THE LOCAL R-DOMINATION NUMBER AND ITS PARAMETER

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Abstract- Let G=(V,E) be a simple ,connected and undirected graph .Let r be a positive integer .A subset S of V(G) is said to be a r-dominating set if for any vertex uεV-S,there is a vεS such that d(u,v)≤r.The minimum cardinality of all r-dominating set of G is called r-domination number of G and its cardinality is denoted by γl_r (G) .We define γl_r (G)=max{γ_r(<N_r(x)>):xεV}.The number γl_r is the local r-domination number of G within a distance r.Also we define γ_l(G)=min{ γl_r (G):1≤r≤diam(G)}.The number γ_l(G) is called the local r-domination number of the graph G.

Keywords:**r-domination number ,r-dominating set,local r-dominating set,local r-domination number.**

I.INTRODUCTION

By a graph mean a finite ,simple ,connected and undirected graph G(V,E),where V denotes its vertex set and E its edge set.Unless otherwise stated ,the graph G has p vertices and q edges.Degree of a vertex v is denoted by d(v),the maximum degree of a graph G is denoted by Δ(G).We denote a cycle on p vertices by $C_{p,}$ a path on P vertices by P_p,and a complete graph on P vertices by K_p.A graph G is connected if any two vertices of G are connected by a path.A maximal connected subgraph of a graph G is called a component of G.The number of component of G is denoted by$\omega(G)$.The component $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G.A tree is connected is acyclic graph.A bipartite (or bigraph)is a graph whose vertices can be divided into two disjoint sets U and V such that every edge connects a vertex in U to one in V.A complete bipartite graph is a special kind of bipartite graph where every vertex of the first set is connected to every vertex of the second set .The complete bipartite graph with partitions of order $|V_1|$=m and $|V_2|$=n,is denoted by $K_{m,n}$.A star ,denoted by $K_{1,p-1}$ is a tree with one root vertex and p-1 pendent vertices.A bistar ,denoted by B(m,n)is the graph obtained by joining the root vertices of the stars $K_{1,m}$ and $K_{1,n}$.

A subset S of V is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S.The domination number $\gamma(G)$of G is the minimum cardinality taken over all dominating sets in G.A dominating set S of a connected graph G is said to be a connected dominating set of G if the induced subgraph <S> is connected.The minimum cardinality taken over all connected dominating set is the connected dominating number and is denoted by γ_c.

Let G =(V,E)be a simple undirected graph .Let Δ(G) =max{deg(u):uεV(G)} for x,yεV.Let δ (G)=min{deg(u):uεV(G)} for x,yεV.The distance d(x,y)is the length of a shortest path in G.The diameter of G is the maximum distance between any two vertices of Gand is denoted by diam(G).

Let r be a positive integer .Let $N_r(x)=\{y \in V: d(x,y) \leq r\}$ denote the open neighbourhood of x$\in V$ and its closed neighbourhood is $N_r[x]=N_r(x)\cup\{x\}$.The r-degree $deg_r(u)$ of u inG is given by $|Nr(u)|$,while Δ_r (G) and δ_r(G) denote the maximum and minimum r- degree among all the vertices of G respectively.

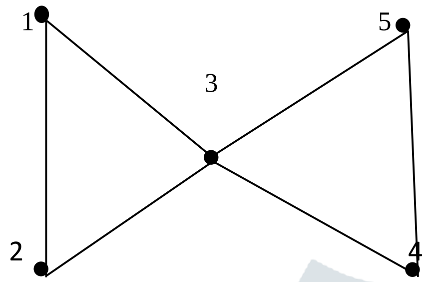
Δ_r(G)=max{deg_r(u):u∈V(G)} and δ_r(G)=min{deg_r(u):u∈V(G)}.A subset S of V(G) is said to be a r-dominating set if for any vertex u∈V-S, there is a vertex v∈S such that uv∈ $E(G)$.The minimum cardinality of all dominating set is called the r-domination number of G and is denoted by γ_r(G).

DEFINITION

The concept of r-domination was introduced in [1,2].A subset S of V(G)is called a r-dominating set if every vertex u∈V-S,there is some vertex v in S which are at a distance ≤r .

The minimum cardinality of r-dominating set of G is called as r-domination number of G and is denoted by δ_r(G).We define γ_{lr}(G)=max{γ_r<N_r(x)>:x∈ V}where N_r(x)={y∈ V:d(x,y)≤ r}.
The number γ_{lr}(G) is called the local r-domination number of a graph G within a distance r.
It is clear that local r-domination number is defined for any graph except totally disconnected graph.

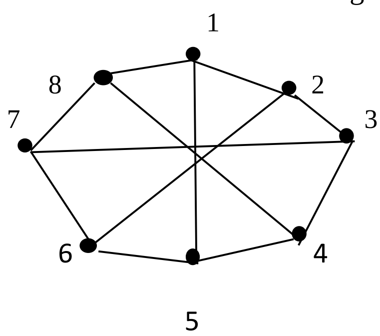
***EXAMPLE*:**consider the following Butterfly graph.



r=1	r=2	r=3
$N_1(1)=\{2,3\}$	$N_2(1)=\{2,3,4,5\}$	$N_3(1)=\{2,3,4,5\}$
$\gamma_1(<N_1(1)>)=1$	$\gamma_2(<N_2(1)>)=1$	$\gamma_3(<N_3(1)>)=1$
$N_1(2)=\{3,1\}$	$N_2(2)=\{1,3,5,4\}$	$N_3(2)=\{1,3,4,5\}$
$\gamma_1(<N_1(2)>)=1$	$\gamma_2(<N_2(2)>)=1$	$\gamma_3(<N_3(2)>)=1$
$N_1(3)=\{1,2,4,5\}$.	$N_2(3)=\{5,4,1,2\}$	$N_3(3)=\{1,2,4,5\}$
$\gamma_1(<N_1(3)>)=2$	$\gamma_2(<N_2(3)>)=2$	$\gamma_3(<N_3(3)>)=2$
$N_1(4)=\{3,5\}$	$N_2(4)=\{3,1,5,2\}$	$N_3(4)=\{1,3,2,5\}$
$\gamma_1(<N_1(4)>)=1$	$\gamma_2(<N_2(4)>)=1$	$\gamma_3(<N_3(4)>)=1$
$N_1(5)=\{3,4\}$	$N_2(5)=\{3,2,4,1\}$	$N_3(5)=\{1,2,3,4\}$
$\gamma_1(<N_1(5)>)=1$	$\gamma_2(<N_2(5)>)=2$	$\gamma_3(<N_3(5)>)=1$
$\gamma_{l1}(G)=2$	$\gamma_{l2}(G)=2$	$\gamma_{l3}(G)=1$

Therefore $\gamma_{lr}(G)=\begin{cases} 2 & if \quad r = 1,2 \\ 1 & if \quad r = 3 \end{cases}$

EXAMPLE:Consider the Wagner graph.



r =1	r=2
$N_1(1)=\{2,5,8\}$	$N_2(1)=\{8,7,2,3,5,6,4\}$
$\gamma_1(<N_1(1)>)=3$	$\gamma_2(<N_2(1)>)=1$
$N_1(2)=\{1,3,6\}$	$N_2(2)=\{1,8,3,4,5,6,7\}$
$\gamma_1(<N_1(2)>)=3$	$\gamma_2(<N_2(2)>)=1$
$N_1(3)=\{2,4,7\}$	$N_2(3)=\{1,2,4,5,7,6,8\}$
$\gamma_1(<N_1(3)>)=3$	$\gamma_2(<N_2(3)>)=1$
$N_1(4)=\{3,5,8\}$	$N_2(4)=\{2,3,5,6,8,7,1\}$
$\gamma_1(<N_1(4)>)=3$	$\gamma_2(<N_2(4)>)=1$
$N_1(5)=\{4,6,1\}$	$N_2(5)=\{3,4,6,7,1,2,8\}$
$\gamma_1(<N_1(5)>)=3$	$\gamma_2(<N_2(5)>)=1$
$N_1(6)=\{2,5,7\}$	$N_2(6)=\{4,5,7,8,2,1,3\}$
$\gamma_1(<N_1(6)>)=3$	$\gamma_2(<N_2(6)>)=1$
$N_1(7)=\{3,6,8\}$	$N_2(7)=\{5,6,1,8,2,3,4\}$
$\gamma_1(<N_1(7)>)=3$	$\gamma_2(<N_2(7)>)=1$
$N_1(8)=\{1,7,4\}$	$N_2(8)=\{1,2,6,7,3,4,5\}$
$\gamma_1(<N_1(8)>)=3$	$\gamma_2(<N_2(8)>)=1$
$\gamma_{l1}(G)=3$	$\gamma_{l2}(G)=1$

In wagner graph,

$$\gamma_{lr}(G)= \begin{cases} 3 \text{ if } r = 1 \\ 1 \text{ if } r = 2 \end{cases}$$

DEFINITION

A spanning tree is a subset of a graph G,which has all the vertices covered with minimum possible number of edges .

THEOREM1

Let G be a simple graph and H be a spanning tree with n≥ 3 and if $1 \leq r \leq diam(G)$ then $\gamma_{lr}(G) \leq \gamma_{lr}(H)$.

Proof

Let G be a simple graph and H be a spanning tree.
Spanning tree H has all the vertices of G.

∴ $\gamma_{lr}(G) \leq \gamma_{lr}(H)$.

RESULT

For any r-regular graph $\gamma_{lr}(G)=1$ if $1 \leq r \leq diam(G)$.

THEOREM2

Let G be a Hamiltonian graph with n≥ 3 then $1 \leq \gamma_{lr}(G) \leq 2$ if $1 \leq r \leq diam(G)$.

Proof

Let G be a Hamiltonian graph and n≥ 3 .
The path visit each vertex at exactly once.
Let r≥ 1 and D be the local r dominating set of G.
Therefore D is the r-dominating set of G .
Since, it has Hamiltonian cycle.
Therefore $\gamma_{lr}(G)$ lies between 1 and 2.

THEOREM 3

If G is an Eulerian graph with n≥ 3 then $1 \leq \gamma_{lr}(G) \leq 2$ when $1 \leq r \leq n$.

Proof

Let G be an Eulerian graph .
Therefore G has an Eulerian cycle
∴Starting and ending points are same and the trail visits the edge exactly once.
∴ The local r domination number lies between 1 and 2.

RESULT AND DISCUSSION :

In this paper ,we have found out the local r-domination number for some standard graphs and special types of graphs.some interesting result and theorems are derived.It would be interesting to check the existence of the local r-domination number for some special types of graphs like banana trees,gracefultrees,honeycomb etc.

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