THERMOELASTICITY INTERACTION WITH HOMOGENEOUS AND INHOMOGENEOUS SOLIDS:ANISOTROPIC, MICRO-STRETCH

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Abstract : In this paper I silhouette the basic governing equations of homogeneous anisotropic generalized thermoelasticity and micropolar thermoelasticity in case of homogeneous isotropic solids.

Index Terms: Thermoelasticity, Micropolar , Anisotropic, Microstretch, GL Theory.

1. Introduction

The Thermoelasticity deals with the study of thermodynamical system of equilibrium stage of bodies which concern with mechanical work and heat exchange and external work. There are different types of thermoelasticity theorylike,Coupled theory, Gernalized theory, Lord-Shulman Theory(LS), Green-Lindsay theory(GL) etc,whereas under Micropolar elasticity a body is supposed to concern with particles in small rigid bodies where transnational and rotational motion undergoing.

2.BASIC EQUATIONS AND CONSTITUTIVE RELATIONS

Anistropic Thermoelasticity

The equations governing thermoelastic interaction in homogeneous anisotropic solids proposed by Dhaliwal and Sherief [1980] and Gree and Lindsay [1972] are as follows:

a) Strain-Displacement relations

$$e_{ij} = (u_{i.j} + u_{j,i})/2 \qquad i, j = 1,2,3 \qquad (1)$$

b) Stress-Strain Temperature relations

$$\sigma_{ij} = c_{ijkl}e_{kl} - \beta_{ij}(T + \delta_{2k}t_1\dot{T}), \qquad i,j,k,l = 1,2,3,4 \qquad (2)$$

c) Equation of Motion

$$c_{ijkl}u_{k,ij} - \beta_{ij}\left(T + t_1\delta_{2k}\dot{T}\right)_{,j} = \rho\ddot{\upsilon}_i - F_i \quad i,j,k,l = 1,2,3 \qquad (3)$$

d) Equation of Heat conduction

$$K_{ij}T_{ij} - \rho C_e\left(T + t_0\ddot{T}\right)$$

$$= T_0\beta_{ij}\left(\dot{u}_{i,j} + \delta_{ik}t_0\ddot{\upsilon}_{i,j}\right) + (Q + t_0Q), i,j = 1,2,3 \qquad (4)$$

Where c_{ijkl} are isothermal elastic parameters, β_{ij} are thermoelastic coupling tensor, K_{ij} are the thermal conductivities, ρ, C_e and t_0, t_1 are respectively, density, specific heat at constant strain and thermal relaxation time, T is the temperature change, (u_l, u_2, u_3) are the displacement components, e_{ij} and σ_{ij} are strain and stress tensors respectively, T_0 is the initial temperature of the medium, F_i are components of the body force Q is the heat source term and $\delta_{ik}, i = 1,2$ is the Kronecker's delta. Here k=1 for LS theory, and k=2 for GL theory. The comma notation is used for spatial derivatives and superposed dot is used for time differentiation can be obtained by taking relaxation times $t_0 = t_1 = 0$ and coupling tensor $\beta_{ij} = 0$, the governing field equations for uncoupled thermoelasticity can be obtained. The parameters in (1) to (4) are assumed to satisfy the following conditions: The thermal conductivity tensor K_{ij} is symmetric and positive definite.

(1) Thermo elastic coupling tensor β_{ij} is non singular.

(2) The isothermal linear elasticities are positive definite in the sense that

$$c_{ijkl}e_{ij}e_{kl} > 0 \qquad (5)$$

The thermal relaxation times t_0 and t_1 satisfies the inequalities

$$t_0 \geq t_1 \geq 0$$

In case of GL theory only. However, it has been proved by Strunin [2001] recently that the inequalities are not mandatory for t_0 and t_1 to follow.

If the material under consideration is elastically and thermal isotropic that the coefficients in the above equations are taken as

$$c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu\left(\delta_{ik} + \delta_{il}\delta_{jk}\right), K_{ij} = K\delta_{ij}, \quad (6)$$

$$\beta_{ij} = (3\lambda + 2\mu)\alpha_t\delta_{ij} = \beta_e\delta_{ij}, \beta_e = (3\lambda + 2\mu)\alpha_t,$$

where λ, μ are Lame's parameters, K is the thermal conductivity and α_t is the coefficient of linear thermal expansion.

MicrostretchThermoelasticity

The equations governing interation inhomogeneous isotropic, micro-stretch generalized solid in the absence of body forces, couple stresses, stretch force and heat sources are given as,

a) Equation of motion

$$(\lambda + 2\mu + K)\nabla\cdot(\nabla, \vec{u}) - (\mu + K)\nabla\times(\nabla\times\vec{u}) +$$

$$K\nabla\times\vec{\phi} - v\left(1 + t_1\delta_{2k}\frac{\partial}{\partial t}\right)\nabla T + \lambda_0\nabla\phi^* = \rho\frac{\partial^2\vec{u}}{\partial^2 t} \quad (7)$$

$$(\alpha + \beta + \gamma)\nabla\cdot(\nabla, \vec{\phi}) - \gamma\nabla\times(\nabla\times\vec{\phi}) + k\nabla\times\vec{u} - 2K\vec{\phi} = \rho\, j\frac{\partial^2\vec{\phi}}{\partial t^2}, \quad (8)$$

$$\frac{2}{3}\alpha_0\nabla^2\phi^* + \frac{2}{9}v_1\left(T + t_1\frac{\partial T}{\partial t}\right) - \frac{2}{9}\lambda_1\phi^* - \frac{2}{9}\lambda_0\nabla.\vec{u} = \rho j\frac{\partial^2 t}{\partial t^2} \quad (9)$$

b) Heat conduction equation

$$\rho C^*\left(\frac{\partial T}{\partial t}+t_0\frac{\partial^2 T}{dt^2}\right)+vT_0\left(\frac{\partial}{\partial t}+\delta_{1k}t_0\frac{\partial^2}{\partial t^2}\right)\nabla\vec{\mu}+vT_0\frac{\partial\phi^*}{\partial t}=K^*\nabla^2 T \qquad (10)$$

c) Constitutive relations, couple stresses and stretch are respectively, given by

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+k\left(u_{j,i}-\in_{ijr}\phi_r\right)$$

$$-v\left(T+t_1\delta_{2k}\frac{\partial T}{\partial t^2}\right)\delta_{ij}+\lambda_0\delta_{ij}\phi^*, \qquad (11)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i} \quad ,$$

$$\lambda_k=\phi^*{}_{,k} \; , \qquad (12)$$

Where

$$v=(3\lambda+2\mu+k)\alpha_{t_1}, \quad ,v_1=(3\lambda+2\mu+k)a_{t_2} \qquad (13)$$

Here $\lambda,\mu,k,\alpha,\beta,\lambda,\alpha_0,\lambda_0,\lambda_1$ are material constants ρ is the density, j is the micro-inertia, K^* is the coefficient of thermal conductivity, C^* is specific heat at constant strain, α_{t_1} and α_{t_2} are coefficients of linear expansion, t_0 and t_1 are the Thermal relaxation times, the constant v and v_1 depend upon mechanical as well as thermal properties of the body, $\vec{u}$ is the displacement vector, $\vec{\phi}$ is the micro rotation vector and ϕ^* is the scalar microstretch. For L-S (Lord-Shulman) theory $t_1=0,k=1$ and GL theory (Green-Lindsay) theory $t_1\rangle 0$ and $k=2$. The thermal relaxation times t_0 and t_1 satisfy the inequality $t_1\geq t_0\geq 0$ for GL theory only.

Conclusion: By above explanation we can conclude that two dimensional problem in thermoelasticity can be studied where angular velocity is uniformed.

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