ON DISPERSION AND SELECTION OF QUADRATIC PERMUTATION COEFFICIENTS

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Abstract: Interleavers play a key role in the performance of the Turbo codes. Recently, Spread spectrum and randomness are the two main factors that influence the performance of an interleaver. Spread spectrum property of Quadratic Permutation Polynomial (QPP) interleaver has been studied in terms of Cycle Correlation Sum (CCS). In this paper, Randomness property of QPP interleaver has been investigated in terms of dispersion factor. A simple metric is defined taking in account of both CCS and dispersion factor. Three different interleaver lengths are considered for studying the proposed metric. Simulations are performed using MATLAB software.

Index Terms: Turbo codes, QPP, CCS, Dispersion Factor

I. INTRODUCTION

Since the discovery of Turbo codes in 1993, they have attained a considerable attention of the researchers (Moon, 2006). Turbo codes are found to have highest error correction capabilities (Heegard et al., 1999). Due to good performance of the turbo codes, they are selected for 3rd generation communication systems. The turbo codes consist of two convolution encoders separated by an interleaver (Vucetic et al., 2007). The role of the interleaver is to shuffle the input bits. The first encoder works on the original input bits while the second encoder works on the shuffled bits obtained from the interleaver and generate the second parity bit sequence. Interleaver plays many other roles in the communication system (Carlos, 2005). Interleaver converts low weight parity sequence of first encoder into high weight parity sequence of second encoder. Hence, the interleaver leads to increase the high weight code words. Also, the bits present close to each other before interleaving should be separated to large distance after interleaving (Takeshita, 2006 and Perez et al., 1996). Moreover, Interleaver should distribute the bits such that interleaved bits should not have any visible pattern (Trifina et al., 2009 and Rosnes, 2012).

There are various interleavers designed till date (Rai et al., 2010). But QPP interleaver has gained much attention due to its low complexity, contention free decoding and good error correction performance (Sun et al., 2005).

A QPP interleaver of length K is defined as

$\pi(x) = (a_0 + a\,x + b\,x^2)\ \mathrm{Mod}(K)$ where $x \in \{0,1,2,3,\ldots\ldots\ldots\ldots K-1\}$

Where, $\pi(x)$ represents the shifted position of the bit present at original position 'x'. a_0 informs about the shift of the permutation elements and 'a' and 'b' are called quadratic permutation coefficients (Takeshita , 2007). In this paper, we will consider the case when $a_0 = 0$

As already mentioned, QPP interleaver have many advantages over the other interleavers. But, the design of QPP interleaver requires a lot of efforts as there are infinite number of possible coefficients of QPP interleaver. It is not possible to simulate interleaver with each combination of coefficients possible and test to find the best among them. To eradicate this problem, (Garg et al., 2018) has discussed the concept of CCS metric. Before going into the details, we will introduce the concept of CCS metric. It has been found that the performance of the turbo code has improved to a large extent due to iterative decoder. In this decoding, the output from the one decoder is feed as input to the other decoder (Xie et al., 2006). This iteration goes on until the correspondence between outbound message and next inbound message is minimum. An interleaver is said to be a good interleaver, if the correspondence between the bits is minimum. To study about the un-correlation between the bits, CCS metric was proposed (Xie, 2011). As the value of uncorrelated messages decrease, CCS value also decreases and hence, performance of the system improves. The correlation between bits p and q is given as $e^{-k|p-q|}$ where 'k' is a parameter, roughly equal to length of the component encoder. Similarly, the correlation between the interleaved bit positions π(p) and π(q) is given by $e^{-k|\pi(p)-\pi(q)|}$. The CCS metric is defined to be

$$CCS= \sum_{p,q\in C} e^{-k(|p-q|+|\pi(p)-\pi(q)|)}$$

Where, C= {0, 1, 2, 3……………….K-1}, K is the interleaver length.

(Garg et al., 2018) studied the QPP interleaver in terms of CCS metric and grouped them according to CCS values. All the QPP coefficients belonging to a sub-group are found to have same CCS value. It is shown in the paper that coefficients predicted on the basis of CCS metric, are in good agreement with that expected on the basis of Turbo code performance. Hence, CCS is found to be a good metric to choose QPP coefficients. But, in order to choose best one among those belonging to a sub-group, we have to compare their Turbo codes performance.

In order to combat this issue, Randomness property of the interleavers has been considered. Randomness property accounts for the dis-orderness in the interleaved bit positions (Trifina et al., 2012). In this paper, we have studied the randomness property of QPP interleaver in terms of dispersion factor. Section 2 depicts that correspondence between elements of minimum CCS value group increases with increase in the length of the interleaver. It is found in section 3 that the results obtained from dispersion factor variation are contradictory to that expected from CCS metric prediction. On the basis of number of simulations performed, a new metric is defined in section 4 to account for both CCS and dispersion factor simultaneously. Section 5 concludes the paper.

II. LIMITATIONS OF CCS METRIC

CCS is found to be a good parameter to study the spread spectrum property of QPP interleaver. (Garg et al., 2018) studied CCS based QPP interleaver of length 128, 256, 512 and 1024. Four sub-groups were formed on the basis of CCS value. In this paper, QPP interleaver of length 1024, 2048 and 4096 has been considered. Table 1-3 depict the variation of CCS metric with QPP coefficients 'a' and 'b' for interleaver length of 1024, 2048 and 4096 respectively. It can be seen from the data obtained that CCS values are identical up to 27 decimal places for 'a' = 31 and 33 with 'b' taking value 64 for interleaver length of 1024. This correspondence between the CCS values increases to 29 decimal places for interleaver length of 2048 and 4096. It can be concluded that correspondence between CCS values for elements of minimum CCS value group increases with interleaver length.

Table 1. Variation of CCS metric for interleaver length of 1024

	b=60	b=62	b=64	b=66
a=27	2.9316e-01	1.5742e-01	5.2353e-07	1.5742e-01
a=29	2.9316e-01	1.5742e-01	9.5889e-09	1.5742e-01
a=31	2.1673e-02	1.5742e-01	**3.1924e-25**	1.5742e-01
a=33	2.1003e-02	1.5742e-01	**3.1892e-25**	1.5742e-01
a=35	2.9316e-01	1.5742e-01	1.7563e-10	1.5742e-01

Table 2. Variation of CCS metric for interleaver length of 2048

	b=60	b=62	b=64	b=66
a=27	2.9316e-01	1.5742e-01	5.2353e-07	1.5742e-01
a=29	2.9316e-01	1.5742e-01	9.6644e-09	1.5742e-01
a=31	2.1673e-02	1.5742e-01	**3.2406e-25**	1.5742e-01
a=33	2.1673e-02	1.5742e-01	**3.2407e-25**	1.5742e-01
a=35	2.9316e-01	1.5742e-01	1.7563e-10	1.5742e-01

Table 3. Variation of CCS metric for interleaver length of 4096

	b=60	b=62	b=64	b=66
a=27	2.9316e-01	1.5742e-01	5.2766e-07	1.5742e-01
a=29	2.9316e-01	1.5742e-01	9.6644e-09	1.5742e-01
a=31	2.1673e-02	1.5742e-01	**3.2663e-25**	1.5742e-01
a=33	2.1673e-02	1.5742e-01	**3.2664e-25**	1.5742e-01
a=35	2.9316e-01	1.5742e-01	1.7701e-10	1.5742e-01

Hence, one cannot rely only on CCS metric for predicting the parameters of QPP interleavers for longer length input bits.

III. DISPERSION FACTOR VARIATION OF QPP INTERLEAVER

Spread spectrum properties of an interleaver solely cannot account for the performance of the interleaver. This is because of the fact that merely distributing the bits at large distance is not sufficient but the bits should be distributed at random also. This is one of the main functions of an interleaver. For example, Row-column interleaver typically has a larger minimum spread than a random interleaver but still for longer length, the performance of a random interleaver is much better than row column interleavers. This is because of the impact of randomness. It is necessary to consider the impact of randomness along with spread spectrum property based CCS metric. Randomness is an important factor that determines the performance of the system to a great extent. For an interleaver to be a good interleaver, the permuted bits should not have a regular pattern among them i.e. there should be complete disorder among the permuted bit positions. The interleaved bits should not have a visible pattern i.e. there should not be any visible or simple rule to regenerate original input bits stream from the interleaved ones. Although, it is very difficult to consider this property for deterministic interleavers. This is because of the fact that, for deterministic interleaver there are specific rules to generate interleaved bits from input bits. Dispersion factor accounts for the degree of randomness or dis-orderness of the interleaved patterns. Dispersion gives the decrease in the multiplicities of low weight code words. So, dispersion factor should be as large as possible. More the value of dispersion, higher will be the randomness content. It is given by the number of distinct differences of the images of the elements that are at same distance before interleaving. For an interleaver π, the dispersion factor is defined as

$$D=\frac{2}{K(K-1)}\left\{\left(p-q,\ \pi(p)-\pi(q)\right)\middle|\ 0\leq q<p<K\right\}$$

Where, K is the interleaver length.

In this paper, we have studied the dispersion factor of QPP interleaver of length 1024, 2048 and 4096. Table 4 depict the values of dispersion factor for QPP interleaver length 1024 with odd values of ‘a’ varying from 27 to 35 and even values of ‘b’ varying from 30 to 66. It can be seen from table 4 that normalized dispersion factor is minimum for b=64 irrespective the value of coefficient ‘a’. We know that according to CCS variation, (33, 64) pair have minimum value of CCS parameter i.e. for this pair the interleaved bits are spread to maximum extent while dispersion factor performance is least.

Table 4.Variation of dispersion factor for QPP interleaver of length 1024

b↓	**a=27**	**a=29**	**a=31**	**a=33**	**a=35**
30	0.7865	0.7939	0.7896	0.7840	0.7905
32	0.4458	0.4477	0.4462	0.4489	0.4441
34	0.7898	0.7928	0.7857	0.7928	0.7927
36	0.6377	0.6371	0.6271	0.6208	0.6372
38	0.7908	0.7952	0.7862	0.7953	0.7958
40	0.5311	0.5352	0.5266	0.5278	0.5297
42	0.7942	0.7966	0.7869	0.7883	0.7887
44	0.6350	0.6337	0.6233	0.6223	0.6353
46	0.7888	0.7911	0.7897	0.7942	0.7904
48	0.4759	0.4764	0.4710	0.4703	0.4755
50	0.7895	0.7870	0.7914	0.7917	0.7900
52	0.6337	0.6362	0.6191	0.6232	0.6353
54	0.7884	0.7932	0.7851	0.7865	0.7907
56	0.5326	0.5331	0.5251	0.5253	0.5325
58	0.7927	0.7918	0.7906	0.7911	0.7938
60	0.6341	0.6400	0.6269	0.6183	0.6306
62	0.7902	0.7926	0.7946	0.7860	0.7876
64	**0.4285**	**0.4271**	**0.4274**	**0.4315**	**0.4300**
66	0.7880	0.7837	0.7875	0.7948	0.7890

Further, we have studied the variation of dispersion factor for QPP interleaver of length 2048 and 4096 using odd values of parameter ‘a’ from 27 to 35 while the variation of parameter ‘b’ is considered for all the even values from 60 to 66. Table 5-6 depicts the results obtained using MATLAB software of interleaver length 2048 and 4096 respectively. Here again, this is very clear that dispersion factor acquire minimum value for b=64, irrespective the value of ‘a’. Hence, there is tradeoff between the results predicted from CCS metric and dispersion factor.

Table 5. Variation of dispersion factor for QPP interleaver of length 2048

	b=60	b=62	**b=64**	b=66
a=27	0.6371	0.7912	**0.4295**	0.7937
a=29	0.6397	0.7946	**0.4312**	0.7928
a=31	0.6270	0.7908	**0.4314**	0.7942
a=33	0.62436	0.7913	**0.4324**	0.7907
a=35	0.6386	0.7921	**0.4318**	0.7930

Table 6. Variation of dispersion factor for QPP interleaver of length 4096

	b=60	b=62	**b=64**	b=66
a=27	0.6393	0.7964	**0.4317**	0.7968
a=29	0.6391	0.7957	**0.4315**	0.7947
a=31	0.6278	0.7959	**0.4320**	0.7933
a=33	0.6282	0.7947	**0.4324**	0.7957
a=35	0.6399	0.7940	**0.4312**	0.7958

It may be concluded from this section that coefficients predicted by CCS metric and dispersion factor are contradictory to each other. It is desired that the interleaved bits should have least Correlation between inbound and outbound bits (Minimum CCS) and maximum randomness (Maximum dispersion factor). But, CCS and Dispersion factor are attaining minimum values for the same set of coefficients. It is summarized from this section that there is a tradeoff between CCS and dispersion factor metrics.

IV. PROPOSED METRIC- 'GAMMA'

As concluded in the section 3, there is a contradiction between results predicted by CCS metric and dispersion factor. So for an interleaver, it is not possible to achieve best values of CCS and dispersion factor simultaneously. Hence, we need to find a optimize metric that can account for both the metrics simultaneously. A number of simulations under different simulating conditions have been performed so as to find the suitable metric. After a large number of simulations, we have found a metric 'γ' that can account for both CCS and Dispersion factor.

$$\gamma = (d)^{50} + c$$

Here,'d' stands for dispersion factor and 'c' stands for CCS value. To predict the accurate QPP coefficients, metric 'γ' should be as small as possible. We have investigated the QPP interleaver of length 1024, 2048 and 4096 in terms of proposed metric. Table 7 represents the variation of metric 'gamma' for interleaver length of 1024. It can be seen from the data obtained that the value of 'gamma' for (31, 64) and (33, 64) pair differ significantly. Moreover, metric 'gamma' attains its minimum value for a=31, b=64 pair. Hence, best suitable QPP coefficients for interleaver length 1024 are (31, 64).

Table 7. Variation of 'gamma' for QPP interleaver of length 1024

	b=60	**b=62**	**b=64**	**b=66**
a=27	2.9316e-01	1.5742e-01	5.2353e-07	1.5742e-01
a=29	2.9316e-01	1.5742e-01	9.5889e-09	1.5742e-01
a=31	2.1673e-02	1.5742e-01	**7.1125e-23**	1.5742e-01
a=33	2.1003e-02	1.5742e-01	**1.2588e-22**	1.5742e-01
a=35	2.9316e-01	1.5742e-01	1.7563e-10	1.5742e-01

Table 8. Variation of 'gamma' for QPP interleaver of length 2048

	b=60	**b=62**	**b=64**	**b=66**
a=27	2.9316e-01	1.5742e-01	5.2353e-07	1.5742e-01
a=29	2.9316e-01	1.5742e-01	9.6644e-09	1.5742e-01
a=31	2.1673e-02	1.5742e-01	**1.2415e-22**	1.5742e-01
a=33	2.1673e-02	1.5742e-01	**1.4260e-22**	1.5742e-01
a=35	2.9316e-01	1.5742e-01	1.7563e-10	1.5742e-01

Table 9. Variation of 'gamma' for QPP interleaver of length 4096

	b=60	**b=62**	**b=64**	**b=66**
a=27	2.9316e-01	1.5742e-01	5.2766e-07	1.5742e-01
a=29	2.9316e-01	1.5742e-01	9.6644e-09	1.5742e-01
a=31	2.1673e-02	1.5742e-01	**1.3492e-22**	1.5742e-01
a=33	2.1673e-02	1.5742e-01	**1.4261e-22**	1.5742e-01
a=35	2.9316e-01	1.5742e-01	1.7701e-10	1.5742e-01

For interleaver length 2048, the value of metric 'gamma' with a=31, b=64 and a= 33, b=64 are **1.2415e-22 and 1.4260e-22** respectively. The values are identical upto 23 decimal places which is far better than the corresponding CCS values that are identical to 29 decimal places. Similar results are obtained for interleaver length 4096. Also, 'gamma' metric is minimum for (31, 64) pair for both the interleaver lengths. This implies that best suited QPP coefficients for interleaver length of 2048 and 4096 are (31, 64).

V. CONCLUSION

In this paper, randomness property of QPP interleavers has been studied in terms of dispersion factor. Dispersion factor for interleaver lengths 1024, 2048 and 4096 has been considered. It is found that there is a trade off between the results obtained from CCS metric and dispersion factor. For a good interleaver, the CCS metric should be minimum and dispersion factor should be maximum. But it is found from the results obtained from MATLAB simulations that there is a trade off between the two metrics. In order to optimise both the factors, a simple metric is proposed in terms of CCS metric and dispersion factor. Variation of proposed metric has been considered for various QPP Coefficients. Proposed metric found to predict accurate QPP coefficients.

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