Bounds for Location-2-Domination in Split Graphs

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Abstract:

This paper finds bounds for location-2-domination for some split graphs like "path graph, cycle graph, wheel graph and any graph without isolated vertex" as $R_2^D\left(S\left(P_n\right)\right)=n+1,\ R_2^D\left(S\left(C_n\right)\right)=n$, $R_2^D\left[S\left(K_{1,n}\right)\right]=2n$, $n\geq 3$ The Location-2-Dominationof non-regular graph are also found.

Key Words*:* 2 Domination, Location- Domination, Split Graph.

1.Introduction:

Throughout this Paper let us follow the terminology and notation of Harary [5]. E. J. Cockayne and S. T. Hedetniemi[2] introduce the concept dominating set A subset S of vertices from V is called a dominating set for G if every vertex of G is either a member of S or adjacent to a member of S. A dominating set of G is called a minimum dominating set if G has no dominating set of smaller cardinality. The cardinality of minimum dominating set of G is called the dominating number for G and it is denoted by $\gamma(G)$ [1].

F.Harary and T.W.Haynes [1] introduced the concepts of double domination in graphs. A dominating set S of G is called double dominating set if every vertex in V-S is adjacent to at least two vertices in S. Given a dominating set S for graph G, for each u in V-S let S(u) denote the set of vertices in S which are adjacent to u. The set S is called locating dominating set, if for any two vertices u and w in V-S one has S(u) not equal to S(w) and the minimum cardinality of Location Domination number is denoted by RD(G) [2].

[6]Duplication of a vertex v of graph G produces a new graph G' by adding a vertex v′ with $N(v')=$ N(v) other words a vertex v is said to be duplication of v if all the vertices which are adjacent to v are now adjacent to v' also, If the vertices of graph G are duplicated altogether then the resultant graph is known as splitting graph of G, which is denoted as S(G)

2.Perlimnaries:

Defination2.1[5]: A subset $S\subseteq V$ is Location – 2 -Dominating set of G if S is 2 Dominating set of G and if for any two vertices $u,v\in V-S$ such that $N(u)\cap S\neq N(v)\cap S$ .The minimum cardinality of Location – 2- Dominating set is denoted by $R_2^D(G)=|S|$

2.1.Location-2-Domination for Simple Graphs:

Theorem 2.1.1[4]: In Location-2-Domination For any graph the vertex $\{v\}$ is an pendent vertex then $\{v\}\in R_2^D(G)$ only.

Theorem 2.1.2[3]: Location-2-Domination number of a Path P_n is

$$R_D^2(P_n)=\begin{cases}\dfrac{n-1}{2}+1 \ ; n \text{ is odd}\\ \dfrac{n}{2}+1 \ ; n \text{ is even}\end{cases}$$

Theorem 2.1.3[3]: For any cycle C_n with $n\neq 4$, $R_D^2(G)=\begin{cases}\dfrac{n}{2}; n \text{ is even}\\ \dfrac{n-1}{2}+1; n \text{ is odd}.\end{cases}$

3. Location-2-Domination for split Graph

Theorem:3.1. Let G be a path on n vertices then Location-2-Domination for Split Graph of Path on n vertices is $n+1$. ie; $R_2^D\left[S\left(P_n\right)\right]=n+1,\ n=2,3,...,\mathrm{n}$

Proof: Let G be a path with n vertices $v_1,v_2,...,v_n$ and the vertex of $S\left(P_n\right)$ be $v_1,v_2,...,v_n,v_1',v_2',...,v_n'$.

Case (i): Suppose that n is odd, now $\left[S\left(P_n\right)\right]$ can be split into two paths P_1 and P_2 each of length n, clearly no two vertices are common with P_1 and P_2 (ie; $P_1\cap P_2=\phi$) where P_1 is a path containing the vertices $v_1,v_2',v_3,v_4',...,v_{n-1},v_n'$ and P_2 contain the vertices $v_1',v_2,v_3',v_4,...,v_{n-1}',v_n$. By the Theorem 2.1.2, $R_2^D\left(P_1\right)=\frac{(n-1)}{2}+1$ and $R_2^D\left(P_2\right)=\frac{(n-1)}{2}+1$.

Therefore $R_2^D\left[S\left(P_n\right)\right]=R_2^D\left(P_1\right)+R_2^D\left(P_2\right)=\frac{n-1}{2}+1+\frac{n-1}{2}+1=\frac{2(n+1)}{2}=n+1.$

Case (ii): Suppose n is even, clearly $d_G(v_1')=\mathrm{d_G}(v_n')=1$ by the Theorem 2.1.1, v_1' and v_n' are the member of Location-2-Dominating set, as per case (i) construct a path P_1 whose vertices are $v_1,v_2',v_3,v_4',...,v_{n-1},v_n'$. Based on proof of the Theorem 2.1.2 collect the Location-2-dominating set of P_1 as S_1 and it contain the vertices $v_1,v_3,...,v_{n-1},v_n'$ and clearly $\left|S_1\right|=\frac{n}{2}+1.$ Now consider the path P_2 whose vertices are $v_1',v_2,v_3',v_4,...,v_{n-1}',v_n$. Based on the proof of the Theorem 2.1.2, Locatiom-2-Dominating set S_2 of P_2 is $\left\{v_1',v_3',...,v_{n-1}',v_n\right\}$. By the Theorem 2.1.1, clearly v_1' is one of the member in Location-2-Dominating set, so $\left\{v_3',...,v_{n-1}',v_n\right\}$ is also members of S_2 but it's not admissible because of $d_G(v_n)=2$ and v_n is clearly adjacent to $v_{n-1}'\in\mathrm{S}_2$ and $v_{n-1}\in S_1$ so no need to be v_n is a one of the member in S_2. Therefore $\left|S_2\right|=\frac{n}{2}$ thus $R_2^D\left[S\left(P_n\right)\right]=R_2^D\left(P_1\right)+R_2^D\left(P_2\right)=\left(\frac{n}{2}+1\right)+\left(\frac{n}{2}\right)=n+1.$

Case (iii): Suppose that the construction of path P_1 is $v_n',v_{n-2}',...,v_2',v_1$. Here $d_G(v_1)=2$ by the choice of the theorem 2.1.2 and hence omit the vertex v_1 from S_1, $v_1\in V-S_1$ but v_1 have only one member in S_1. In this way construct second path P_2 as per $v_1',v_2,v_3',v_5',...,v_{n-1}',v_n$ this situation gives that v_1 having another member form S_2 therefore $\left|S_1\right|+\left|S_2\right|=\frac{n}{2}+\frac{n}{2}+1=n+1$ or one may construct the second path P_2. As per the theorem 2.1.2, $v_n,v_{n-2},v_{n-4},...,v_2$ this situation gives that v_1 having another member form S_2 and v_1 having a degree one by theorem 2.1.1. Also v_1 is one of the member in Location-2-Dominating set and therefore $\left|S_1\right|+\left|S_2\right|=\left(\frac{n}{2}\right)+\left(\frac{n}{2}\right)+1=n+1.$

Theorem:3.2 Let G be a cycle of Length n then Location-2-Domination for Split graph of Cycle of length n is n ie; $R_2^D\left[S\left(C_n\right)\right]=n, n=5,6,...,\mathrm{n}$.

Proof: Let G be a cycle on n vertices and $S\left(C_n\right)$ be the splitting graph of C_n. Label the vertices of $S\left(C_n\right)$ namely $v_1,v_2,...,v_{n-1},v_n,\mathrm{v}_1',\mathrm{v}_2',...,v_{n-1}',v_n'$. By the observation of vertices $d_{S(C_n)}(v_i)=4,\ \ 1\le i\le n$ and $d_{S(C_n)}(v_i')=2,\ \ 1\le i\le n$ no two vertices in v_i' $1\le i\le n$ are adjacent, but each $v_i'(1\le i\le n)$ is adjacent to v_{i-1} and v_{i+1}, $i=2,3,...,n-1$, v_1 is adjacent to v_2,v_n and v_n is adjacent to v_{n-1},v_1. Based on theorem 2.1.3, in both cases of Location -2-Dominating set containing the vertices $v_1,v_3,...,v_{n-2},v_n$ where n is odd and $v_1,v_3,...,\mathrm{v}_{n-3},v_{n-1}$ where n is even, $v_2,v_4,...,v_{n-1},v_n$ satisfy the condition of Location-2-Domination but not $\mathrm{v}_1',\mathrm{v}_2',...,v_{n-1}',v_n'$. Here $\mathrm{v}_1',\mathrm{v}_2',...,v_{n-1}',v_n'$ are all has only one member from Location-2-Domination and it will contradict our definition of location-2-domination. By the observation of the location-2-Domination set which is defined as $S=\{v_1,v_2,...,v_{n-1},v_n\}$ clearly $N(v_i')\cap\mathrm{S}\ne\mathrm{N}(\mathrm{v_j}')$, $\mathrm{i}\ne\mathrm{j}$ one can obtain $\left|S\right|=n$. Therefore $R_2^D\left[S\left(C_n\right)\right]=\left|S\right|=n, n=5,6,...,\mathrm{n}$.

Case (i): Suppose that $|S| < n,\ n = 5,6,\ldots,$n. let $v_k \notin S$ but $v_{k-1}, v_{k+1} \in S$ therefore $N(v_k) \cap S$ is exists. On the other hand, one can find out vertices such that $N(v_{k-1}') = \{v_{k-2}, v_k\}$ and $N(v_{k+1}') = \{v_k, v_{k+2}\}$, $N(v_{k-1}') \cap \mathrm{S} = \{v_{k-2}\}$ and $N(v_{k+1}') \cap \mathrm{S} = \{v_{k+2}\}$ which will contradict the definition of Location-2-Domiantion and therefore, $R_2^D\left[S\left(C_n\right)\right] = |S| = n, n = 5,6,\ldots,$n.

Case (ii): Suppose that $|S| > n,\ n = 5,6,\ldots,$n. Then, $|S|$ will not be the minimum cardinality of Location -2-Domiantioning set.

Theorem:3.3 Let G be a Star graph on n vertices then Location-2-Domination for split graph of star graph is 2n, that is $R_2^D\left[S\left(K_{1,n}\right)\right] = 2n$, $n \geq 3$

Proof: Label the vertices of $S\left(K_{1,n}\right)$ as $v, v_1, v_2, \ldots, v_{n-1}, v_n, v', v_1', v_2', \ldots, v_{n-1}', v_n'$ and S be the location-2-Dominating set, clearly $d(v_1') = d(v_2') = \ldots = d(v_{n-1}') = d(v_n') = 1$. By theorem 2.1.1, $v_1', v_2', \ldots, v_{n-1}', v_n'$ are the members of S –set only. Now $S\left(K_{1,n}\right)$ can be rearranged into n distinct path of length 3. Possibly the paths may be like $P_1 = \{v_1', v, v_1, v'\}$, $P_2 = \{v_2', v, v_2, v'\}, \ldots, P_n = \{v_n', v, v_n, v'\}$. Here every path is passing through v, v'. By the theorem 2.1.2 Location-2-domiantion for each path is $R_2^D(P_1) = 3$, $R_2^D(P_2) = 3$, $R_2^D(P_3) = 3, \ldots, R_2^D(P_n) = 3$. But by the proof of the theorem 2.1.2, degree of origin and terminus vertices are one so it must be a one of the member, but here $d(v') = n$ also v' is adjacent to $v_1, v_2, \ldots, v_n$, by the theorem 2.1.1 $v_1', v_2', \ldots, v_n' \in S$ therefore the vertex v' no need to be a member of S.

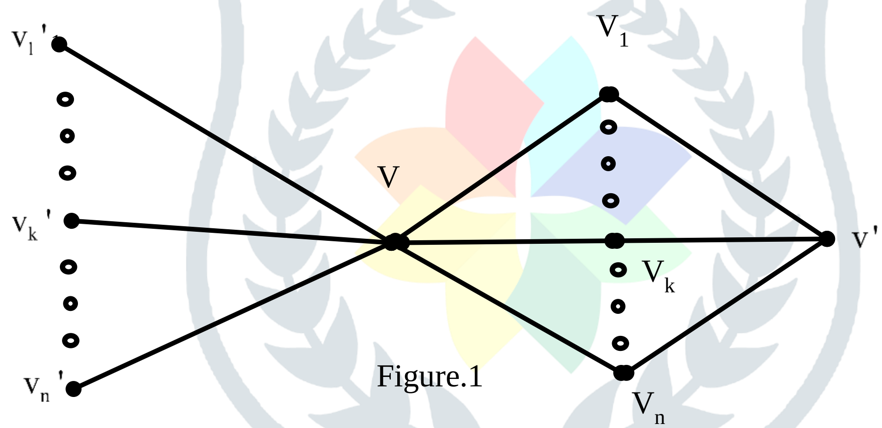

Figure.1

$$R_2^D\left[S(k_1, n)\right] = R_2^D\left(P_1\right) + R_2^D\left(P_2\right) + \ldots + R_2^D\left(P_n\right) - n = 3 + 3 + \ldots + 3 - n = 2n$$

Theorem:3.4 Let G be non-regular graph with n vertices, more than $\frac{n}{2}$ vertices with same degree (or) more than $\frac{n}{2}$ vertices with same degree which are adjacent to same set of vertices, then $R_2^D\left(G\right) \geq n - d$, d is the number of distinct degrees of vertex.

Proof: Let S be the location-2-Dominating set.

Case(i): Suppose there are more than $\frac{n}{2} + 1$ vertices are adjacent to same set of vertices , from G have two different set of degree of vertices like that v_1, v_2, v_3 are adjacent to v_j for $j = 1,2,3,\ldots,$n and $\mathrm{i} \neq \mathrm{j}$ clearly $d_G(v_1) = d_G(v_2) = d_G(v_3) = \mathrm{n} - 1$, take $d_1 = d_G(v) = n - 1$, and each $v_i, i = 4,5,\ldots,n$ are adjacent to v_1, v_2, v_3, $d_G(v_i) = 3$ for $i = 4,5,6,\ldots,n$ also take $d_2 = d_G(v) = 3$, clearly d=2 choose only one vertices from different each set of different degree of vertices , take these are in V-S only , evidently $N(v_1) \cap \mathrm{S} \neq \mathrm{N}(\mathrm{v}_4) \cap S$, otherwise $N(v_i) \cap \mathrm{S} = \mathrm{N}(\mathrm{v}_j) \cap S$ if $i, j \in d_1$ also $i, j \in d_2$ if G has more number of distinct set of degree of vertices will follow the same process in this cases we get equality that is $R_2^D\left(G\right) = n - d$

Case(ii): Suppose that $|S| < n-d$, let $\frac{n}{2}+1$ vertices are having same degree with same adjacent vertices and reaming vertices having different adjacency set of vertices , by convenient choose v_i for $i = r, r+1, ..., n$ where $r = \frac{n}{2}-1$ place are adjacent to v_1, v_2, v_3 that $\frac{n}{2}+1$ vertices having degree three are labeled the degree as d_1. Clearly the 3 vertices v_1, v_2, v_3 has degree $\frac{n}{2}+1$ this degree set is labeled as d_2 therefore $\frac{n}{2}+4$ vertices containing two distinct degree of vertices. Without loss generality, assume that remaining $\frac{n}{2}-4$ vertices are having distinct degrees. Then it is clear that if $d = \{d_1, d_2, ..., d_{(n/2)-2}\}$, then $|S| < n - \frac{n}{2} + 2 = \frac{n}{2} + 2$ vertices only which will contradict the fact $N(v_i) \cap S = N(v_j) \cap S$ if $i, j \in d_k$ for $k \in d_1, d_2, ..., d_{(n/2)-2}$. Therefore, $R_2^D(G) \geq n-d$.

Case(iii): Suppose that more than $\frac{n}{2}$ vertices are having same degree with different adjacent vertices, in this case the result follow the above cases.

Theorem:3.5 Let G be a non-regular graph with isolated vertex, then Location-2-Domiantion for G is $R_2^D(G) \geq \left\lceil \frac{n}{2} \right\rceil$.

Proof: Let G be the graph with n vertices and S set be the Location-2-Dominating set. Label the vertex of G as $\{v_1, v_2, ..., v_i, v_j, v_k, ..., v_n\}$. By assumption $d_G(v_i) = d_G(v_j) = d_G(v_k) = 1$ $d_G(v_r) > 1, r \neq i, j, k$. By the theorem2.1.1, v_i, v_j, v_k are one of the member of S_1, where $|S_1|$ = number of isolated vertex from G. On observing adjacent vertices, v_j, v_i, v_k has exactly one neighbor from S_1- set, possibly take v_l, v_m, v_n which are the adjacent vertices with degree greater than one. Then, clearly $v_l, v_m, v_n \in V - S$. From G remove whose vertex which is adjacent to the isolated vertex. This gives a new graph $G_1 = G - \{v_l, v_m, v_n\}$, here the adjacent vertices of v_l, v_m, v_n are changed as isolated vertex. Without loss of generality, we take $d_G(v_e) = d(v_f) = 1, e, f \in G_1$. Again by the theorem 2.1.1, v_e, v_f are one of the member in location-2-Dominating set. clearly S-set contains isolated vertex from G and isolated vertex from G_1 i.e; $|S_2| = |S_1| +$ isolated vertex from G_l other than G. Also v_l, v_m, v_n satisfies the definition of location-2-Dominating set. Form G_1 eliminate the adjacent vertex of v_e, v_f, this gives $G_2 = G_1 - \{v_t, v_u, ...\}$. Continuing the same processes till the graph can reduces to empty graph one can get $|S| = |S_{n-1}| \geq \left\lceil \frac{n}{2} \right\rceil$ vertices.

Suppose that v_i, v_j, v_k are adjacent to v_s only, then v_s produces a single vertex or more than one vertex , then removal of v_s gives a graph G_1 with isolated vertex then following the above processes or removal of v_s we get a graph G_1 with non-isolated vertex. Here choose any one of adjacent vertex form v_s suitably by the definition and remove the neighbors. This yields a new graph G with isolated vertex or without isolate vertex. If necessary repeat the above process.

Let us consider the graph $G = \{v_1, v_2, ..., v_{12}\}$

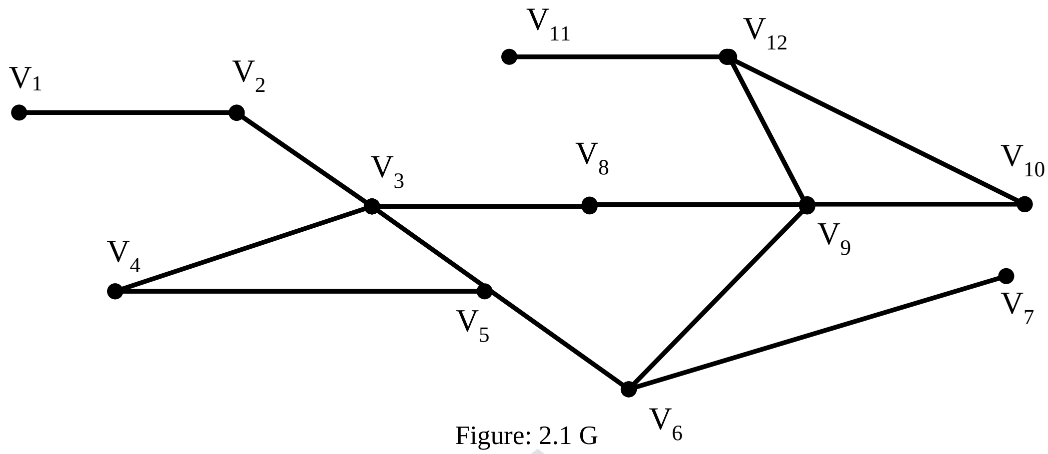

Figure: 2.1 G

By the theorem 2.1.1, clearly $S_1 = \{v_1, v_7, v_{11}\}$, now construct $G_1 = G - \{v_1, v_7, v_{11}\}$

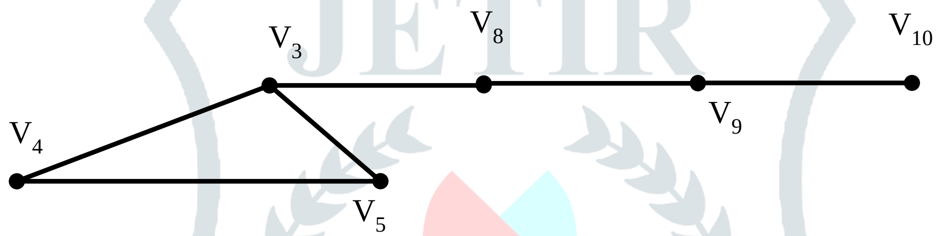

Figure: 2.2

Again by the theorem2.1.1 $S_2 = |S_1| + \{v_{10}\}$, now G_1 can be reduces to $G_2 = G_1 - \{v_{10}\}$

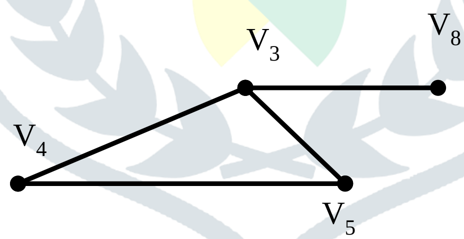

Figure: 3 G_2

Again by the same theorem 2.1.1, $S_3 = |S_2| + \{v_8\}$ now G_2 can be reduces to $G_3 = G_2 - \{v_8\}$

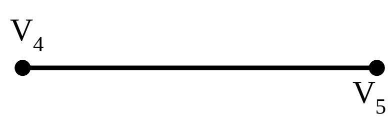

Figure: 4 G_3

Again by the theorem 2.1.1, $S_4 = |S_3| + \{v_4, v_5\}$

Clearly, $|S| = |S_4| = |S_3| + \{v_4, v_5\} = |S_2| + \{v_8\} + \{v_4, v_5\} = |S_1| + \{v_{10}\} + \{v_8\} + \{v_4, v_5\}$

$=\{v_{10}\}+\{v_8\}+\{v_4,v_5\}+\{v_1,v_7,v_{11}\}=\{v_1,v_4,v_5,v_7,v_8,v_{10},v_{11}\}$

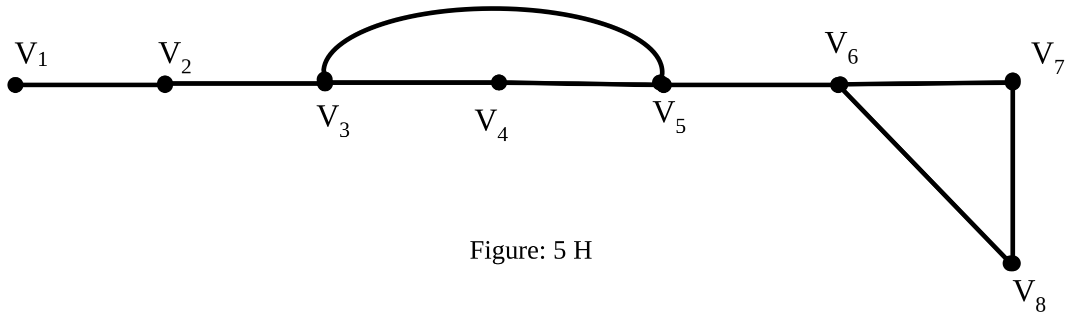

Figure: 5 H

By the theorem 2.1.1 from H $S_1=\{v_1\}$ and v_1 is adjacent to v_2 now H can reformed as $H_1=H-\{v_2\}$

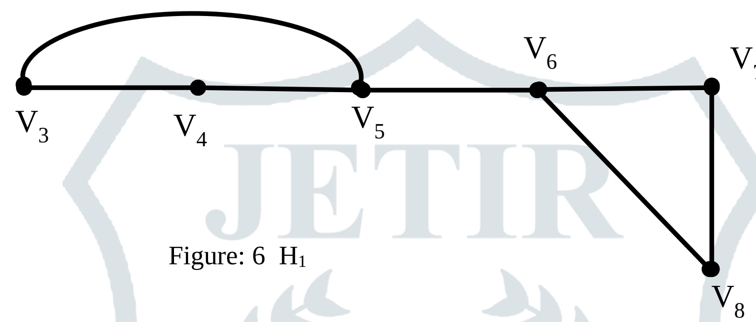

Figure: 6 H_1

Clearly H_1 has no isolated vertices which follows from theorem 2.1.1. Choose the vertex $S_2=|S_1|+\{v_5\}$, v_5 is adjacent to v_4 and v_6 now H_1 can be changed as $H_2=H_1-\{v_4,v_6\}$

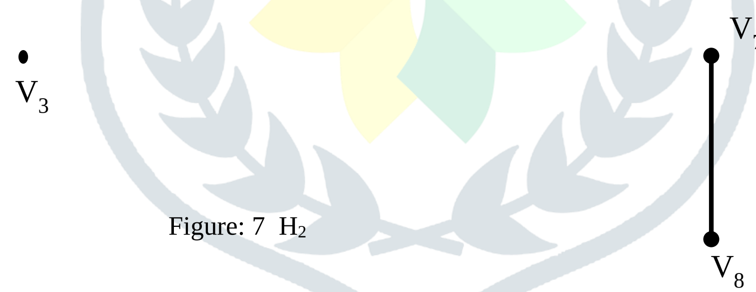

Figure: 7 H_2

Here all are isolated vertices then $S_3=|S_2|+\{v_3,v_7,v_8\}$ and therefore, $|S|=\{v_1,v_3,v_5,v_7,v_8\}$.

Theorem:3.6 Let G be a non-regular graph and without isolated vertex, multiple edges, cycle of length four, then Location-2-Domiantion for G is $R_2^D(G)\le\begin{cases}\frac{n+1}{2}, & n \text{ is odd}\\ \frac{n}{2}+1, & n \text{ is even}\end{cases}$

Proof: Let G be a graph with n vertices and S be location-2-Dominating set. Clearly, $d_G(v_i)>1$, $i=1,2,...,n$, let us choose graph in such a way that there is an edge from $v_1v_2, v_2v_3, v_3v_4,...,v_{n-1}v_n$ also v_i, $i=1,2,...,n$ is adjacent to any vertex $v_j, v_{j+1},...$ for $j=1,2,...,n$ and no two vertices having multiple edges .

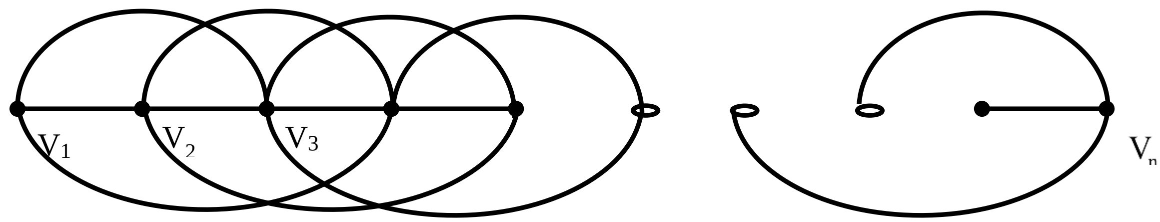

Figure: 8

In this case, following the theorem 2.1.2, if one collects alternate set of vertices, it is the best location-2-dominating graph. Suppose if we take $S=\{v_1,v_3,v_7,...,v_n\}$ and $v_5 \in V-S$ then $N(v_4)\cap S = N(v_2)\cap S$. This contradicts the location-2-Domiantion and hence in this situation from theorem 2.1.2, $|S|\le \frac{n+1}{2}$, when n is odd and $|S|\le \frac{n}{2}+1$, *when n is even*

Case (a): For the edge v_1v_2 and v_2v_3, v_1v_3 are edges with no more edges in between these vertices. In this case choose $v_1, v_3 \in S$ such that $v_2 \in V-S$ but $d(v_3)>2$. From v_3 one can draw at least two distinct paths of length one, not covering the vertices v_1, v_2. Clearly these two paths terminal vertices are adjacent. In this case any one of these two vertex is a member of S-set, because already v_1, v_3 are member of S so we need only one adjacent, clearly both are not a member of S. If possible it will contradicts the definition of Location-2-Domination, continue the same process for any path covering the vertices, in this way it yields $|S|\le \frac{n+1}{2}$, for n is odd, and $|S|\le \frac{n}{2}+1$, for n is even.

Case (b): Without regular formation there is a graph, there is just start the first vertex $v_1 \in S$ by hypothesis $d_G(v_1)>1$, $d_G(v_2)>1$, $d_G(v_3)>1$ etc., no need to v_2 be member of S choose the adjacent vertex of v_2 as a member in S and in this way, choose alternate set of vertices based on theorem 2.1.2. From v_2 one can draw distinct path that are possible. Choose all the paths and collect alternate set of vertices it will gives the result. Suppose that $P_1=\{v_5, v_6, v_7,....., v_{14}\}, P_2=\{v_{21}, v_{22},......, v_{30}\}$ are possible paths and there is edge from v_{10} to v_{25} is single edge here also will follow the theorem 2.1.2 or more edges in between P_1 and P_2 choose alternate set vertices form one path and second. Choose alternate but no need choose start or end of the path of vertices. Continue the same process for every vertex,

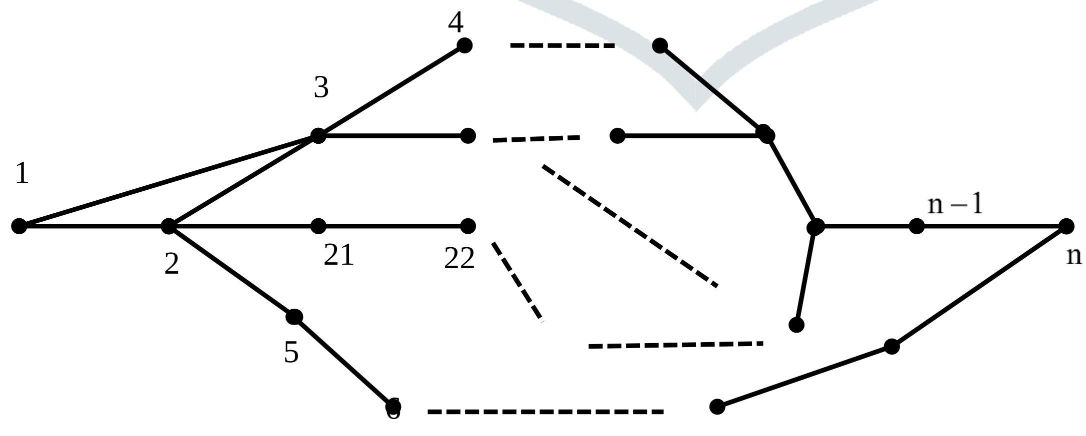

Figure:9

This gives $|S| \le \frac{n+1}{2}, n \text{ is odd}$, $|S| \le \frac{n}{2}+1, n \text{ is even}$ not for v_2 whatever the vertices will follow the above processes will yield a better result.

Theorem:3.7 Let G be a non-regular graph with or without isolated vertex, then Location-2-Domiantion for Split graph of G has $R_2^D(S(G)) \ge n$

Proof: Label the vertices of $S(G)$ as $\{v_1, v_2, ..., v_n, v_{1'}, v_{2'}, ..., v_{n'}\}$, where $v_1, v_2, ..., v_n$ are vertices from G and $v_{1'}, v_{2'}, ..., v_{n'}$ is duplication of vertices. Clearly it has 2n vertices. Now vertices of S(G) can divided into two groups $D_1 = \{v_1, v_2, ..., v_n\}$ and $D_2 = \{v_{1'}, v_{2'}, ..., v_{n'}\}$. In D_1 every vertex is adjacent to at least one vertex in D_2 and also adjacent with at least on v_i, $i = 1, 2, 3, ...,$ but D_2 has no adjacency from $v_{i'}$ to $v_{j'}$ for any i, j and every vertices in $v_{i'}$ is adjacent at least on v_i for $i = 1, 2, 3, ...$ Assume contrary that $R_2^D[S(G)] = |S| < n$, and possibly choose $S = \{v_1, v_2, ..., v_{n-1}\}$.

Case(i): Suppose that any one vertices in $v_{i'}$, $i = 1, 2, 3, ...$ of degree one it will contradiction to theorem 2.1.2 therefore $R_2^D[S(G)] = |S| \ge n$

Case(ii): suppose that let us consider the edges in G are such that v_1 is adjacent to v_2, v_n and v_n is adjacent to v_1, v_{n-1} further no more vertices are adjacent to v_1, v_n and remaining $n-2$ vertices are adjacent to any order, then edges in $S(G)$ are v_1 is adjacent to $v_2, v_n, v_{2'}, v_{n'}$ and v_n is adjacent to $v_1, v_{n-1}, v_{1'}, v_{n-1'}$ further more it will satisfies $N(v_n) \cap S \ne N(v_i) \cap S$ for i=1,2,3,…,n-1 or $N(v_n) \cap S \ne N(v_{i'}) \cap S$ for i=1,2,3,…,n-1, but there is a question on v_1 that is $N(v_1) \cap S = \{v_2\}$ only it will contradicts the definition of Location-2-Domination in this cases either $v_1 \in S$ or $v_n \in S$ therefore $R_2^D[S(G)] = |S| \ge n$.

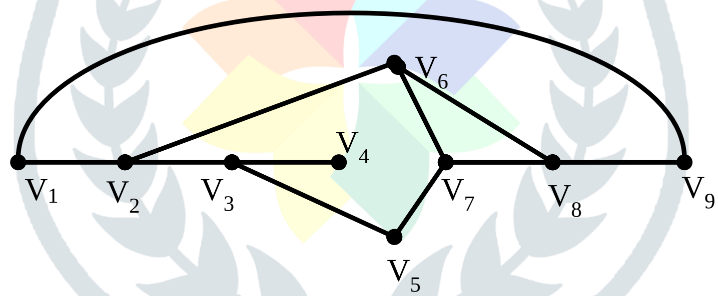

Figure: 10

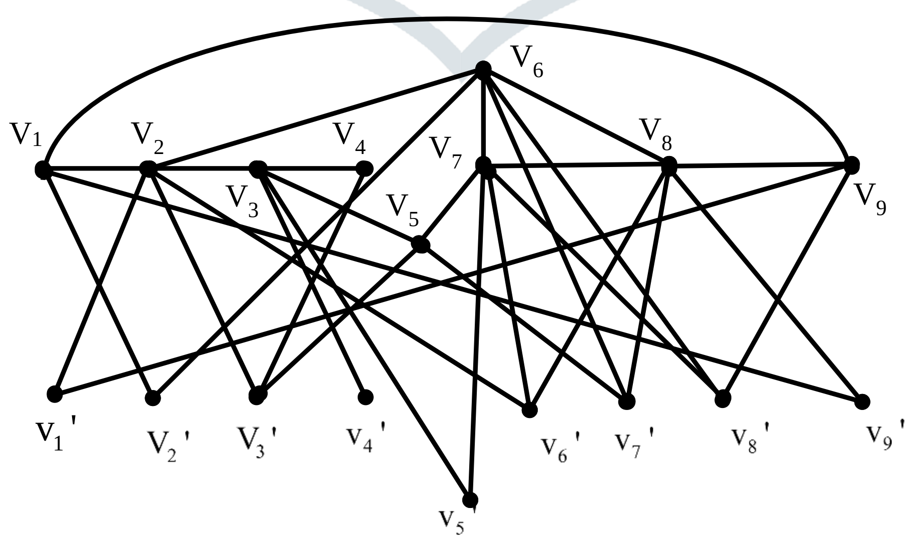

Figure: 11 S(G)

Theorem:3.8 Let G be a path on n vertices then $R_2^D\left[S\left(P_n\right)\right]=2R_2^D\left(P_n\right)$, *n is odd*

Proof: Obvious from the theorem 3.1.

Theorem:3.9 Let G be a path on n vertices then $R_2^D\left[S\left(P_n\right)\right]=2R_2^D\left(P_n\right)-1$, where n is even

Proof: Obvious from the theorem 3.1.

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