STABILITY BOUNDARY LOCUS APPROACH FOR THE CONTROLLER DESIGN OF DC-DC BUCK CONVERTER

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Abstract : This paper deals with the design of a Proportional Integral (PI) controller for DC-DC Buck converter, using stability boundary locus approach, to stabilize the output during load current and line voltage disturbances. Stability boundary locus approach is used to set the parameters of PI controller to satisfy the minimum phase margin requirement. The non idealities of the Buck converter are included in its mathematical modeling. Control to output transfer function (i.e. duty cycle to output transfer function) of non ideal Buck converter is obtained by State-space averaging technique. Finally, a model is created in SIMULINK and its behavior is tested

Keywords **– PI Controller, Buck converter, Phase margin, SIMULINK**

I. INTRODUCTION

The power Buck converter is the most widely used DC/DC converter with variety of applications in DC motor drives, renewable energy system, electric vehicle, medical equipments and many more. Buck converter is basically used where we need to lower the input DC voltage. This converter converts the high level DC voltage signal to low level stabilized DC voltage signal. Modeling and design of controller for the DC/DC Buck converter are the important aspects which contain important research scope. To obtain the transfer function of DC/DC Buck converter we need to model it. This transfer function is further utilized to design the proper controller for the converter. There are various modeling techniques, to model the DC/DC Buck converter out of which state space averaging method is widely used due to its ease of implementation.

Different elements of a Buck converter such as inductor, capacitor, diode and switch have internal resistances with them, which make the Buck converter a non-ideal one. In this paper these resistances have been included in modeling of Buck converter. Further, many literatures present various linear and non-linear techniques to control the Buck converter. The non linear techniques such as sliding mode control and H-infinity control face issues of chattering, non-uniform switching and mathematical complexities. Also non linear techniques are more useful for higher order converters. While the linear techniques, which are conventional and in which the DC/DC Buck converters are controlled by voltage mode and current mode control methods (such as proportional integral and proportional integral derivatives control) are simple, low cost controllers and easy to implement in physical world. PI and PID are always in demand for most industrial applications. In this paper liner approach has been taken. Further we need a suitable tuning method to set the parameters of PI controller. There are several time domain and frequency domain based methods of tuning. In this paper a frequency domain based tuning approach; named stability boundary locus approach is used. For DC/DC Buck converter phase margin greater than 60 degree provides good performance, so the PI controller is tuned to improve the phase margin of Buck converter [8]. Further the performance of SIMULINK model is tested under variations in load current and input voltage.

The organization of this paper is as follows. In section II mathematical model of non ideal buck converter is obtained followed by stability boundary locus approach for tuning in section III. Thereafter SIMULINK result in section IV and conclusion in section V followed by references.

II. MATHEMATICAL MODEL OF NON IDEAL DC-DC BUCK CONVERTER

Fig.1 illustrates the basic circuit diagram of a Non-ideal DC-DC Buck converter. The circuit has a power MOSFET switch S, diode D_d, inductor L , capacitor C, and load resistance R. This is to be noted that in place of MOSFET as a switch, GTO, IGBT, BJT can be substituted, if desired. Non idealities of Buck converter, i.e. equivalent series resistance of inductor, equivalent series resistance of capacitor, resistance of switch in ON condition, forward resistance and forward voltage drop of diode, have been considered. The converter is operating in continuous conduction mode. Duty cycle is D and switching frequency is f. In continuous conduction mode, a DC/DC converter operates in two modes. First when switch is on (time interval $0 < t \leq DT$) and secondly when switch is off (time interval $DT < t \leq T$).

When the MOSFET is ON, the KVL and KCL equations are given as-

$$L\frac{di_L(t)}{dt} = -(r_{sw} + r_L + \frac{r_c R}{R+r_c})\, i_L(t) - \frac{R}{R+r_c} v_c(t) + v_g(t) \quad (1)$$

$$C\frac{dv_c(t)}{dt} = \frac{R}{R+r_c} i_L(t) - \frac{1}{R+r_c} v_c(t) \quad (2)$$

$$v_o(t) = \frac{r_c R}{R+r_c} i_L(t) + \frac{R}{R+r_c} v_c(t) \quad (3)$$

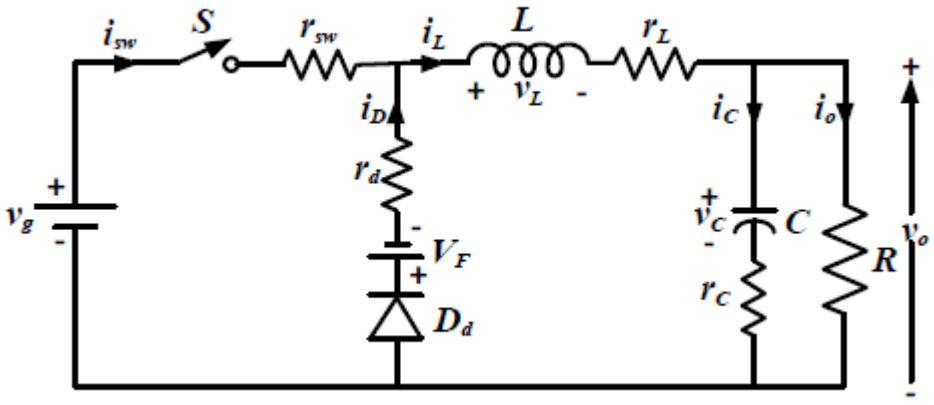

Fig. 1 Circuit of Non ideal DC/DC Buck converter

When the MOSFET is off, the KVL and KCL equations are given as-

$$L\frac{di_L(t)}{dt} = -(r_d + r_L + \frac{r_c R}{R+r_c})\, i_L(t) - \frac{R}{R+r_c} v_c(t) + v_F \qquad (4)$$

$$C\frac{dv_c(t)}{dt} = \frac{R}{R+r_c} i_L(t) - \frac{1}{R+r_c} v_c(t) \qquad (5)$$

$$v_o(t) = \frac{r_c R}{R+r_c} i_L(t) + \frac{R}{R+r_c} v_c(t) \qquad (6)$$

Here $i_L, v_c, v_o and\ v_g$ have their standard meaning as shown in the above fig.

State space averaging technique is used here to get the mathematical model of Buck converter. In state space averaging method, the two set of equations, eq. (1) - (3) and eq. (4) - (6) are first averaged with respect to their duration in the switching period. This averaging gives non linear large signal averaged model. This non linear large signal averaged model is linearized around a DC operating point using small signal perturbations, which finally gives small signal transfer function. We can find this method in many textbook. The detailed step by step procedure to find the transfer function is skipped here because of being lengthy. Readers can find it in [4]. The small signal, duty cycle to output voltage transfer function is obtained as –

$$G_{vd}(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)} = \frac{\frac{R(V_g + V_F - (r_{sw} - r_d) I_L)}{LC(R+r_C)}(r_C Cs+1)}{s^2 + \left(\frac{1}{L}\left(r_x + r_L + \frac{r_C R}{R+r_C}\right) + \frac{1}{C}\left(\frac{1}{R+r_C}\right)\right)s + \frac{r_x + r_{L+R}}{LC(R+r_C)}} \qquad (7)$$

A suitable controller is now designed and simulated based on the above transfer function.

III. PI CONTROLLER DESIGN BASED ON THE STABILITY BOUNDARY LOCUS APPROACH

This section covers the algorithms for the determination of parameters of PI controller. Block diagram of closed loop PI control of DC/DC Buck converter is shown in Fig. 3. As shown in the Fig., output v_o is compared with desired output reference voltage (V_{oref}). The generated error (V_{err}) acts as the input to the PI controller. PI controller generates an output signal ($V_{control}$) which is further compared with a fixed frequency saw tooth waveform in PWM modulator. The output of this PWM modulator is the switching pulses of desired pulse width. Transfer function of PWM modulator is –

$$G_{pwm}(s) = \frac{1}{v_{sw}} \qquad (8)$$

v_{sw} is the peak of saw-tooth waveform.

Substituting the parameters of non-ideal Buck converter given in Table I, the duty cycle to output voltage small-signal transfer function of Buck converter is obtained as-

$$G_{vd}(s) = \frac{\tilde{v}_o(s)}{\tilde{d}(s)} = \frac{4452s + 1.760\times10^8}{s^2 + 1532s + 1.068\times10^7} \qquad (9)$$

So, the uncompensated Buck converter has the transfer function

$$T_{uncomp} = G_{pwm}(s)\, G_{vd}(s) = \frac{4452s + 1.760\times10^8}{s^2 + 1532s + 1.068\times10^7} \qquad (10)$$

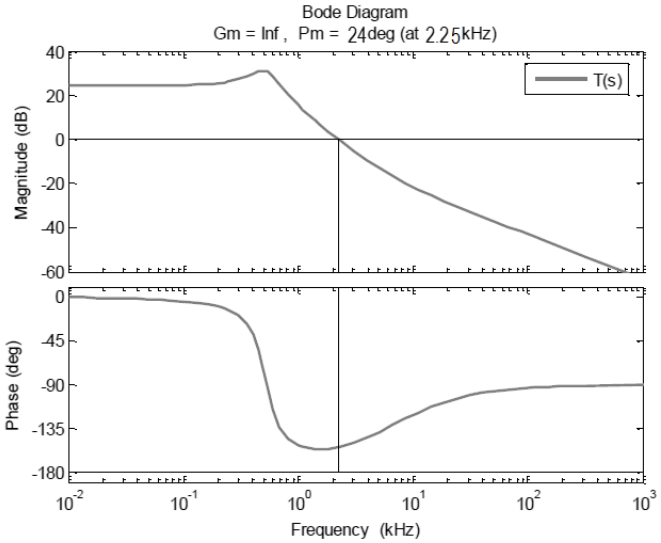

Fig.2 Frequency response of uncompensated DC/DC Buck converter

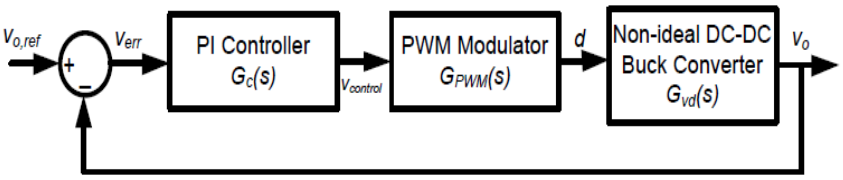

Fig. 3 Closed loop PI control of DC/DC Buck converter

TABLE I. Parameter Values of Buck Converter

Variable	Parameters	Values
v_g	Input Voltage	15-25 V
v_o	Output Voltage	10 V
R	Load resistance	10-20 Ω
L/r_L	Inductance	1.0 mH/0.15 Ω
C/r_c	Capacitance	83.5 µF/0.31Ω
r_{sw}	Resistance of switch in ON mode	0.040 Ω
r_d	Forward resistance of diode	0.024 Ω
v_F	Forward voltage of diode	0.7 V
f	Switching frequency	27.4 kHz
v_{sw}	Saw tooth wave peak	1

The frequency response of uncompensated Buck converter is shown in Fig. 2. The Fig. shows, that the gain margin is infinite and phase margin is 24 degree. Gain crossover frequency is 2.25 kHz. Such insufficient phase margin results in poor transient response and large overshoot. Therefore the phase margin of the system need to be improved which can be done by shifting the gain crossover frequency to a lower value. Also the uncompensated Buck converter, being a type 0 system, shows steady state error for the step signal. This error can be improved by adding a pole at the origin. Therefore a suitable PI controller is designed.

A. Algorithm for setting the parameters of PI controllers

Stability boundary locus approach is used in this paper for setting the parameters of non-ideal Buck converter. Since the Buck converter is having infinite gain margin, our prime importance is to set the parameters of PI controller to get the desired phase margin. The procedure of boundary locus approach is as follows [11]:

1. Consider a general transfer function of uncompensated Buck converter as

$$T'(s) = \frac{B_1(s) + B_0}{A_2 s^2 + A_1 s + A_0} \quad (11)$$

2. Consider the transfer function of designed PI controller is

$$G_c(s) = K_p + \frac{K_i}{s} \quad (12)$$

K_p, K_i : parameters of PI controller

3. Consider the Buck converter is having a minimum desired phase margin of Ø degree. So the characteristics equation of closed loop DC/DC converter system for the phase margin of Ø degree will be

$$1 + e^{-j\emptyset}\, T'(s) G_c(s) = 0 \quad (13)$$

4. Now put eq. (11) and (12) into eq. (13). This gives

$$1+ (\cos\emptyset - j\sin\emptyset)\left(\frac{B_1(s)+B_0}{A_2s^2+A_1s+A_0}\right)\left(K_p + \frac{K_i}{s}\right) = 0 \qquad (14)$$

5. Put s = jω in above equation ,we get

$$(K_pX_1 + K_iX_2) + j\,(K_pX_4 + K_iX_5) = X_3 + jX_6 \qquad (15)$$

Where, $X_1 = [-\omega^2 B_1\cos\emptyset + \omega B_0 sin\emptyset]$ $X_2 = [B_0\cos\emptyset + \omega B_1 sin\emptyset\ ]$ $X_3 = [\omega^2 A_1]$

$X_4 = [\omega^2 B_1\sin\emptyset + \omega B_0 cos\emptyset]$ $X_5 = [-B_0\sin\emptyset + \omega B_1 cos\emptyset\ ]$ $X_6 = [\omega^3 A_2 - \omega A_0]$

6. Separate real and imaginary parts in eq. (15) and simplify for K_p and K_i. The equation for K_p and K_i will be in terms of A_0, A_1, A_2, B_0, B_1 and ω.

The solutions is obtained for K_p and K_i and pair of values for K_p and K_i, chosen within stability limit ,would give the minimum phase margin. As discussed earlier, the uncompensated Buck converter has poor phase margin. Also for DC/DC Buck converter, phase margin greater than 60 degree provides good performance. So consider the compensated system has phase margin of 70 degree. So by putting B_1= 4452, $B_0 = 1.760\times10^8$,$A_2 = 1$,$A_1 = 1532$, $A_0 = 1.068 \times 10^7$ and Ø = 70 degree in the equation of K_p and K_i , obtained in step 6 of the *Algorithm* above and by making ω variable, the stability boundary locus is plotted as shown in Fig. 4 below.

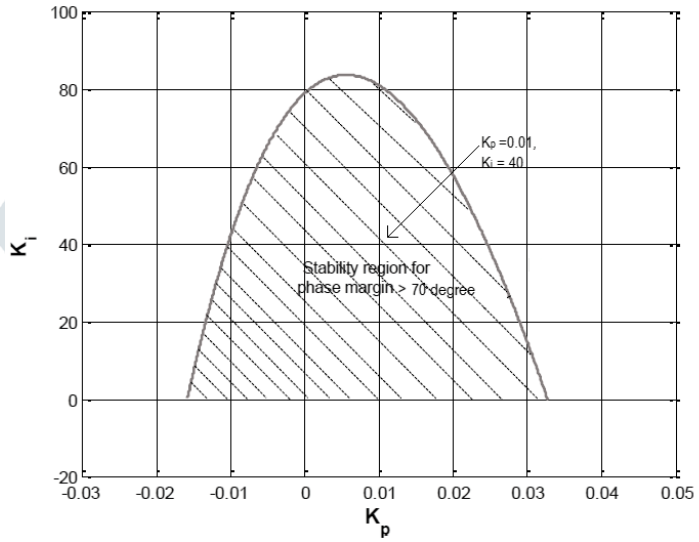

Fig. 4 Stability boundary locus plot for DC/DC Buck Converter

K_p and K_i chosen within the stability region would come up with phase margin greater than or equal to 70 degree.

IV. SIMULATION AND RESULTS DISCUSSION

Based on the above algorithm of stability boundary locus approach, parameters of K_p and K_i are obtained. Based on these parameters values, a SIMULINK model is created in MATLAB. The frequency response is tested, if the phase margin of closed loop buck converter is increased and steady state error is improved or not.

SIMULINK model is shown in Fig. 5 and the frequency response of compensated Buck converter is shown in Fig. 6.

V. CONCLUSION

It is clear from the frequency response plot of compensated Buck converter that for the selected values of K_p and K_i , the phase margin has increased to 103 degree (which is greater than 60 degree requirement for good performance). The increased value of phase margin reduces overshoot. Thus the SIMULINK model performs well.

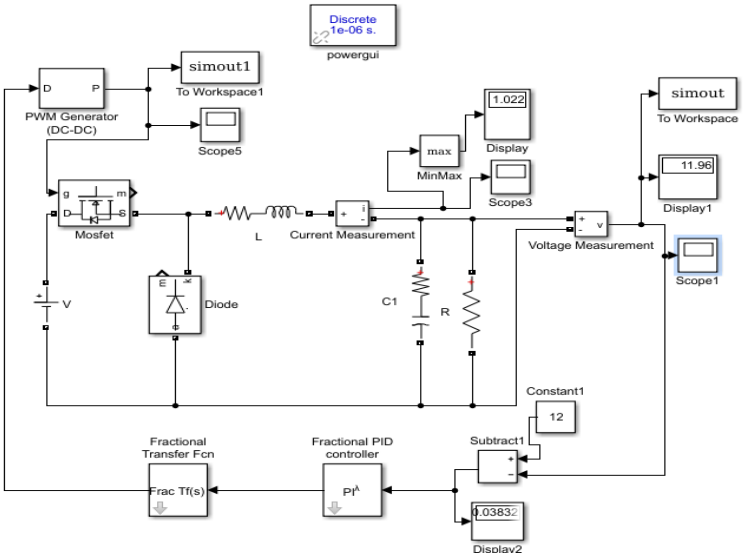

Fig. 5 SIMULINK model of non ideal DC/DC Buck converter

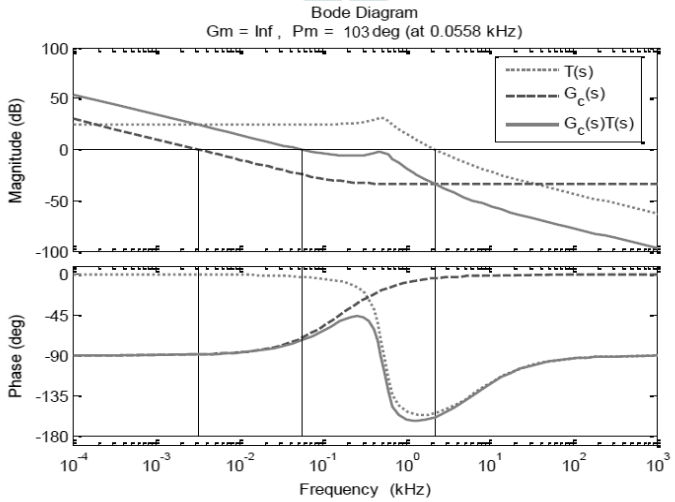

Fig. 6 Frequency Response of compensated Buck converter

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