CUBIC STRONG BI-IDEALS IN NEAR SUBTRACTION SEMIGROUPS

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Abstract

Dheena discussed and derived some properties of near subtraction s. The concept of fuzzy set was first initiated by Zadeh. In this paper we introduced the notation of Cubic strong bi-ideal in near- subtraction s.

Key Words : Near subtraction semigroups,strong bi-ideals,fuzzy strong bi-ideals, Cubic strong bi-ideals,homomorphism of cubic strong bi-ideals.

1 Introduction

B.M.Schein [15] considered systems of the form (X; ∘; /), where X is a set of functions closed under the composition ”∘ ” of functions (and hence (X; ∘) is a function semigroup) and the set theoretic subtraction ”/” (and hence (X; /) is a subtraction algebra in the sense of [6]). Y.B. Jun et. al. [10] introduced the notation of ideals in subtraction algebras and discussed the characterization of ideals In [10], Y.B.Jun and H.S. Kim established the ideal generated by a set, and discussed related results. The concept of fuzzy set was first initiated by Zadeh[17].

Narayanan et al.[14] defined the concept of generalized fuzzy ideals of near-rings. Mahalakshmi et. al. [11] studied the notation of bi-ideals in near subtraction s. Manikandan [12] studied fuzzy fuzzy bi-ideals in near-rings. Chinnadurai et. al. [2] defined the concept of fuzzy bi-ideals of near-subtraction s. Chinnadurai et. al. [3] studied the notation of fuzzy weak bi-idelas of near rings and established. Some of its properties motivated by this concept, we introduced Cubic strong bi-ideals in ear-subtraction s and some of its properties.

2 Preliminaries

Definition 2.1. A nonempty set X together with a binary operation − is called **near subtraction algebra** if it satisfying the following:

(i) $x - (y - x) = x$

(ii) $x - (x - y) = y - (y - x)$

(iii) $(x - y) - z = (x - z) - y$

Definition 2.2. A nonempty set X together with two binary operation − and · is said to be **subtraction semigroup** if it satisfying the following:

(i) (x, −) is a subtraction algebra.

(ii) (x, ·) is a semigroup.

(iii) $x(y - z) = xy - xz$ and $(x - y)z = xz - yz$ for every x, y, z □ X

Definition 2.3. A near- subtraction semigroup X is called **zero-symmetric**, if $x0 = 0$, for all x in X.

Definition 2.4. A non empty subset S of a subtraction algebra X is said to be a **Subalgebra** of X, if $x - y \in S$.

Note 2.5. Let X be a near- subtraction semigroup. Given two subsets A and B of X,

$A \cdot B = \{ab/a \in A, b \in B\}$. Also we define another operation " *"

$A * B = \{ab - a(a' - b)/a , a' \in A, b \in B\}$.

Definition 2.6. A function A from a non-empty set X to the unit interval. [0, 1] is called a fuzzy subset of X. [14]

Definition 2.7. A Sub algebra B of X is called bi-ideal if BXB ∩ BX □ B ⊆ B.In case of zero Symmetric, BXB ⊆ B.

Notation 2.8. Let μ and λ be two fuzzy subsets of a semigroup X. We define the relation ⊆ between μ and λ , the union, intersection and product of μ and λ, respectively as follows:

1. $\mu \subseteq \lambda$ if $\mu(x) \le \lambda(x)$, for all $x \in X$,
2. $(\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}$, for all $x \in X$,
3. $(\mu \cap \lambda)(x) = \min\{\mu(x), \lambda(x)\}$, for all $x \in X$.
4. $$(\mu - \lambda)(x) = \begin{cases} \sup_{x=yz} \min\{\mu(y), \lambda(z)\} & \text{if } x=y-z, \text{for all } y,z \in X \\ 0 & \text{otherwise} \end{cases}$$
5. $$(\mu\lambda)(x) = \begin{cases} \sup_{x=yz} \min\{\mu(y), \lambda(z)\} & \text{if } x=yz, \text{for all } y,z \in X \\ 0 & \text{otherwise} \end{cases}$$
6. $$(\mu * \lambda)(x) = \begin{cases} \sup_{x=ac-a(b-c)} \min\{\mu(y), \lambda(z)\} \\ 0 & \text{otherwise} \end{cases}$$

Definition: 2.9: A Fuzzy subalgebra μ of X is called **fuzzy bi-ideal** of X,if $(\mu X \mu) \cap (\mu X * \mu) \subseteq \mu$.

In case of zero symmetric if $\mu X \mu \subseteq \mu$.

Definition: 2.10: Let (X,-,.) be a near subtraction semigroup.A non-emty subset I of X is called

(i) A Left ideal if I is a subalgebra of (X,-) and xi-$x(y-i)$∈I for every x,y∈X.

(ii) A right ideal if I is a subalgebra of (X,-) and IX⊆I.

(iii) An Ideal if I is both a left and right.

Definition: 2.11: A Fuzzy subset μ of X is called **fuzzy ideal** if it satisfying the following conditions:

(i) μ(x-y)≥min{ μ(x), μ(y)}

(ii) μ(xi-x(y-i) ≥ μ(i).

(iii) μ(xy) ≥μ(x),for every x,y ЄX.

(iv) μ((x+z)y-xy ≥μ(x),for every x,y,z ЄX.

A fuzzy subset with (i) and (ii) is called a fuzzy left ideal of X,Whereas a fuzzy subset with (i),(ii) and (iv) is called a fuzzy right ideal of X.

Definition: 2.12: A Fuzzy subset μ of X is called **fuzzy X subalgebra** if it satisfying the following conditions:

(i) μ is a fuzzy subalgebra of (X,-).

(ii) μ(xy) ≥μ(x).

(iii) μ(xy) ≥μ(y),for every x,y ЄX.

A fuzzy subset with (i) and (ii) is called a fuzzy right subalgebra of X ,Whereas a fuzzy subset with (i),(ii) and (iii) is called a fuzzy left subalgebra l of X.

3 Cubic Strong Bi-ideals in Near Subtraction Semigroups

Definition 3.1. A cubis set A={ μ̄, ω} of X is called cubic sub algebra of X if

(i) $\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$

(ii) $\omega(x - y) \leq \max\{\omega(x), \omega(y)\}$ for all x, y □ X.

Definition 3.2. A cubic sub algebra A = { μ̄, ω} of X is called strong bi-ideal of X if

(i) $\bar{\mu}(xyz) \geq \min\{\bar{\mu}(y), \bar{\mu}(z)\}$

(ii) $\omega(xyz) \leq \max\{\omega(y), \omega(z)\}$ for all x, y, z Є X

Example 3.3. Let X = {0, a, b, c} be a near subtraction semigroup with two binary operation − and · are defined as follows

--	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	0	b	b
c	0	0	b	b

Then (X, −, ·) is a near subtraction semigroup.

Let $\bar{\mu} : X \to D[0, 1]$ be an interval valued fuzzy subset defined by $\bar{\mu}(a) = [0.5, 0.6]$, $\bar{\mu}(b) = [0.7, 0.8]$ and $\bar{\mu}(c) = [0.1, 0.2] = \bar{\mu}(d)$.

Then $\bar{\mu}$ is an inter-valued fuzzy strong bi-ideal of X

Let $\omega : X \to [0, 1]$ be a fuzzy subset defined by $\omega(a) = 0.2$, $\omega(b) = 0.4$ and $\omega(c) = 0.8 = \omega(d)$.

Then ω is a fuzzy strong bi-ideal of X.

Definition 3.4. Let A_i be cubic strong bi-ideals of near subtraction semigroups X_i for i = 1, 2, 3, . . . , n. Then the cubic direct product of A_i (i = 1, 2, 3, . . . , n) is a function $\mu_1 \times \mu_2 \times \ldots \times \mu_n : X_1 \times X_2 \times \ldots \times X_n \to D[0, 1]$, $\omega_1 \times \omega_2 \times \ldots \times \omega_n : X_1 \times X_2 \times \ldots \times X_n \to [0, 1]$ defined by $(\bar{\mu}_1 \times \bar{\mu}_2 \times \ldots \times \bar{\mu}_n)(x_1, x_2, \ldots, x_n) = \min\{\bar{\mu}_1(x_1), \bar{\mu}_2(x_2), \ldots, \bar{\mu}_n(x_n)\}$ and $(\omega_1 \times \omega_2 \times \ldots \times \omega_n)(x_1, x_2, \ldots, x_n) = \max\{\omega_1(x_1), \omega_2(x_2), \ldots, \omega_n(x_n)\}$.

Definition 3.5. Let $\mathcal{A}_1 = \bar{\mu}_1, \omega_1$ and $\mathcal{A}_2 = \bar{\mu}_2, \omega_2$ be any two cubic subsets of X. Then $\mathcal{A}_1 \mathcal{A}_2$ is cubic subsets of X defined by

$$(\mathcal{A}_1 \mathcal{A}_2)(x) = \begin{cases} (\bar{\mu}_1, \bar{\mu}_2)(x) = \begin{cases} \sup_{x=yz} \{\bar{\mu}_1(y), \bar{\mu}_2(z)\} & \text{if } x=yz \text{ for all } x,y,z \in X \\ [0,0] & \text{otherwise} \end{cases} \\ \omega_1, \omega_2(x) = \begin{cases} \inf_{x=yz} \max\{\omega_1(y), \omega_2(y)\} & \text{if } x=yz \text{ for all } x,y,z \in X \\ [0,0] & \text{otherwise} \end{cases} \end{cases}$$

Theorem 3.6. Let $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ be a cubic subalgebra of X. Then $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic Strong bi-ideal of X ⇔ $XA A \subseteq A$. (ie)., $X \bar{\mu} \bar{\mu} \subseteq \bar{\mu}$ and $X\omega\omega \subseteq \omega$.

Proof. Assume that $\mathcal{A}$ = ‹ $\bar{\mu}$, ω › is a cubic strong bi-ideal of X. Let x, y, z, p, q Є X such that x = yz and z = pq.

$$X\bar{\mu}\bar{\mu}(x) = \sup_{x=yz}\{\min\{X(y)\bar{\mu}\bar{\mu}(z)\}\}$$

$$= \sup_{x=yz}\min\{X(y)\sup_{z=pq}\min\{\bar{\mu}(p),\bar{\mu}(q)\}\}$$

$$= \sup_{x=yz}\sup_{z=pq}\{\min X(y)\{\min\{\bar{\mu}(p),\bar{\mu}(q))\}\}$$

$$= \sup_{x=yz}\sup_{z=pq}\min\{\bar{\mu}(p),\bar{\mu}(q)\}$$

$$= \sup_{x=ypq}\min\{\bar{\mu}(p),\bar{\mu}(q)\}$$

$$\leq \sup_{x=ypq}\bar{\mu}(ypq)$$

$$X\bar{\mu}\bar{\mu}(x) = \sup_{x=yz}\bar{\mu}(yz)$$

$$= \bar{\mu}(x)$$

If x cannot be expressed as x = yz. Then $X\bar{\mu}\bar{\mu} = 0 \leq \bar{\mu}(x)$

In both cases $X\bar{\mu}\bar{\mu} \subseteq \bar{\mu}$

$$(X\omega\omega)(x) = \inf_{x=yz}\{\max\{X(y)\,\omega\,\omega\,(z)\}\}$$

$$= \inf_{x=yz}\max\{X(y)\inf_{z=pq}\max\{\omega(p),\omega(q)\}\}$$

$$= \inf_{x=yz}\inf_{z=pq}\{\max X(y)\{\max\{\omega(p),\omega(q)\}\}\}$$

$$= \inf_{x=yz}\inf_{z=pq}\max\{\omega(p),\omega(q)\}$$

$$\geq \inf_{x=ypq}\omega(ypq)$$

$$(X\omega\omega)(x) = \inf_{x=yz}\omega(yz)$$

$$= \omega(x)$$

If x cannot be expressed as x = yz then (Xωω)(x) = 1 ≥ ω(x).

In both cases Xωω ⊇ ω.

Hence XA A ⊆ A .

Conversely, assume that XA A ⊆ A holds.

To prove that A = ‹ $\bar{\mu}$, ω › is a cubic strong bi-ideal of X.

For any x, y, z, a ЄX such that a = xyz. Then

$$\bar{\mu} = \bar{\mu}(a) \geq X\bar{\mu}\bar{\mu}(a)$$
$$= \sup_{a=bc} \{\min\{X\bar{\mu}(b), \bar{\mu}(c)\}$$
$$= \sup_{a=bc} \{\min\{\sup_{b=pq} \min\{X(p), \bar{\mu}(q), \bar{\mu}(c)\}\}$$
$$= \sup_{a=pqc=xyz} \{\min\{\bar{\mu}(q), \bar{\mu}(c)$$
$$\bar{\mu}(a) \geq \min\{\bar{\mu}(y), \bar{\mu}(z)\}$$

Therefore, $\bar{\mu}(a) \geq \min\{\bar{\mu}(y), \bar{\mu}(z)\}$

$$\omega(xyz) = \omega(a) \leq (X\omega\omega)(a)$$
$$= \inf_{a=bc} \{\max\{X\omega(b), \omega(c)\}$$
$$= \inf_{a=bc} \{\max\{\inf_{b=pq} \max\{X(p), \omega(q), \omega(c)\}\}$$
$$= \inf_{a=pqc=xyz} \{\max\{\omega(q), \omega(c)\}$$
$$\omega(xyz) \leq \max\{\omega(y), \omega(z)\}$$

Hence $\mathcal{A} = \langle \bar{\mu}, \omega \rangle$ is a cubic strong bi-ideal of X.

Theorem 3.7. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be two cubic strong bi-ideals of X. Then the product $\mathcal{A}_1 \mathcal{A}_2$ is a cubic strong bi-ideals of X

Proof. Let $\mathcal{A}_1 = \bar{\mu}_1, \omega_1$ and $\mathcal{A}_2 = \bar{\mu}_1, \omega_2$ be two cubic strong bi-ideals of X.
Since $\bar{\mu}_1$and $\bar{\mu}_2$ are interval-valued fuzzy strong bi-ideal of X. Then

$$\bar{\mu}_1\bar{\mu}_2)(x-y) = \sup_{x-y=pq} \min\{\bar{\mu}_1(p), \bar{\mu}_2(q)\}$$
$$\geq \sup_{x-y=p_1q_1-p_2q_{2\leq(p_1-p_2)(q_1-q_2)}} \min\{\bar{\mu}_1(p_1-p_2), \bar{\mu}_2(q_1-q_2)\}$$
$$\geq \sup \min\{\min\{\bar{\mu}_1(p_1), \bar{\mu}_1(p_2)\}, \min\{\bar{\mu}_2(q_1), \bar{\mu}_2(q_2)\}\}$$
$$= \sup \min\{\min\{\bar{\mu}_1(p_1), \bar{\mu}_2(q_1)\}, \min\{\bar{\mu}_1(p_2), \bar{\mu}_2(q_2)\}\}$$
$$= \min\{\sup_{x=p_1q_1} \min\{\bar{\mu}_1(p_1), \bar{\mu}_2(q_1)\}, \sup_{y=p_2q_2} \min\{\bar{\mu}_1(p_2), \bar{\mu}_2(q_2)\}\}$$
$$= \min\{(\bar{\mu}_1\bar{\mu}_2)(x), (\bar{\mu}_1\bar{\mu}_2)(y)\}$$

It follows that $((\bar{\mu}_1\bar{\mu}_2)$ is an interval-valued fuzzy subalgebra of X. Further

$$X(\bar{\mu}_1\,\bar{\mu}_2)(\,(\bar{\mu}_1\bar{\mu}_2)(x) = \sup_{x=yz} \min\{X(y), ((\bar{\mu}_1\bar{\mu}_2)(\,(\bar{\mu}_1\bar{\mu}_2)(z)\}$$

$$= \sup_{x=yz} \{X(y) \sup_{z=pq} \min\{(\bar{\mu}_1\bar{\mu}_2\,(p), \bar{\mu}_1\bar{\mu}_2\,(q)\}\}$$

$$= \sup_{x=yz} \sup_{z=pq} \{\min X(y)\{\min((\bar{\mu}_1\bar{\mu}_2)(p)(\,(\bar{\mu}_1\bar{\mu}_2)(q)\}\}$$

$$X(\bar{\mu}_1\,\bar{\mu}_2)(\,(\bar{\mu}_1\bar{\mu}_2)(x) = \sup_{x=yz} \sup_{z=pq} \min\{((\bar{\mu}_1\bar{\mu}_2)(p)(\,(\bar{\mu}_1\bar{\mu}_2)(q)\}\}$$

$$\leq \sup_{x=ypq} \min\{(\,\bar{\mu}_1\bar{\mu}_2)(p)(\,\bar{\mu}_1\bar{\mu}_2)(q)\}$$

$$\leq \sup_{x=ypq} (\bar{\mu}_1\bar{\mu}_2)(ypq)$$

$$= \bar{\mu}_1\bar{\mu}_2\,(x)$$

Therefore $(\bar{\mu}_1\bar{\mu}_2)$ is an interval-valued fuzzy strong bi-ideals of X. Since $\omega_1\,\omega_2$ are fuzzy strong bi-ideals of X, then

$$(\omega_1\,\omega_2)(x-y) = \inf_{x-y=pq} \max\{\,\omega_1(p), \omega_2(q)\}$$

$$\leq \inf_{x-y=p_1q_1-p_2q_2\leq(p_1-p_2)(q_1-q_2)} \max\{\,\omega_1(p_1-p_2), \omega_2(q_1-q_2)\}$$

$$\leq \inf \max\{\max\{\,\omega_1(p_1), \omega_1(p_2)\},\max\{\,\omega_2\,(q_1), \omega_2\,(q_2)\}\}$$

$$= \inf \max\{\max\{\,\omega_1(p_1), \omega_2\,(q_1)\},\max\{\,\omega_1(p_2), \omega_2\,(q_2)\}\}$$

$$= \max\{\inf_{x=p_1q_1} \max\{\,\omega_1(p_1), \omega_2\,(q_1)\}, \inf_{x=p_2q_2} \max\,\{\,\omega_1(p_2), \omega_2\,(q_2)\}\}$$

$$= \max\{(\,\omega_1\,\omega_2)(x), (\omega_1\,\omega_2)(y)\}$$

It follows that $(\omega_1\,\omega_2)$ is an interval-valued fuzzy subgroup of X. Further

$$X(\omega_1\,\omega_2)(\,\omega_1\,\omega_2)(x) = \inf_{x=yz} \max\{X(y), (\omega_1\,\omega_2\,)(\,\omega_1\,\omega_2\,)(z)\}$$

$$= \inf_{x=yz} \{X(y) \inf_{z=pq} \max\{\,\omega_1\,\omega_2\,(p), \omega_1\,\omega_2\,(q)\}\}$$

$$= \inf_{x=yz} \inf_{z=pq} \{\max X(y)\{\max(\omega_1\,\omega_2)(p)(\,\omega_1\,\omega_2)(q)\}\}$$

$$= \inf_{x=yz} \inf_{z=pq} \max\{(\,\omega_1\,\omega_2)(p)(\,\omega_1\,\omega_2)(q)\}$$

$\geq \inf_{x=ypq} \max\{(\omega_1\, \omega_2)(p)(\omega_1\, \omega_2)(q)\}$

$\geq \inf_{x=ypq} (\omega_1\, \omega_2)(ypq)$

$= \omega_1\, \omega_2\, (x)$

Thus $(\omega_1\, \omega_2)$ is a fuzzy strong bi-ideals of X. Hence $\mathcal{A}_1\mathcal{A}_2 = \prec (\bar{\mu}_1\bar{\mu}_2), (\omega_1\, \omega_2\,) \succ$ is a cubic strong bi-ideal of X.

Remark 3.8. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be two cubic strong bi-ideals of X. Then the product $\mathcal{A}_2 \cdot \mathcal{A}_1$ is also a cubic strong bi-ideal of X.

Theorem 3.9. Let $\mathcal{A} = \prec \bar{\mu}, \omega \succ$ be a cubic strong bi-ideal of X, then the set $X_{\mathcal{A}} = \{x \in X / \mathcal{A}(x) = \mathcal{A}(0)\}$

(ie)., $X_{\mathcal{A}} = \{x \in X / \bar{\mu}(x) = \bar{\mu}(0)$ and $\omega(x) = \omega(0)\}$ is a strong bi-ideal of X.

Proof. Let $\mathcal{A} = (\bar{\mu}, \omega)$ be a cubic strong bi-ideal of X. Then $\mathcal{A}(x) = \mathcal{A}(0)$ and $\mathcal{A}(y) = \mathcal{A}(0)$ (i.e.,) $\bar{\mu}(x) = \bar{\mu}(0)$, $\omega(x) = \omega(0)$ and $\bar{\mu}(y) = \bar{\mu}(0)$, $\omega(y) = \omega(0)$. Since $\bar{\mu}$ is an interval-valued fuzzy strong bi-ideal of X. We have $\bar{\mu}(x) = \bar{\mu}(0)$ and $\bar{\mu}(y) = \bar{\mu}(0)$, $\bar{\mu}(x-y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\} = \min\{\bar{\mu}(0), \bar{\mu}(0)\} = \bar{\mu}(0)$ and ω is a fuzzy strong bi-ideal of X, we have $\omega(x) = \omega(0)$ and $\omega(y) = \omega(0)$. Then $\omega(x-y) \leq \max\{\omega(x), \omega(y)\} = \max\{\omega(0), \omega(0)\} = \omega(0)$. Thus $x - y \in X_{\mathcal{A}}$.

For every x, y, z $\in X_{\mathcal{A}}$. Then $\mathcal{A}(x) = \mathcal{A}(0)$, $\mathcal{A}(y) = \mathcal{A}(0)$ and $\mathcal{A}(z) = \mathcal{A}(0)$. Since $\bar{\mu}$ is an interval-valued fuzzy strong bi-ideal of X. We have $\bar{\mu}(x) = \bar{\mu}(0)$, $\bar{\mu}(y) = \bar{\mu}(0)$ and $\bar{\mu}(z) = \bar{\mu}(0)$. Then $\bar{\mu}(xyz) \geq \min\{\bar{\mu}(y), \bar{\mu}(z)\} = \min\{\bar{\mu}(0), \bar{\mu}(0)\}$ and ω is a fuzzy strong bi-ideal of X. We have $\omega(x) = \omega(0)$, $\omega(y) = \omega(0)$, $\omega(z) = \omega(0)$ and $\omega(xyz) \leq \max\{\omega(x), \omega(y), \omega(z)\} = \max\{\omega(0), \omega(0), \omega(0)\} = \omega(0)$. Thus $xyz \in X_{\mathcal{A}}$.

Hence $X_{\mathcal{A}}$ is a cubic strong bi-ideal of X.

Theorem 3.10. Let $\{\mathcal{A}_i\}_{i\varepsilon\wedge} = \prec \bar{\mu}_i, \omega : i \in \wedge \succ$ be a family of cubic strong bi-ideals of X Then $\cap_{i\varepsilon\wedge} \mathcal{A}_i = \prec \cap_{i\varepsilon\wedge} \mu_i, \cup_{i\varepsilon\wedge} \omega_i \succ$ is also a family of cubic strong bi-ideals of X, where $\wedge$ is any index set.

Proof. Let $\{\mathcal{A}_i\}_{i\varepsilon\wedge} = \prec \bar{\mu}_i, \omega : i \in \wedge \succ$ be a family of cubic strong bi-ideals of X.
Let x, y, z $\in$ X and $\cap_{i\varepsilon\wedge} \bar{\mu}_i(x) = (\inf_{i\varepsilon\wedge}) \bar{\mu}_i(x)$

$$= \inf_{i\varepsilon\wedge} \bar{\mu}_i(x)$$

$$\cup_{i\varepsilon\wedge} \omega_i(x) = (\sup_{i\varepsilon\wedge})\omega_i(x)$$

$$= \sup_{i\varepsilon\wedge} \omega_i(x)$$

Since $\bar{\mu}_i$ is a family of interval-valued fuzzy strong bi-ideals of X we have

$$\cap_{i\varepsilon\wedge} \bar{\mu}_i(x - y) = \inf_{i\varepsilon\wedge} \bar{\mu}_i(x\text{-}y)$$

$$\geq \inf_{i\varepsilon\wedge} \min\{\bar{\mu}_i(x), \bar{\mu}_i(y)\}$$

$$= \min\{\inf_{i\varepsilon\wedge} \bar{\mu}_i(x), \inf_{i\varepsilon\wedge} \bar{\mu}_i(y)\}$$

$$= \min\{\cap_{i\varepsilon\wedge} \bar{\mu}_i(x), \cap_{i\varepsilon\wedge} \bar{\mu}_i(y)\}$$

And ω_i is a family of fuzzy strong bi-ideals of X. We have

$$\cup_{i\varepsilon\wedge} \omega_i(x - y) = \sup_{i\varepsilon\wedge} \omega_i(x\text{-}y)$$

$$\geq \sup_{i\varepsilon\wedge} \max\{\omega_i(x), \omega_i(y)\}$$

$$= \max\{\sup_{i\varepsilon\wedge} \omega_i(x), \sup_{i\varepsilon\wedge} \omega_i(y)\}$$

$$= \max\{\cup_{i\varepsilon\wedge} \omega_i(x), \cup_{i\varepsilon\wedge} \omega_i(y)\}$$

Thus $\cap_{i\varepsilon\wedge} \mathcal{A}_i$ is a cubic subalgebra of X.
Again

$$\cap_{i\varepsilon\wedge} \bar{\mu}_i(x\,yz) = \inf_{i\varepsilon\wedge} \bar{\mu}_i(xyz)$$

$$\geq \inf_{i\varepsilon\wedge} \min\{\bar{\mu}_i(y), \bar{\mu}_i(z)\}$$

$$= \min\{\inf_{i\varepsilon\wedge} \bar{\mu}_i(y), \inf_{i\varepsilon\wedge} \bar{\mu}_i(z)\}$$

$$= \min\{\cap_{i\varepsilon\wedge} \bar{\mu}_i(y), \cap_{i\varepsilon\wedge} \bar{\mu}_i(z)\}$$

$$\cup_{i\varepsilon\wedge} \omega_i(xyz) = \sup_{i\varepsilon\wedge} \omega_i(xyz)$$

$$\geq \sup_{i\varepsilon\wedge} \max\{\omega_i(y), \omega_i(z)\}$$

$$= \max\{\sup_{i\varepsilon\wedge} \omega_i(y), \sup_{i\varepsilon\wedge} \omega_i(z)\}$$

$$= \max\{\cup_{i\varepsilon\wedge} \omega_i(y), \cup_{i\varepsilon\wedge} \omega_i(z)\}$$

Hence $\cap_{i\varepsilon\wedge} \mathcal{A}_i = \prec \cap_{i\varepsilon\wedge} \bar{\mu}_i, \cup_{i\varepsilon\wedge} \omega_i \succ$ is also a family of cubic strong bi-ideals of X.

Theorem 3.11. Let H be a non empty subset of X and $\mathcal{A} = \prec \bar{\mu}_i, \omega \succ$ be a cubic subset of X defined by

$$\mathcal{A}(x) = \begin{cases} \bar{\mu} = \begin{cases} [p_1, p_2] & \text{if } x \in H \\ [q_1, q_2] & \text{Otherwise} \end{cases} \\ \omega(x) = \begin{cases} 1-p & \text{if } x \in H \\ 1-q & \text{Otherwise} \end{cases} \end{cases}$$

For all $x \in X$, $[p_1, p_2], [q_1, q_2] \in D[0, 1]$ and $p, q \in [0, 1]$ with $[p_1, p_2] > [q_1, q_2]$, $p > q$. Then H is a strong bi-ideal of X $\Leftrightarrow \mathcal{A} = \prec \bar{\mu}, \omega \succ$ is a cubic strong bi-ideal of X.

Proof. Assume that H is a strong bi-ideal of X. Let $x, y \in H$ we consider four cases.

(i) $x \in H$ and $y \in H$

(ii) $x \in H$ and $y \notin H$

(iii) $x \notin H$ and $y \in H$

(iv) $x \notin H$ and $y \notin H$

Case(i) If $x \in H$ and $y \in H$. Then $\bar{\mu}(x) = [p_1, p_2] = \bar{\mu}(y)$ and $\omega(x) = 1-p = \omega(y)$. Since H is a strong bi-ideal of X then $x - y \in X$. Thus $\bar{\mu}(x - y) = [p_1, p_2] = \min\{[p_1, p_2], [p_1, p_2]\} = \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and $\omega(x-y) = 1-p = \max\{1-p, 1-p\} = \max\{\omega(x), \omega(y)\}$

Case(ii) If $x \in H$ and $y \notin H$. Then $\bar{\mu}(x) = [p_1, p_2]$, $\bar{\mu}(y) = [q_1, q_2]$ and $\omega(x) = 1 - p$, $\omega(y) = 1 - q$. So $\min\{\bar{\mu}(x), \bar{\mu}(y)\} = [q_1, q_2]$ and $\max\{\omega(x), \omega(y)\} = 1 - q$. Now $\bar{\mu}(x - y) = [p_1, p_2]$ or $[q_1, q_2]$, according as $x - y \in H$ or $x - y \notin H$.

By assumption that $[p_1, p_2] > [q_1, q_2]$ and $p > q$, we have

$\bar{\mu}(x - y) \geq \min\{\bar{\mu}(x), \bar{\mu}(y)\}$ and $\omega(x - y) \leq \max\{\omega(x), \omega(y)\}$

Similarly we can prove that Case(iii).

Case(iv) If $x \notin H$ and $y \notin H$. Then $\bar{\mu}(x) = [q_1, q_2] = \bar{\mu}(y)$ and $\omega(x) = 1-q = (y)$. So $\min\{\bar{\mu}(x), \bar{\mu}(y)\} = [q_1, q_2]$ and $\max\{\omega(x), \omega(y)\} = 1-q$. Next $\bar{\mu}(x-y) = [p_1, p_2]$ or $[q_1, q_2]$ and $\omega(x - y) = 1 - p$ or $1 - q$, according as $x - y \in H$ or $x - y \notin H$. So $\mathcal{A} = \prec \bar{\mu}, \omega \succ$ is a cubic subalgebra of X. Now let $x, y, z \in H$. We have following eight cases:

(i) x ∈ H, y ∈ H and z ∈ H

(ii) x ∉ H, y ∈ H and z ∈ H

(iii) x ∈ H, y ∉ H and z ∈ H

(iv) x ∈ H, y ∈ H and z ∉ H

(v) x ∉ H ,y ∉ H and z ∈ H

(vi) x ∈ H, y ∉ H and z ∉ H

(vii) x ∉ H, y ∈ H and z ∉ H

(viii) x ∉ H, y ∉ H and z ∉ H

These cases can be proved by similar arguments of the cubic cases above.

Hence $\bar{\mu}$ (xyz) ≥ min{ $\bar{\mu}$ (y), $\bar{\mu}$ (z)} and ω(xyz) ≤ max{ ω (y), ω (z)}.

Therefore $\mathcal{A}$ =≺$\bar{\mu}$,ω ≻ is a cubic strong bi-ideal of X.

Conversely, assume that $\mathcal{A}$ =≺$\bar{\mu}$,ω ≻ is a cubic strong bi-ideal of X.

Let x, y, z ∈ H be such that $\bar{\mu}$ (x) = $\bar{\mu}$ (y) = $\bar{\mu}$ (z) = 1 − p. Since $\bar{\mu}$ is an intervalvalued fuzzy strong bi-ideal of X, we have $\bar{\mu}$ (x−y) ≥ min{ $\bar{\mu}$ (x), $\bar{\mu}$ (y)} = $[p_1, p_2]$ and ω is a fuzzy strong bi-ideals of X, we have ω (x − y) ≤ max{ ω (x), ω (y)} = 1 − p.

Again, $\bar{\mu}$ (xyz) ≥ min{ $\bar{\mu}$ (y), $\bar{\mu}$ (z)} = $[p_1, p_2]$ and ω (xyz) ≤ max{ ω (y), ω (z)} = 1 − p so x − y, xyz ∈ H.

Hence H is a cubic strong bi-ideal of X.

Theorem 3.12. The direct product of cubic strong bi-ideals of near subtraction semigroup is also a cubic strong bi-ideals

Proof. Let $\mathcal{A}_i$ =≺$\bar{\mu}$,ω ≻ be cubic strong bi-ideals of near subtraction semigroup R_i for i = 1, 2, 3, · · · , n.

Let x = $(x_1, x_2, \cdots, x_n)$, y = $(y_1, y_2, \cdots, y_n)$ and z = $(z_1, z_2, \cdots, z_n) \in R_1 \times R_2 \times \cdots \times R_n$.

$\bar{\mu}_i(x - y) = \bar{\mu}_i((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n))$

$= \bar{\mu}_i((x_1 - y_1, x_2 - y_2, \cdots, x_n - y_n)$

$= \min\{\bar{\mu}_1 (x_1 - y_1), \bar{\mu}_2 (x_2 - y_2), \ldots\ldots\ldots, \bar{\mu}_n (x_n - y_n)\}$

$\geq \min\{\min\{\bar{\mu}_1 (x_1), \bar{\mu}_1 (y_1)\}, \min\{\bar{\mu}_2 (x_2), \bar{\mu}_2 (y_2)\}, \ldots\ldots, \min\{ \bar{\mu}_n (x_n), \bar{\mu}_n (y_n)\}\}$

$= \min\{\min\{\bar{\mu}_1 (x_1), \bar{\mu}_2 (x_2), \ldots\ldots, \bar{\mu}_n (x_n)\}, \min \{\bar{\mu}_1 (y_1), \bar{\mu}_2 (y_2), \bar{\mu}_n(y_n)\}\}$

$= \min\{(\bar{\mu}_1 \times \bar{\mu}_2 \times \cdots \times \bar{\mu}_n)(x_1, x_2, \cdots, x_n), (\bar{\mu}_1 \times \bar{\mu}_2 \times \cdots \times \bar{\mu}_n)(y_1, y_2, \cdots, y_n)\}$

$= \min\{\mu_i(x), \mu_i(y)\}$

$\omega_i(x - y) = \omega_i((x_1, x_2, \cdots, x_n) - (y_1, y_2, \cdots, y_n))$

$= \omega_i(x_1 - y_1, x_2 - y_2, \cdots, x_n - y_n)$

$= \max\{\omega_1(x_1 - y_1), \omega_2(x_2 - y_2), \cdots, \omega_n(x_n - y_n)\}$

$\leq \max\{\max\omega_1\{(x_1), \omega_1(y_1)\}, \max\{\omega_2(x_2), \omega_2(y_2)\}, \cdots, \max\{\omega_n(x_n), \omega_n(y_n)\}\}$

$= \max\{\max\{\omega_1(x_1), \omega_2(x_2), \cdots, \omega_n(x_n)\}, \max\{\omega_1(y_1), \omega_2(y_2), \cdots, \omega_n(y_n)\}\}$

$= \max\{(\omega_1 \times \omega_2 \times \cdots \times \omega_n)(x_1, x_2, \cdots, x_n), (\omega_1 \times \omega_2 \times \cdots \times \omega_n)(y_1, y_2, \cdots, y_n)\}$

$= \min\{\omega_i(x), \omega_i(y)\}$

And

$\bar{\mu}_i(xyz) = \bar{\mu}_i((x_1, x_2, \ldots, x_n)(y_1, y_2, \cdots, y_n)(z_1, z_2, \cdots, z_n))$

$= \bar{\mu}_i(x_1y_1z_1, x_2y_2z_2, \cdots, x_ny_nz_n)$

$\bar{\mu}_i(xyz) = \min\{\bar{\mu}_1(y_1z_1), \bar{\mu}_2(y_2z_2), \cdots, \bar{\mu}_n(y_nz_n)\}$

$\geq \min\{\min\{\bar{\mu}_1(y_1), \bar{\mu}_1(z_1)\}, \min\{\bar{\mu}_2(y_2), \bar{\mu}_2(z_2)\}, \cdots, \min\{\bar{\mu}_n(y_n), \bar{\mu}_n(z_n)\}\}$

$= \min\{\min\{\bar{\mu}_1(y_1), \bar{\mu}_2(y_2), \cdots, \bar{\mu}_n(y_n)\}, \min\{\bar{\mu}_1(z_1), \bar{\mu}_2(z_2), \cdots, \bar{\mu}_n(z_n)\}\}$

$= \min\{(\bar{\mu}_1 \times \bar{\mu}_2 \times \cdots \times \bar{\mu}_n)(y_1, y_2, \cdots, y_n), (\bar{\mu}_1 \times \bar{\mu}_2 \times \cdots \times \bar{\mu}_n)(z_1, z_2, \cdots, z_n)\}$

$= \min\{\bar{\mu}_i(y), \bar{\mu}_i(z)\}$

$\omega_i(xyz) = \omega_i((x_1, x_2, \ldots, x_n)(y_1, y_2, \cdots, y_n)(z_1, z_2, \cdots, z_n))$

$= \omega_i(x_ny_1z_1, x_ny_2z_2, \cdots, x_ny_nz_n)$

$= \max\{\omega_1(y_1z_1), \omega_2(y_2z_2), \cdots, \omega_n(y_nz_n)\}$

$\leq \max\{\max\{\omega_1(y_1), \omega_1(z_1)\}, \max\{\omega_2(y_2), \omega_2(z_2)\}, \cdots, \max\{\omega_n(y_n), \omega_n(z_n)\}\}$

$= \max\{\max\{\omega_1(y_1), \omega_2(y_2), \cdots, \omega_n(y_n)\}, \max\{\omega_1(z_1), \omega_2(z_2), \cdots, \omega_n(z_n)\}\}$

$= \max\{(\omega_1 \times \omega_2 \times \cdots \times \omega_n)(y_1, y_2, \cdots, y_n), (\omega_1 \times \omega_2 \times \cdots \times \omega_n)(z_1, z_2, \cdots, z_n)\}$

$= \max\{\omega_i(y), \omega_i(z)\}$

4. Homomorphism of cubic strong bi-ideals in near subtraction semigroups

Definition 4.1. [5] Let X and Y be near subtraction semigroup. A map $\theta : X \to Y$ is called (near subtraction semigroup) homomorphism if θ (x−y) = θ (x)− θ (y) and θ (xy) = θ (x) θ (y) for all x, y ∈ X.

Definition 4.2. Let f be a mapping from a set X to Y and A = ($\bar{\mu}$, λ)_ be a cubic set of X then the image of X $C_f(\mathcal{A}) = \prec C_f(\bar{\mu}), C_f(\lambda) \succ$ is a cubic set of Y defined by

$$C_f(A)(y) = \begin{cases} C_f(\bar{\mu})(y) = \begin{cases} \sup\limits_{f(x)=y} \bar{\mu}(x) & \text{if } f^{-1}(y) \neq \theta \\ [0,0] & \text{otherwise} \end{cases} \\ C_f(\lambda)(y) = \begin{cases} \inf\limits_{f(x)=y} \lambda(x) & \text{if } f^{-1}(y) \neq \theta \\ 1 & \text{otherwise} \end{cases} \end{cases}$$

and let f be a mapping from a set X to Y and $C_f(\mathcal{A}) = (\bar{\mu}, \lambda)$ is a cubic set of Y , then the preimage of Y $C_f^{-1}(\mathcal{A}) = \prec C_f^{-1}(\bar{\mu}), C_f^{-1}(\lambda) \succ$ is a cubic X is defined by

$$C_f(A)(y) = \begin{cases} C_f^{-1}(\bar{\mu})(x) = \bar{\mu}(f(x)) \\ C_f^{-1}(\lambda)(x) = \lambda(f(x)) \end{cases}$$

Theorem 4.3. *Let* f : X → X_1 *be a homomorphism between two near subtraction semigroup* X *and* X_1. *If* $\mathcal{A} = \prec \bar{\mu}, \omega \succ$ *is a cubic strong bi-ideal of* X_1 *,then* $C_f^{-1}(\mathcal{A}) = \prec C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) \succ$ *is a cubic strong bi-ideal of* X.

Proof. Let $\mathcal{A} = \prec \bar{\mu}, \omega \succ$ be a cubic strong bi-ideal of X_1 .
Let x, y, z ∈ X. Then $C_f(x), C_f(y), C_f(z) \in X_1$, we have $\bar{\mu}$ is an interval-valued fuzzy strong bi-ideal of X_1.

$$\begin{aligned} C_f^{-1}(\bar{\mu})(x-y) &= \bar{\mu}(f(x-y)) \\ &= \bar{\mu}(f(x)-f(y)) \\ &\geq \min\{\bar{\mu}(f(x)), \bar{\mu}(f(y))\} \\ &= \min\{C_f^{-1}(\bar{\mu})(x), C_f^{-1}(\bar{\mu})(y)\} \end{aligned}$$

And ω is a fuzzy strong bi-ideal of X_1.

$$\begin{aligned} C_f^{-1}(\omega)(x-y) &= \omega(f(x-y)) \\ &= \omega(f(x)-f(y)) \end{aligned}$$

$$\leq \max\{\omega(f(x)), \omega(f(y))\}$$
$$= \max\{C_f^{-1}(\omega)(x), C_f^{-1}(\omega)(y)\}$$

$C_f^{-1}(\mathcal{A}) = \prec C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) \succ$ is a cubic subalgebra of X. Again

$$C_f^{-1}(\bar{\mu})(xyz) = \bar{\mu}(f(xyz))$$

$$= \bar{\mu}(f(x)f(y)f(z))$$

$$C_f^{-1}(\bar{\mu})(xyz) \geq \min\{\bar{\mu}(f(y)), \bar{\mu}(z))\}$$

$$= \min\{C_f^{-1}(\bar{\mu})(y), C_f^{-1}(\bar{\mu})(z)\}$$

and

$$C_f^{-1}(\omega)(xyz) = \omega(f(xyz)$$

$$= \omega(f(x)f(y)f(z))$$

$$C_f^{-1}(\omega)(xyz) \leq \max\{\omega(f(y)), \omega(z))\}$$

$$= \max\{C_f^{-1}(\omega)(y), C_f^{-1}(\omega)(z)\}$$

Hence $C_f^{-1}(\mathcal{A}) = \prec C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) \succ$ is a cubic strong bi-ideal of X.

Remark 4.4. We can also state the converse of the theorem by strengthening the condition of f as follows:

Theorem 4.5. *Let* $f : X \to X_1$ be a homomorphism between two near subtraction semigroup X *and* X_1. *Let* $\mathcal{A} = \prec \bar{\mu}, \omega \succ$ *is a cubic subset of* X_1. *If* $C_f^{-1}(\mathcal{A}) = \prec C_f^{-1}(\bar{\mu}), C_f^{-1}(\omega) \succ$ *is a cubic strong bi-ideal of* X, *then* $\mathcal{A} = \prec \bar{\mu}, \omega \succ$ *is a cubic strong bi-ideal of* X_1.

Proof. Let $\mathcal{A} = \prec \bar{\mu}, \omega \succ$ is a cubic subset of X_1 and x, y, z $\in X_1$. Then f(a) = x, f(b) = y, f(c) = z for some a, b, c $\in$ X. It follows that $\bar{\mu}$ is an interval-valued fuzzy strong bi-ideal of X_1.

$$\bar{\mu}(x - y) = \bar{\mu}(f(a) - f(b))$$
$$= \bar{\mu}(f(a - b))$$
$$= (C_f^{-1}(\bar{\mu}))(a\text{-}b)$$

$$\bar{\mu}(x - y) \geq \min\{C_f^{-1}(\bar{\mu})(a), C_f^{-1}(\bar{\mu})(b)\}$$
$$= \min\{(\bar{\mu})f(a)), (\bar{\mu})f(b))\}$$
$$= \min\{\bar{\mu}(x), \bar{\mu}(y)\}$$

And

$$\omega (x - y) = \omega (f(a) - f(b))$$
$$= \omega (f(a - b))$$
$$= (C_f^{-1} (\omega)) (a\text{-}b)$$

$$\omega (x - y) \leq \max\{C_f^{-1} (\omega)(a), C_f^{-1} (\omega)(b)\}$$
$$= \max\{(\omega)f(a)), (\omega)f(b))\}$$
$$= \max\{ \omega (x), \omega (y)\}$$

Again

$$\bar{\mu} (xyz) = \bar{\mu} (f(a)f(b)f(c))$$
$$= \bar{\mu} (f(abc))$$
$$= (C_f^{-1} (\bar{\mu}))(abc)$$
$$\geq \min\{C_f^{-1} (\bar{\mu})(b), C_f^{-1} (\bar{\mu})(c)\}$$
$$= \min\{ \bar{\mu}(f(b)), \bar{\mu}(f(c))\}$$
$$= \min\{ \bar{\mu}(y), \bar{\mu}(z)\}$$

And

$$\omega(xyz) = \omega (f(a)f(b)f(c))$$
$$= \omega (f(abc))$$
$$= (C_f^{-1} (\omega))(abc)$$
$$\leq \max\{C_f^{-1} (\omega)(b), C_f^{-1} (\omega)(c)\}$$
$$= \max\{ \omega (f(b)), \omega (f(c))\}$$
$$= \max\{ \omega (y), \omega (z)\}$$

Hence $\mathcal{A} = \prec \bar{\mu}, \omega \succ$ is a cubic bi − ideal of X_1

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