{N}(MR_{i,pre} - MR_{i,exp_{mean}})^2}{\sum_{i=1}^{N}(MR_{i,exp} - MR_{i,exp_{mean}})^2} \quad (5)$$

2.2.6 Moisture ratio (M_R):

$$M_R = \frac{M - M_{exp}}{M_0 - M_{exp}} \quad (6)$$

III. Result and Discussion:

Figure 2 shows the variation of moisture ratio of *Tor-Putitora* with drying time at various constant temperatures. This gives information about the exponential drying behavior of dried fish and less drying time consumption at higher temperature as usual. The variation of drying rate with drying time is shown in Figure 3. This shows removal of moisture is so rapid at higher temperature in comparison to lower one. Estimated model constants and statistical parameters at 50°C, 60°C and 70°C of drying-*Tor-Putitora s*ummarized in Table 2, Table3 and in Table4 respectively. From these evaluated data and after being the comparison study one may predict that the, Midilli model has the highest EF value as 0.9996 at 50 ° C,0.9993 at 60 ° C and 0.999627 at 70 ° C and the lowest χ^2 value as 4E-05 at 50° C, 8E-05 at 60° C and 4.92453E-05 at 70° C and the lowest RMSE value as 0.0057 at 50 ° C, 0.0079 at 60 ° C and 0.00607734 at 70 ° C. Figure 4, Figure 5 and Figure 6 shows the variation of predicted data with experimental of the Midilli model for *Tor-Putitora* at 50° C, 60° C, 70° C respectively. This shows that in the case of Midilli model the values are more close to the straight line in comparison to the other considered models.

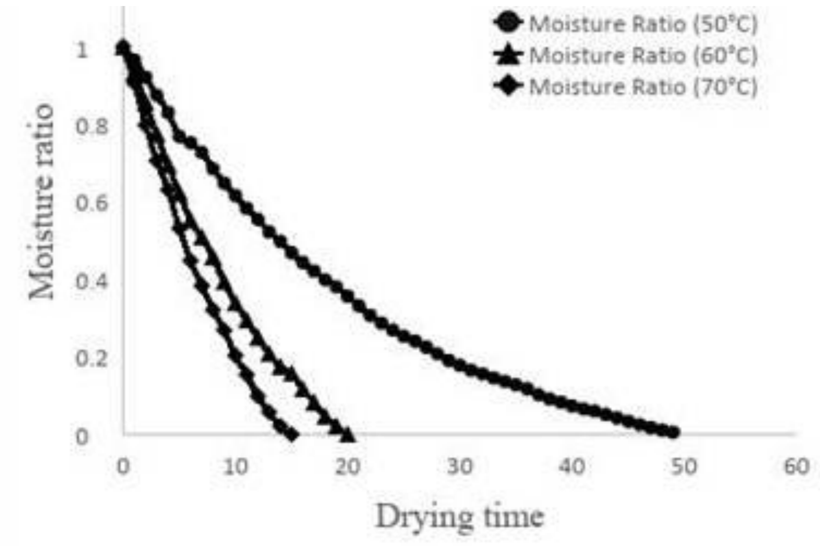

Figure 2 Variation of Moisture Ratio of *Tor-Putitora* with drying time.

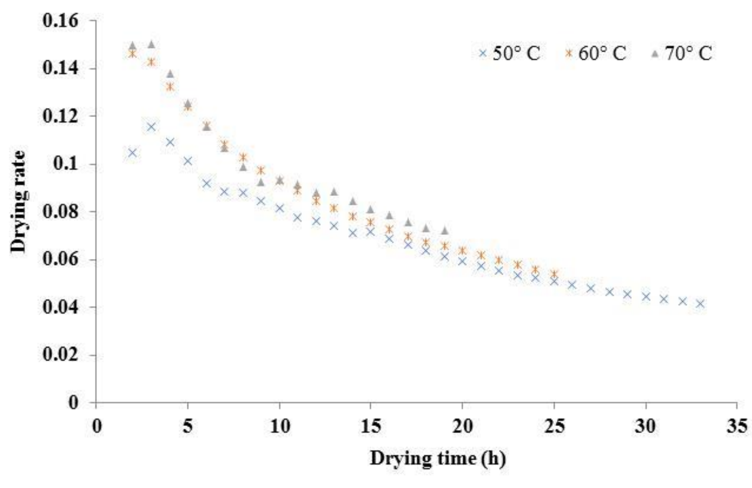

Figure 3 variation of drying rate of *Tor-Putitora* with drying time.

Table 2: Model constants and statistical parameters estimated at 50°C of drying- *Tor-Putitora*.

Model	Model constant	Correlation Coefficient (R)	Mean Bias Error (MBE)	Root Mean Square Error (RMSE)	Reduced ChiSquare(χ^2)	Efficiency (EF)
Lewis	k = 0.0547	0.99375	-0.003	0.0321	0.0011	0.985
Page	k = 0.0312, n = 1.1837	0.99851	-0.004	0.0157	0.0003	0.997
Modified and Pabis	k = 0.2339, n = 0.2339	0.99375	-0.003	0.0321	0.0011	0.985
Henderson and Pabis	a = 1.0541, k = 0.0577	0.99552	-0.007	0.0272	0.0008	0.9904
Yagcloglu et al.	a = 1.1465, k = 0.0426, c = -0.136	0.99983	-2.4E-05	0.037	3E-05	0.9997
Two- term	a = 0.5271, k_0 = 0.0577, b = 0.5271, k_1 = 0.0577	0.99552	-0.007	0.1923	0.0008	1.0096
Two- term exponential	a = 1.7065, k = 0.0729	0.99848	-0.004	0.0158	0.0003	0.9969
Wang and Singh	a = -0.041, b = 0.0004	0.99849	-0.003	0.0158	0.0003	0.9971
Diffusion approach	a = -9.331, k = 0.0924, b = 0.9425	0.99869	-0.003	0.0147	0.0002	0.9973
Verma et al.	a = -6.166, k = 0.0938, g = 0.086	0.99868	-0.004	0.0148	0.0002	0.9973
Modified Handerson and Pabis	a = 0.3514, k = 0.05777, b = 0.3514, g = 0.0576, c = 0.3514, h = 0.0577	0.99552	-0.007	0.0272	0.0008	0.9904
Midilli and Kucuk	a = 1.014, k = 0.218, m = 0.218, b = -0.002	0.99980	-3.3E-05	0.0057	4E-05	0.9996

Table 3: Model constants and statistical parameters estimated at 60°C of drying- *Tor-Putitora*.

Model	Model constant	Correlation Coefficient (R)	Mean Bias Error (MBE)	Root Mean Square Error (RMSE)	Reduced ChiSquare(χ^2)	Efficiency (EF)
Lewis	k = 0.1113	0.98635	0.0005	0.0492	0.0026	0.9645
Page	k = 0.0551, n = 1.3114	0.99766	-0.004	0.0204	0.0005	0.9953
Modified and Pabis	k = 0.3336, n = 0.3336	0.98635	0.0005	0.0492	0.0027	0.9645
Henderson and Pabis	a = 1.0716, k = 0.1197	0.99016	-0.008	0.0418	0.0019	0.9785
Yagcloglu et al.	a = 1.3927, k = 0.0656, c = -0.378	0.99954	-3.11E-07	0.0091	1E-04	0.9991
Two- term	a = 0.5358, k_0 = 0.1197, b = 0.5358, k_1 = 0.1197	0.99016	-0.008	0.0418	0.0022	0.9785
Two- term exponential	a = 1, k = 0.1113	0.98635	0.0005	0.0492	0.0027	0.9645
Wang and Singh	a = -0.082, b = 0.0016	0.99948	0.0006	0.0097	0.0001	0.9989
Diffusion approach	a = -6.449, k = 0.219, b = 0.8971	0.99746	-0.004	0.0213	0.0005	0.9948
Verma et al.	a = 0.0997, k = 0.1113, g = 0.1113	0.98635	0.0005	0.0492	0.0029	0.9645
Modified Handerson and Pabis	a = 0.3572, k = 0.1197, b = 0.3572, g = 0.1197, c = 0.3572, h = 0.1197	0.99016	-0.008	0.0418	0.0025	0.9785
Midilli and Kucuk	a = 1.0038, k = 0.0697, n = 1.1005, b = -0.007	0.99965	6E-06	0.0079	8E-05	0.9993

Table 4: Model constants and statistical parameters estimated at 70°C of drying- *Tor-Putitora*.

Model	Model constant	Correlation Coefficient (R)	Mean Bias Error (MBE)	Root Mean Square Error (RMSE)	Reduced ChiSquare (χ^2)	Efficiency (EF)
Lewis	k = 0.149014	0.98113	-0.00172	0.060836	0.003948	0.948937
Page	k = 0.06986, n = 1.391717	0.99645	-0.00718	0.026503	0.000803	0.992832
Modified and Pabis	k = 0.386023, n = 0.386023	0.98113	-0.00172	0.060836	0.00423	0.948937
Henderson and Pabis	a = 1.077991, k = 0.160716	0.98537	-0.011	0.053623	0.003286	0.966951
Yagcloglu et al.	a = 1.495693, k = 0.077649, c = -0.48189	0.99966	-1.6124E-08	0.00819304	8.26166E-05	0.999321
Two- term	a = 0.538995, k_0 = 0.160716, b = 0.538995, k_1 = 0.160716	0.98537	-0.011	0.053623	0.003834	0.966951
Two- term exponential	a = 1.897063, k = 0.221752	0.99498	-0.0078	0.031474	0.001132	0.989465
Wang and Singh	a = -0.10551, b = 0.002563	0.99978	0.000843	0.006603506	4.98358E-05	0.999555
Diffusion approach	a = -10.2258, k = 0.298464, b = 0.925779	0.99569	-0.00721	0.029166	0.001047	0.991042
Verma et al.	a = 0.331445, k = 0.149014, g = 0.149014	0.98113	-0.00172	0.060836	0.004555	0.948937
Modified Handerson and Pabis	a = 0.35933, k = 0.160716, b = 0.35933, g = 0.160716 c = 0.35933, h = 0.160716	0.98537	-0.011	0.053623	0.004601	0.966951
Midilli and Kucuk	a = 0.999737, k = 0.084816, n = 1.134119, b = -0.01132	0.99981	-3.9365E-06	0.00607734	4.92453E-05	0.999627

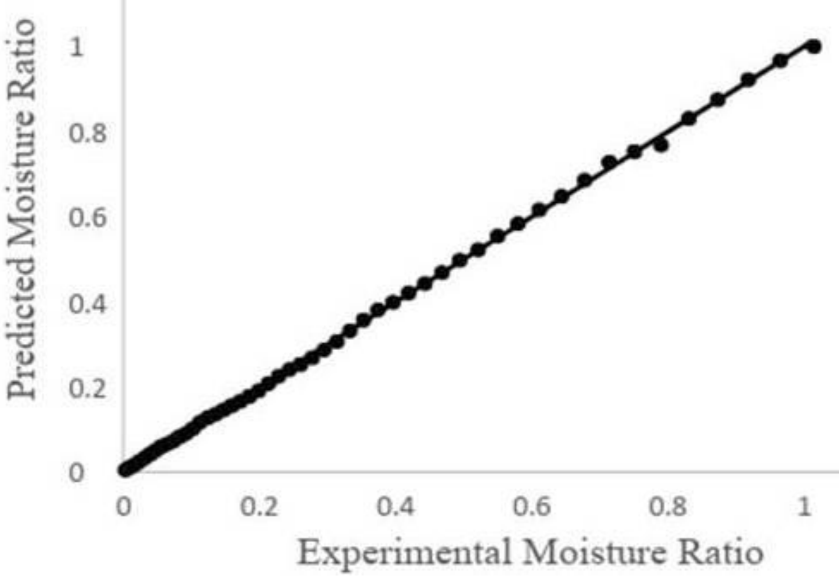

Figure 4 Relationship among experimental and predicted moisture Ratio at 50° C in case of Midilli

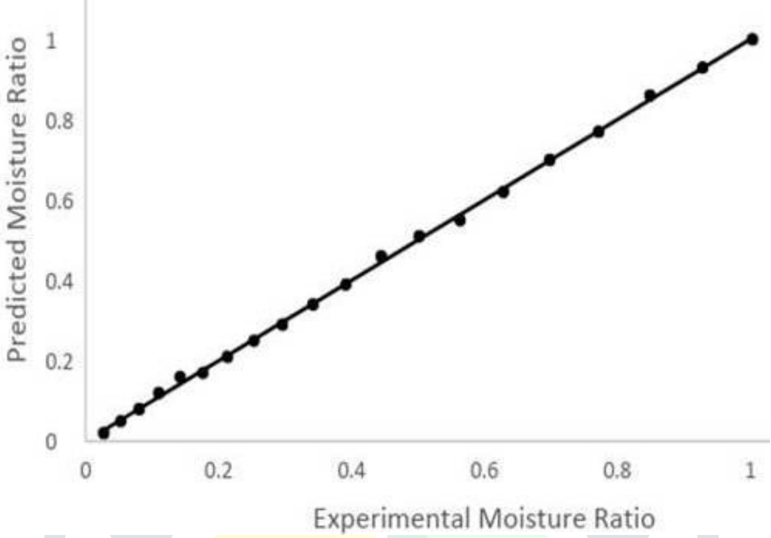

Figure 5 Relationship among experimental and predicted moisture Ratio at 60° C in case of Midilli

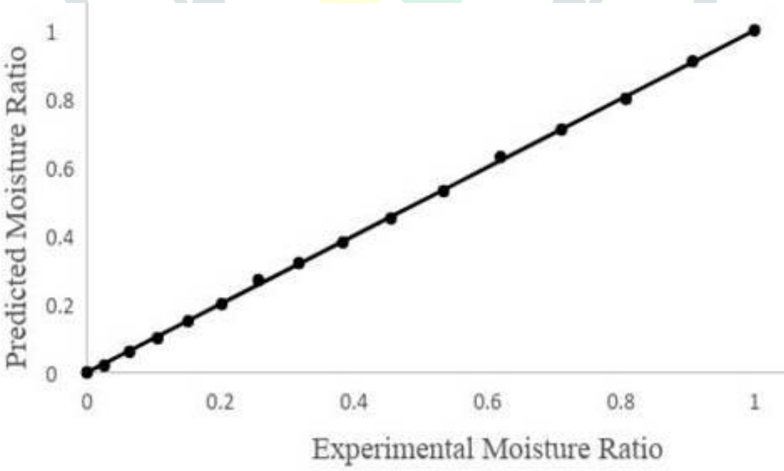

Figure 6 Relationship among experimental and predicted moisture Ratio at 70° C in case of Midilli

IV. Conclusion

The present study was proposed to find the drying characteristics by means of the establishment of a suitable model of Tor-Putitora. In this perspective, on the basis of the estimated parameters based on twelve selected drying models, midilli model was found best.

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