SOLVING FULLY FUZZY ASSIGNMENT PROBLEM USING BRANCH AND BOUND TECHNIQUE

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ABSTRACT

In this paper, we define fuzzy set, fuzzy assignment problem is various classifications of Basic definitions. This paper is organized as follows In section1 we adopt some usual terminology, notations and conventions which will be used later in the section we establish the Branch and Bound Techniques . The fully fuzzy assignment problems is using basic notations of fuzzy numbers and branch and bound techniques have also been presented.

Key words: Fuzzy set, fuzzy Number , Triangular fuzzy number, Basic notations of Assignment problem. .

1.1 INTRODUCTION

The main objective of Assignment problem which is also a special type of linear programming problem is to obtain the optimum assignment for number of tasks (jobs), that is equal to the number of resources (workers) at a minimum cost or maximum profit. This plays an vital role in assigning persons to jobs,classes to rooms, operators to machines etc. An optimal solution to assignment problems, can be achieved using various algorithm such as linear programming, Hungarian algorithm, Neural networks genetic algorithm,etc. In 1965, Zadeh, has introduced fuzzy sets which provided a new mathematical tool to deal with uncertainty of information.

1.2 PRELIMINARIES

1.2.1 Fuzzy set

A fuzzy set is characterized by a membership function mapping the elements of a domain, space, or universe of discourse x to the unit interval [o,1],

$$A=\{(x,\mu_A(x):x\in X\},$$

where $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy is a mapping called the degree of membership function of the fuzzy set A. and $\mu_A(x)$ is called the membership value of $x\in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1].

A which maps *x* to [0, 1].

1.2.2 DEFINITION FUZZY NUMBER

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \to [0,1]$ has the following characteristics

i) A is normal. It means that there exists an $x\in R$ such that $\mu_A(x)=1$

ii) A is convex. It means that for every $x_1, x_2 \in R,$

iii) $\mu_A\left(\lambda x_1+(1-\lambda)x_2\right)\geq \min\left\{\mu_A(x_1),\mu_A(x_2)\right\},\ x\in[0,1]$

iv) μ_A is upper semi – continues

v) Suppose A is bounded in R.

Arithmetic Operation of Triangular fuzzy number

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two fuzzy triangular fuzzy numbers. then

1. $(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2. $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3) \quad for \ \ k \geq 0$
3. $k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1) \quad for \ \ k < 0$
4. $(a_1, a_2, a_3) \otimes (b_1, b_2, b_3) = \begin{cases} (a_1b_1, a_2b_2, a_3b_3) & for \ a_1 \geq 0 \\ (a_1b_1, a_1b_1, a_1b_1) & for \ a_1 < 0, a_3 \geq 0 \\ (a_1b_3, a_2b_2, a_3b_1) & for \ a_3 < 0 \end{cases}$

1.3 MATHEMATICAL FORMULATION

1.3.1 The general assignment problem

Suppose there are 'n' people and 'n' jobs. Each job must be done by exactly one person; also each person can do, at most, one job. The problem is to assign jobs to the people so as to minimize the total cost of completing all of the jobs. The general assignment problem can be mathematically stated as follows:

$$Minimize \ \ Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1 \ , \qquad for \qquad j = 1, 2....n.$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad for \qquad i = 1, 2...n$$

$$where \ \ x_{ij} = 0 \ \ or \ \ 1 \ .$$

2.4 Fuzzy Assignment Problem

$$Minimize \ \ Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1 \ , \qquad for \qquad j = 1, 2....n.$$

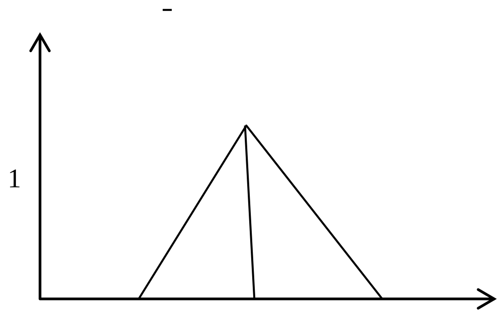
$$\sum_{j=1}^{n} x_{ij} = 1, \qquad for \qquad i = 1, 2...n$$

$$where \ \ x_{ij} = 0 \ \ or \ \ 1 \ .$$

1.4 Triangular fuzzy number

A fuzzy number A is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its membership function $\mu_A(x)$ is given below.

$$\mu_A(x) = \begin{cases} \dfrac{(x - a_1)}{(a_2 - a_1)} & if \quad a_1 \leq x \leq a_2 \\ 1 \ \ if \ \ x = a_2 & \\ \dfrac{(a_3 - x)}{(a_3 - a_2)} & if \quad a_2 \leq x \leq a_3 \\ 0 & if \quad otherwise \end{cases}$$



Triangular fuzzy number $A = (a_1, a_2, a_3)$

1. 5. Branching methodology

1. At level k, the column marked as k of the assignment problem, will be assigned with the best row of the assignment problem.
2. If there are tie on the upper bound, then the terminal node at the upper most level is to be considered for further branching.
3. If the maximum upper bound happens to be at any one of the terminal nodes at the (n -1)th level, the optimality is reached. Then the assignments on the path from the root node to that node along with the missing pair of row-column combination will form the optimum solution.

1.6 Numerical Example:

Let us consider a fuzzy assignment problem with rows representing four Machines and column representing four Jobs

Machines/ Job	Job 1	Job 2	Job 3	Job 4
Machine 1	(1,3, 6)	(8,12, 16)	(20,22, 24)	(34,38, 40)
Machine 2	(8,12,16)	(2,3,5)	(23,26,28)	(27,30, 32)
Machine 3	(13,15,17)	(34,38,40)	(2, 3, 5)	(1, 3, 6)
Machine 4	(27,30,32)	(8,12,16)	(19,22,24)	(19,22,24)

Find an optimal assignment of machines to jobs that will maximize the total profit.(profit in rupees).

Solution:

Consider the fully fuzzy assignment problem and we solve it by branch and bound techniques to get the optimal solution.

Branching

The four different sub problems under the root nodes and upper bounds are shown in fig.1.

$$V_{\in} = \sum_{i,j\in X} c_{i,j} + \sum_{i\in X} \sum_{j\in Y} \max c_{i,j}$$

Where ε =(1,1) A=(1,1) X=(2,3,4) Y=(2,3,4)

$$V_{11} = c_{11} + \sum_{i\in(2,3,4)} \sum_{j\in(2,3,4)} \max c_{i,j}$$

Machines/ Job	Job 1	Job 2	Job 3	Job 4
Machine 1	~~(1,3, 6)~~	~~(8,12, 16)~~	~~(20,22, 24)~~	~~(34,38, 40)~~
Machine 2	~~(8,12,16)~~	(2,3,5)	(23,26,28)	(27,30, 32)
Machine 3	~~(13,15,17)~~	(34,38,40)	(2, 3, 5)	(1, 3, 6)
Machine 4	~~(27,30,32)~~	(8,12,16)	(19,22,24)	(19,22,24)

$$P_{11}^{\ 1} = c_{11} + \{\max C_{22} + \max C_{33} + \max C_{44}\}$$
$$= (1,3,6) + \{(27,30,32) + (34,38,40) + (19,22,24)\}$$

Repeated Value has a maximum value

$$P_{11}^{\ 1} = (81,93,102)$$

Machines/ Job	Job 1	Job 2	Job 3	Job 4
Machine 1	~~(1,3, 6)~~	(8,12, 16)	(20,22, 24)	(34,38, 40)
Machine 2	~~(8,12,16)~~	~~(2,3,5)~~	~~(23,26,28)~~	~~(27,30, 32)~~
Machine 3	~~(13,15,17)~~	(34,38,40)	(2, 3, 5)	(1, 3, 6)
Machine 4	~~(27,30,32)~~	(8,12,16)	(19,22,24)	(19,22,24)

$$P_{21}^{\ 1} = c_{21} + \{\max C_{33} + \max C_{44} + \max C_{11}\}$$
$$= (8,12,16) + \{(34,38,40) + (19,22,24) + (34,38,40)\}$$

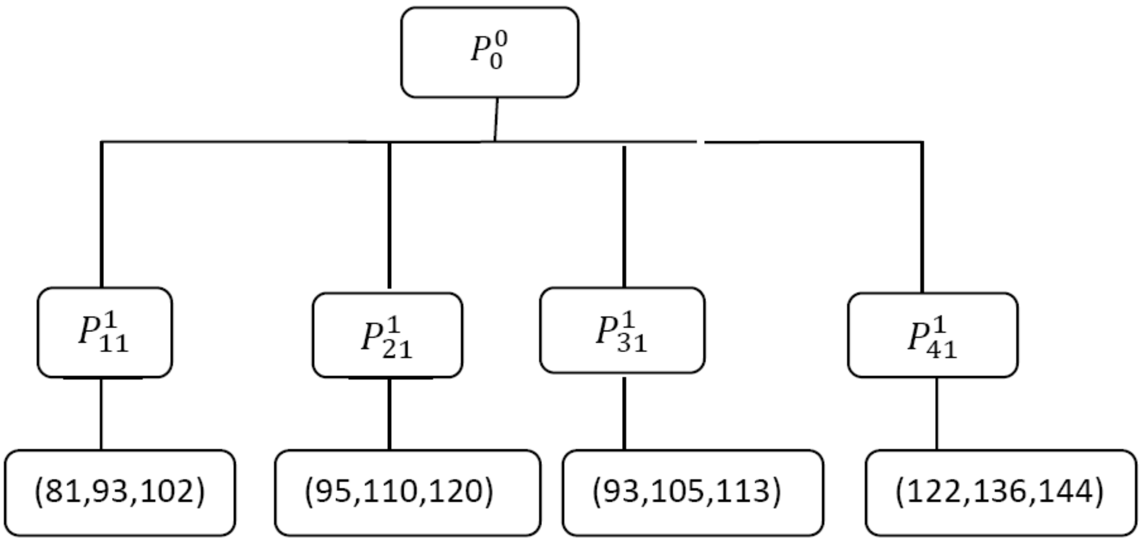
$$P_{21}^{\ 1} = (95,110,120)$$

Machines/ Job	Job 1	Job 2	Job 3	Job 4
Machine 1	~~(1,3, 6)~~	(8,12, 16)	(20,22, 24)	(34,38, 40)
Machine 2	~~(8,12,16)~~	(2,3,5)	(23,26,28)	(27,30, 32)
Machine 3	~~(13,15,17)~~	~~(34,38,40)~~	~~(2, 3, 5)~~	~~(1, 3, 6)~~
Machine 4	~~(27,30,32)~~	(8,12,16)	(19,22,24)	(19,22,24)

$$P_{31}^{\ 1} = c_{31} + \{\max C_{44} + \max C_{11} + \max C_{22}\}$$
$$= (13,15,17) + \{(19,22,24) + (34,38,40) + (27,30,32)\}$$
$$P_{31}^{\ 1} = (93,105,113)$$

Machines/ Job	Job 1	Job 2	Job 3	Job 4
Machine 1	~~(1,3, 6)~~	(8,12, 16)	(20,22, 24)	(34,38, 40)
Machine 2	~~(8,12,16)~~	(2,3,5)	(23,26,28)	(27,30, 32)
Machine 3	~~(13,15,17)~~	(34,38,40)	(2, 3, 5)	(1, 3, 6)
Machine 4	~~(27,30,32)~~	~~(8,12,16)~~	~~(19,22,24)~~	~~(19,22,24)~~

$$P_{41}^{\ 1} = c_{41} + \{\max C_{11} + \max C_{22} + \max C_{33}\}$$
$$= (27,30,32) + \{(34,38,40) + (27,30,32) + (34,38,40)\}$$
$$P_{41}^{\ 1} = (122,136,144)$$



Further branching

Further branching is done from the terminal node which has the greatest upper bound. At this stage, the nodes $P_{11}^{\ 1}, P_{21}^{\ 1}, P_{31}^{\ 1}, P_{41}^{\ 1}$ are the terminal nodes. The node $P_{41}^{\ 1}$ has the greatest upper bound. Eliminate fourth row and first column. Hence further branching from this node is shown as follows.

Machines/ Job	Job 2	Job 3	Job 4
Machine 1	(8,12, 16)	(20,22, 24)	(34,38, 40)
Machine 2	(2,3,5)	(23,26,28)	(27,30, 32)
Machine 3	(34,38,40)	(2, 3, 5)	(1, 3, 6)

$$V_{22} = c_{41} + c_{i2} \sum_{i\in(3,4)} \sum_{j\in(3,4)} \max c_{i,j}$$

Machines/ Job	Job 2	Job 3	Job 4
Machine 1	~~(8,12, 16)~~	~~(20,22, 24)~~	~~(34,38, 40)~~
Machine 2	~~(2,3,5)~~	(23,26,28)	(27,30, 32)
Machine 3	~~(34,38,40)~~	(2, 3, 5)	(1, 3, 6)

$$P_{12}^{\ 2} = c_{41} + c_{12} + \{\max C_{22} + \max C_{33}\}$$

$$=(27,30,32)+\{(8,12,16)+(27,30,32)+(2,3,5)\}$$

$$P_{12}{}^{2}=(64,75,85)$$

Machines/ Job	Job 2	Job 3	Job 4
Machine 1	~~(8,12, 16)~~	(20,22, 24)	(34,38, 40)
Machine 2	~~(2,3,5)~~	~~(23,26,28)~~	~~(27,30, 32)~~
Machine 3	~~(34,38,40)~~	(2, 3, 5)	(1, 3, 6)

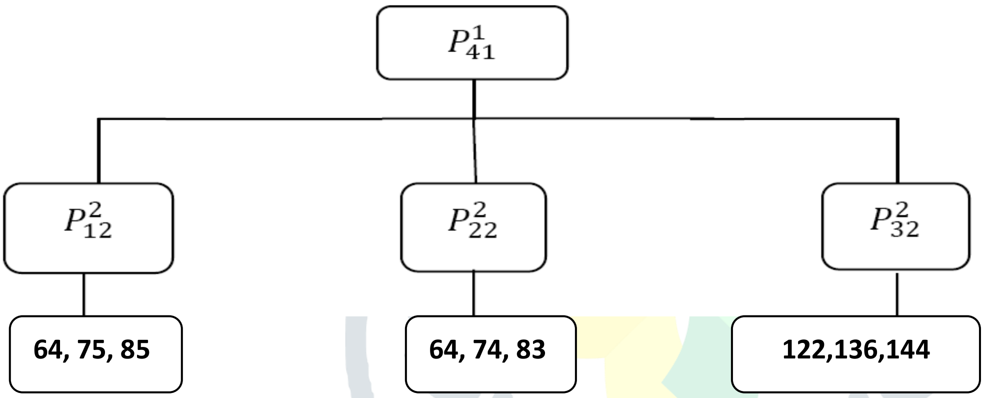
$$P_{22}{}^{2}=c_{41}+c_{22}+\{\max C_{33}+\max C_{11}\}$$
$$=(27,30,32)+(2,3,5)+\{(1,3,6)+(34,38,40)\}$$

$$P_{22}{}^{2}=(64,74,83)$$

Machines/ Job	Job 2	Job 3	Job 4
Machine 1	~~(8,12, 16)~~	(20,22, 24)	(34,38, 40)
Machine 2	~~(2,3,5)~~	(23,26,28)	(27,30, 32)
Machine 3	~~(34,38,40)~~	~~(2, 3, 5)~~	~~(1, 3, 6)~~

$$P_{32}{}^{2}=c_{41}+c_{32}+\{\max C_{11}+\max C_{22}\}$$
$$=(27,30,32)+(34,38,40)+\{(34,38,40)+(27,30,32)\}$$

$$P_{32}{}^{2}=(122,136,144)$$



Further branching

Further branching is done from the terminal node which has the greatest upper bound. At this stage, the nodes $P_{41}{}^{1}, P_{12}{}^{2}, P_{22}{}^{2}, P_{32}{}^{2}$ are the terminal nodes. The node $P411$ has the greatest upper bound. Eliminate third row and second column.

$$V_{33}=c_{41}+c_{32}+c_{i3}\sum_{i\in(4)}\sum_{j\in(4)}\max c_{i,j}$$

Machines/ Job	Job 3	Job 4
Machine 1	(20,22, 24)	(34,38, 40)
Machine 2	(23,26,28)	(27,30, 32)

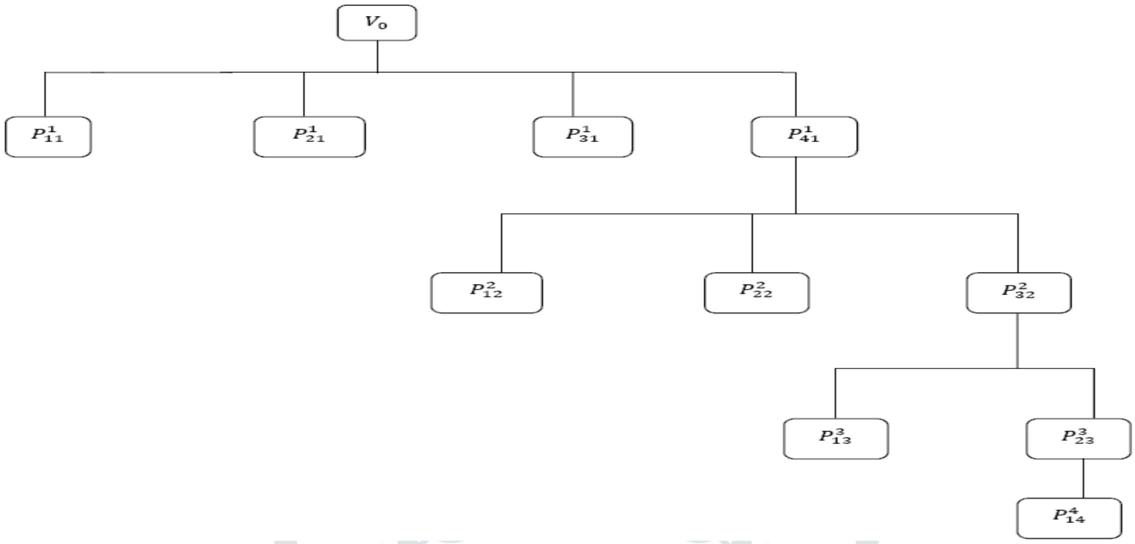
Machines/ Job	Job 3	Job 4
Machine 1	~~(20,22, 24)~~	~~(34,38, 40)~~
Machine 2	~~(23,26,28)~~	(27,30, 32)

$$P_{13}{}^{3}=c_{41}+c_{32}+c_{13}+\{\max C_{22}\}$$
$$=(27,30,32)+(34,38,40)+(20,22,24)+\{(27,30,32)\}$$

$$P_{13}{}^{3}=(108,120,128)$$

Machines/ Job	Job 3	Job 4
Machine 1	~~(20,22, 24)~~	(34,38, 40)
Machine 2	~~(23,26,28)~~	~~(27,30, 32)~~

$$P_{23}^{\ 3} = c_{41} + c_{32} + c_{23} + \{\max C_{11}\}$$
$$= (27,30,32) + (34,38,40) + (23,26,28) + \{(34,38,40)\}$$
$$P_{23}^{\ 3} = (118,132,140)$$
$$P_{14}^{\ 4} = c_{41} + c_{32} + c_{23} + c_{41}$$
$$= (27,30,32) + (34,38,40) + (23,26,28) + (34,38,40)$$
$$P_{14}^{\ 4} = (118,132,140)$$



The optimal assignment

Person 4 → Job1, Person 3 → Job 2, Person 2 → Job 3, Person1 → Job 4.

The optimal cost = (27,30,32) (34,38,40)+(23,26,28)+(34,38,40)
= (118,132,140)

CONCLUSION

In this paper we obtain fuzzy assignment problem was Branch and Bound Method is introduced. This method is effectives and easy to understand because of its natural similarity to classical method of solving optimal assignment problems. The method of branch and bound technique is shows in this paper guarantees the correctness and effectiveness of the working produce of the method. The solutions of optimal assignment problems. This method of branch and bound is also easy to apply and understand.

REFERENCES

1. **Anil, D. Gomare., Khot, P.G,** 2016,"Solution of fuzzy assignment problem by using Branch and Bound Technique with application of linguistic variable," *International Journal of Computers and Technology*. 15(4), pp.6649-6653.
2. **Anchal Choudhary.,Jat.R.N., Sharma.S.C.,Sanjay Jain**.,"A New algorithm for solving fuzzy assignment problem using Branch and Bound Method",International Journal of Mathematical Archive,(2016),vl.7,no.3,pp.5-11.
3. **Bai. X.J. Liu Y.K. and Shen. S.Y.** Fuzzy generalized assignment problem with credibility constraints, Proceedings of the Eighth International Conference on Machine Learning and Cybernetics, Baoding, (2009) 657-662.
4. **Chi-Jen Lin, Ue-Pyng Wen**, A Labelling algorithm for the fuzzy assignment problem, fuzzy Sets and system 142 (2004), 373-391.
5. **Chen M.S**, On a fuzzy assignment problem, *Tamkang J*, 22 (1985), 407-411.
6. **Elias Munapo**.,"A Transportation branch and bound algorithm for solving the generalized assignment problem",6th International Conference on Applied Operational Research, Proceedings,(2014),vol.6,pp.150-158.
7. **Fortemps P and.Roubens M** "Ranking and defuzzification methods based area compensation" fuzzy sets and systems Vol.82,(1996),PP 319-330,
8. **Kalaiarasi. K , Sindhu.S, and Arunadevi.M**, Optimization of fuzzy assignment model with triangular fuzzy numbers IJISET Vol. 1 ISSN, 3, (2014). 2348 - 7968
9. **Nagarajan.R and solairaju.A,** A computing improved fuzzy optimal Hungarian. Assignment problem with fuzzy cost under Robust ranking technique, internal journal of computer application volume .6, No.4, (2010). pp 6-13
10. **Srinivas.B.,Ganesan.G**.,"Method for solving Branch and Bound Techniques for assignment problems using Triangular and Trapezoidal fuzzy numbers", International Journal in Management and social Science.,(2015), vol.3, no.3.167-176.
11. **Terry Ross.G.,Richard M.Soland**.,"A Branch and Bound Algorithm for the generalized assignment problem",Mathematical programming., (1975), 8, pp.91-103.
12. **Wang.X,** Fuzzy optimal assignment problem, *Fuzzy Mathematics*, 3 (1987), 101-108
13. **Yager R.R**, " A procedure for ordering fuzzy subsets of the unit interval, Information Sciences, 24,(1981), 143- 161.
14. **Zadeh L.A.,** Fuzzy sets, *Information and Control*, 8 (1965), 338-353