Some fixed point theorems in compact 2-Metric spaces

Rajesh Shrivastava, Swati Khare*
Professor & Head, Department of Mathematics, Govt. Dr. Shyma Prasad Mukharjee Science & Commerce College Bhopal
*Research Scholar(Mathematics)Govt. Dr. Shyma Prasad Mukharjee Science & Commerce College Bhopal

Abstract : In the present paper some fixed point theorems are obtained for compact -2 metric spaces, which are generalizations of well-known results

Keywords: Fixed point, compact 2-metric spaces

I. INTRODUCTION

There are several genralizations of classical contraction mapping theroem of Banach.In 1961 Edelstein [5] established the existence of a unique fixed point of a self map T of a compact metric space satisfying the inequality d(T(x), T(y)) <d (x, y) which is genralization of Banach. In the past few years a number of authers worked on this spaces.Applying the same concept and using the basic definition of 2-metric spaces some results are obtained in compact 2- metric space

Before starting the main results first we are giving some basic concepts.

Definition (2.1): A class {Gi} of open subset of X is said to be an open cover of X, if each point in X belongs to one Gi that is U$_i$ Gi = X. A subclass of an open cover which has at least an open cover is called a sub cover.

Definition (2.2) A compact space is that space in which every open cover has finite sub cover.

THEOREM (2. 3): In 1961 Edelstein [5] established the existence of a unique fixed point of a self map T of a compact metric space satisfying the inequality

d (T(x), T(y)) <d (x, y), which is genralization of Banach

2.4 Main Results:

THEOREM (6.4. 1): Let T be a continuous mapping of a compact 2-metric space X into it self satisfying the conditions:

(6.4.1a)

$$d\left(T(x),T(y),a\right)\prec \alpha\,\frac{d\left(x,T(x),a\right)d\left(y,T(x),a\right)+d(y,T(y),a)}{d\left(x,T(x),a\right)+d\left(y,T(x),a\right)+d\left(y,T(y),a\right)d(x,T(y),a)}$$

$$+\gamma\,\frac{d\left(x,T(x),a\right)d\left(x,T(y),a\right)+d(y,T(x),a)\,d(y,T(y),a)}{d\left(x,T(x),a\right)+d\left(x,T(y),a\right)+d\left(y,T(x),a\right)+d\left(y,T(y)\right),a\right)}$$

$$+\delta\, d\left(x,y,a\right)$$

for all x, y∈X, x≠y and α+γ+2δ ≤2, whereα,γ, δ are non negative real then T has a unique fixed point.

PROOF: First we define a function F on X as follows:

F(x) =d(x, T(x),a), for all x∈X. Since d and T are continuous on X, F is also continuous on X. From compactness of X, there exists a point P∈X, such that

F (P) =inf {F(x): x∈X} ----------- (2.4.1b)

If F (P) ≠ 0, it follows that P ≠T (P)

And so F (T (P)) =d (T (P),T2 (P),a)

$$d\left(T(p),T\left(T(p)\right),a\right)\prec$$

$$\alpha\,\frac{d\left(p,T(p),a\right)d\left(T(p),T(p),a\right)+d\left(T(p),T\left(T(p),a\right)d\left(p,T\left(T(p)\right)\right),a\right)}{d\left(p,T(p),a\right)+d\left(T(p),T(p),a\right)+d\left(T(p),T\left(T(p),a\right)+d\left(p,T\left(T(p)\right)\right),a\right)}$$

$$+\ \delta\, d\left(p,T(p),a\right)$$

$$+\gamma\,\frac{d\left(p,T(p),a\right)d\left((p),T(T(p)),a\right)+d\left(T(p),T\left(T(p)\right),a\right)d\left(T(p),T(p),a\right)}{d\left(p,T(p),a\right)+d\left((p),T(T(p)),a\right)+d\left(T(p),T\left(T(p)\right),a\right)+d\left(T(p),TT(p),a\right)}$$

so d(T(P), T2(P),a) < α/2 d(T(P), T2(P),a) +(γ/2+δ) d(P, T(P),a)

That is

d(T(P), T2(P), a) [1- α/2] < (γ/2+δ) d(P, T(P),a)

d (T (P), T2(P),a) < (γ/2+δ)/[1- α/2] d(P, T(P),a)

That is

T (T (P)) < S F (P) where S= (γ/2+δ)/[1- α/2] ≤1

Because α+γ+2δ ≤2

This is a contradiction to the condition (24.1b)

And hence P=T (P) consequently, P is a fixed point of T.

Uniqueness: Now we shall prove the uniqueness of P. Let if possible Q≠P be another fixed point of T.

Now d (P,Q) = d(T(P), T(Q))

$$d(T(P),T(Q),a) \prec \alpha \frac{d(P,T(P),a)d(Q,T(P),a)+d(Q,T(Q),a)d(P,T(Q),a)}{d(P,T(P),a)+d(Q,T(P),a)+d(Q,T(Q),a)+d(P,T(Q),a)}$$

$$+\gamma \frac{d(P,T(P),a)d(P,T(Q),a)+d(Q,T(P),a)d(Q,T(Q),a)}{d(P,T(P),a)+d(P,T(Q),a)+d(Q,T(P),a)d(Q,T(Q),a)}$$

$$+\delta d(P,Q,a)$$

That is d (P, Q) < (δ) d (P, Q)

This is a contradiction because η < 2

Hence P is unique fixed point of F.

REMARK:

If we put δ = 1, α = γ = 0, then we get the result of Edelstein [5] for compact metric form.

Now we are taking another type rational expression for compact 2-metric spaces.

THEOREM (2.4.2): Let T be a continous mapping of a compact 2-metric space X into itself setisfying the conditions ;

$$d(T(x),T(y),a) \prec$$

$$\alpha\left[\frac{d(x,T(y),a)d(y,T(x),a)d(x,T(x),a)+d(y,T(y),a)+d(x,y,a)}{1+d(x,T(y),a)d(y,T(x),a)d(x,T(x),a)d(y,T(y),a)+d(y,T(x),a)d(x,y,a)}\right]$$

$$+\delta d(x,y,a)$$

$$\textit{for all } x,y\,\varepsilon\, X, x\neq y \textit{ where } \alpha,\delta \textit{ are non negative reals such that } 2\alpha+\delta \prec 1,$$

$$\textit{Then } T \textit{ has unique fixed point.}$$

THEOREM (2.4.3): Let T be a continous mapping of a compact 2-metric space X into itself setisfying the conditions ;

$$d(T(x),T(y),a) \prec \gamma\left[\frac{d(x,T(y),a)d(y,T(x),a)d(y,T(y),a)+d(x,T(x),a)+d(x,y,a)}{1+d(x,T(y),a)d(y,T(x),a)d(y,T(y),a)d(x,T(x),a)+d(y,T(x),a)d(x,y,a)}\right]$$

$$+\delta d(x,y,a)$$

$$\textit{for all } x,y\,\varepsilon\, X, x\neq y \textit{ where } \gamma,\delta \textit{ are non negative reals such that } 2\gamma+\delta \prec 1$$

$$\textit{Then } T \textit{ has unique fixed point.}$$

THEOREM (2.4.4): Let T be a continuous mapping of a compact 2- metric space to itself satisfying the conditions:

$$d(T(x),T(y),a) \prec$$

$$\alpha\left[\frac{d(x,T(y),a)d(y,T(x),a)d(x,T(x),a)+d(y,T(y),a)+d(x,y,a)}{1+d(x,T(y),a)d(y,T(x),a)d(x,T(x),a)d(y,T(y),a)+d(y,T(x),a)d(x,y,a)}\right]$$

$$+\gamma\left[d(y,T(x),a)+d(y,T(y),a)\right]+\delta\, d(x,y,a),$$

$$\textit{for all } \alpha,\gamma,\delta \textit{ are non negative real such that } 2\alpha+\gamma+\delta \prec 1, \textit{then } T \textit{ has fixed point}$$

Note: Above results can be proved for integral type mappings also.

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