EFFECTIVENESS OF NURSE SCHEDULING USING INTEGER PROGRAMMING PROBLEM

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Abstract : This paper focuses on the construction of efficient nurse schedule methods based on the integrations of Constraint Programming (CP) and Integer Programming (IP) to tackle combinatorial optimisation problems from the nurse rostering problems. The properties of the problems are investigated firstly and the problems are decomposed accordingly in certain ways; then the suitable solution techniques are integrated to solve the problem based on the properties of MS Excel technique.

Keywords: Nurse scheduling problem, Integer programming, Shift Schedule.

I. INTRODUCTION

Nurse Scheduling is convoluted combinatorial optimization problem with increasing healthcare costs, and a shortage of trained staff it is becoming increasingly important for hospital management to make good operational decisions. A major element of hospital expenditure is staff cost. In order to help Hospitals to make decisions about staffing and work scheduling, a simulation model was created to scrutinize the impact of alternate work schedules and investigate the optimum balance between the staffing levels of the ward and the ability to achieve worthy quality schedules.

The Nursing Scheduling Problems (NSPs) are defined as the identification of that staffing pattern which specifies the number of nursing personnel of each skill class to be scheduled among the nursing shifts of a scheduling period, satisfies total nursing personnel capacity, integral assignment, and other relevant constraints, and minimizes a "shortage cost" of nursing care services provided for the scheduling period. Two different communities have conducted research on appropriate models and solution methods for these problems. The Operations Research community mainly used integer programming techniques, in particular branch-and-bound methods.

Nurse scheduling has received considerable attention in their search community during the last sixty years. It is a highly complex planning problem and as such usually decomposed into the interrelated sub-problems of nurse staffing, nurse rostering, and nurse rerostering. Nurse staffing is the long term planning that determines the required number of nurses of different qualification levels. Nurse rostering can be described as the task of finding a duty roster for a set of nurses such that the rosters observe with work regulations and meet the management's requirement. The day to day planning where nurses that call in sick and other day to day changes have to be considered is referred to as nurse rerostering. The objective of rerostering is to create a duty roster for all nurses such that all required shifts are still covered and the new rosters are as close to the pre-planned rosters as possible. The main aim of this study is to evaluate the nurse rostering and posits viable solutions through which the rostering is systematised.

1. Definitions of nurse scheduling

The strategic nurse scheduling is simply referred to as nurse scheduling, but 'nurse shift scheduling' by Chen & Yeung, or 'nursing staff scheduling' by Okada& Okada, 1988 are also used. Nurse scheduling is the procedure for providing nursing care by assigning shifts to nursing personnel. To be more specific, nurse scheduling is the process of determining when each nurse of a nursing unit will be on or off duty, which shift will be worked, by whom, and how weekends, the number of consecutive days worked, requests, and vacations are taken into by Bailey.J.

2. Integer Programming Techniques for the Nurse Rostering Problem

A significant amount of research has been devoted to the computational solution of the Nurse Rostering Problem. However, the specific case studies, focusing in attentions of certain institutions. For these works, comparison between different search strategies is a very difficult task. An instance set was proposed and a significant number of different algorithms have been empirically evaluated using it. As a result, best known solutions have been updated since then. In this work we present a Integer Programming formulation, i.e. a formulation with a polynomial number of constraints and variables.

The nurse rostering problem can be described by a nurse-day view, a nurse task view, or a nurse-shift pattern view. In the nurse-day view, allocations are indexed for each nurse and each day. This way, a solution can be directly represented by a matrix where each cell $m_{i,j}$ contains a set of shifts to be performed by the day$i$ in the nurse j. Broadly speaking this set may haveany number of shifts, but the International Nurse Rostering Competition problem and most practical cases a nurse performs only one shift per day - which may include morning shift (M), evening shift (E), night shift (N), day-off (-), among others. The below table presents part of a weekly roster which indicates the shifts allocated to the nurses, in a nurse-day view.

Example of an NRP solution in a nurse-day view

Day/Nurse	N_1	N_2	N_3
Monday	*M*	*E*	-
Tuesday	*M*	*N*	*E*
Wednesday	*N*	*E*	*M*
Thursday	-	*E*	-
Friday	-	*M*	*M*

Saturday	*E*	-	-
Sunday	*E*	-	*N*

In the nurse-task view, the decision variable is indexed for each nurse and each task that the nurse performs in the scheduling period. This decision variable may assume a value of 1 if the nurse is assigned to the task or 0 otherwise. In the nurse-shift pattern view, the decision variable is indexed for each nurse and each pattern of shifts available. Cheang et al. presents a bibliographic survey of the many models and methodologies available to solve the (Nurse Rostering problem) NRP.
In this work, we address the problem defined in the first International Nurse Rostering Competition, sponsored by the leading conference in the Automated Timetabling domain. Competitors were allowed to submit a specific technique for each instance type. Here follow brief descriptions of approaches that had succeeded in the competition. Valouxis et al. , winners of the challenge, developed a two phase strategy where in the first phase the workload for each nurse and for each day of the week was decided while in the second phase the specific daily shifts were assigned.
Since the competition imposed quality and time constraint requirements, they partitioned the problem instances into sub-problems of manageable computational size which were then solved sequentially using Integer Mathematical Programming. Also, they applied local optimization techniques for searching across combinations of nurses' partial schedules. This sequence was repeated several times depending on the available computational time.
Burke and Curtois apply an ejection chain based method for sprint instances and a branch and price algorithm for medium and long instances. Problem instances have been converted to the general staff rostering model proposed and documented by the same team. Then, their software Roster Booster which has included the above mentioned algorithmic approaches is used. An IP formulation is first used to solve the sub problem which includes the full set of hard constraints and a subset of soft constrains.

3. An Integer Programming Formulation for the Problem

In this, we present an Integer Programming formulation which successfully models all constraints considered in instances of the International Nurse Rostering Competition.

4.1 Input Data

N	set of nurses
C	set of contracts
$\widetilde{c_n}$	contract of nurse n
S	set of shifts
$\tilde{S}$	set of night shifts
D	set of days with elements sequentially numbered from 1
π	set of all ordered pairs $(d_1, d_2) \in D \times D: d_1 \leq d_2$representing windows in the planning horizon
$\widetilde{W_c}$	set of weekends in the planning horizon according to the weekend definition of contract c, with elements numbered from 1 to $\widetilde{W_c}$
$\widetilde{D}_{ic}$	set of days in the i^{th} weekend of contract *c*
$\tilde{r}_{ds}$	number of required nurses at day *d* and shift *S*
$\grave{P}_c$	set of unwanted working shift patterns for contract *C*
$\hat{P}_c$	set of unwanted working days patterns for contract *C*

4.2 Decision variables

The main decision variables are the three indexed x_{nsd} binary variables:

$$n \rightarrow nurse, s \rightarrow Shift, d \rightarrow day$$

$$x_{nsd} = \begin{cases} 1 & if\ nurse\ \boldsymbol{n} is\ allocated\ to\ shift\ \boldsymbol{s} and\ day\ \boldsymbol{d} \\ 0 & otherwise \end{cases}$$

additionally, there are the following auxiliary variables:

$$y_{nsd} = \begin{cases} 1 & if\ nurse\ n\ works\ at\ weekend\ i \\ 0 & otherwise \end{cases}$$

$$w_{nd_1d_2} = \begin{cases} 1 & if\ nurse\ n\ works\ from\ day\ d_1\ until\ day\ d_2 \\ 0 & otherwise \end{cases}$$

$$r_{nd_1d_2} = \begin{cases} 1 & if\ nurse\ n\ rest\ from\ day\ d_1\ until\ day\ d_2 \\ 0 & otherwise \end{cases}$$

$$z_{ni_1i_2} = \begin{cases} 1 & if\ nurse\ n\ works\ from\ weekend\ i_1\ until\ weekend\ i_2 \\ 0 & otherwise \end{cases}$$

To simplify the statement of constraints we consider additional variables y_{n0} which are always fixed to zero.

4.3 Objective Function

The objective function we remark that some slack variables (and their respective constraints) do not need to be explicitly included. This is the case of constraints which are directly linked to the selection of a specific working/resting window from the set $\pi$by activating variables $w_{nd_1d_2}$and$r_{nd_1d_2}$, respectively. The every activation of$w_{nd_1d_2}$finishing /starting in the middle of a weekend must be penalized.
We denote by $\sigma_{nd_1d_2}$ andτ_{cd_2} the weighted penalty of all violations incurred fromworking (resting) continuously in a block starting at day *d₁*and finishing at day d_2 for nurses of contract *c*, respectively. Soft constraints are also directly penalized in x_{nsd}variables with coefficientsv_{nsd}. Analogously,soft constraint below is penalized in variables$z_{ni_1i_2}$with coefficients $\varphi_{ni_1i_2}$

$$Minimize: \sum_{n \in N} \Big[\sum_{(d_1,d_2) \in \pi} (\sigma_{\tilde{c}nd_1d_2} w_{nd_1d_2} + \tau_{\tilde{c}nd_1d_2} \tau_{nd_1d_2}] + \sum_{s \in S} \sum_{d \in D} v_{nsd} x_{nsd} + \overline{w}^1_{\tilde{c}_n} \overline{\alpha}^1_n \underline{w}^1_{\tilde{c}_n} \underline{\alpha}^1_n \sum_{i \in \{1 \dots \widetilde{w}_c\}} (w^4_{\tilde{c}_n} \alpha^4_{ni} + \underline{w}^8_{\tilde{c}_n} \underline{\alpha}^8_{ni} + \underline{w}^9_{\tilde{c}_n} \underline{\alpha}^9_{ni}) + \sum_{i_1 i_2 \in \widetilde{W}_{\tilde{c}_n} : i_1 \leq i_2} \varphi_{ni_1i_2} z_{ni_1i_2} + \sum_{d \in D} \underline{w}^6_{\tilde{c}_n} \underline{\alpha}^6_{ni} + \sum_{\grave{p} \in \grave{P}_{\tilde{c}n}} \alpha^{11}_{n\grave{P}} + \sum_{\grave{p} \in \grave{P}_{\tilde{c}n}} \alpha^{12}_{n\grave{P}} \Big]$$

4.4 Constraints

Constraints are presented in the following.

$$\sum_{n\in N} x_{nsd} = \tilde{r}_{ds} \,\forall\, d \in D, s \in S$$

$$\sum_{s\in S} x_{nsd} \le 1 \forall\, n \in N, dD$$

The two hard constraints are to provide sufficient coverage of nurses for every day and shift and to limit working shifts for nurses to a maximum of one per day.

$$y_{ni} \ge \sum_{s\in S} x_{nsd} \,\forall\, n \in N, i \in \widetilde{w}_{\tilde{c}_n}, d \in \widetilde{D}_{i\tilde{c}_n}$$

$$y_{ni} \le \sum_{s\in S, d\in \widetilde{D}_{i\tilde{c}_n}} x_{nsd} \,\forall\, n \in N, i \in \widetilde{w}_{\tilde{c}_n}$$

Are the constraints link the activation of variables x with the activation of y variables which indicate working weekends.

4. Management view

Managers are sometimes hesitant to adopt 12-hour shifts because they believe alertness and productivity decline after 8 hours of work. The more you read on the subject, the more confused you get. Some researchers have concluded that the longer shifts were detrimental to productivity and safety; others say that things got better after workers changed to 12-hour shifts.

We believe that the differing conclusions stem from comparing dissimilar situations. Researchers may have compared one group on an 8-hour fixed shift schedule with another group on a 12-hour rotating shift schedule. They didn't factor in different schedule variables, such as the on-off work pattern, the amount of overtime, shift start times, and so on. They also didn't consider worker characteristics or the work environment. Problems they attribute to working longer shifts most likely are the result of these other factors in combination with the shift length. Based our experience with organizations that have adopted longer shifts, we have not found problems as long as the schedule is well designed, with sufficient time off and reasonable limits on the number of consecutive days worked.

Our survey shows that over a period of time, shift workers on 12-hour shifts actually get more sleep than those on 8-hour shifts. They may get less sleep on the days they work the 12-hour shifts, but they get twice as many days off. Since everyone sleeps longer on their days off, the average sleep with 12-shift is greater than the average sleep with 8-shift.

Average Hours of Sleep over a Four-Week Period:
8-Hour vs. 12-Hour Shifts

Schedule	Shift Length	Days of Work	Hours of Sleep	Days off	Hours of Sleep	Average
Days only	8 hours	21	6.5	7	7.5	6.8
	12 hours	14	6.3	14	7.5	6.9
Nights only	8 hours	21	6.1	7	7.5	6.5
	12 hours	14	6.2	14	7.5	6.9
Rotating shifts	8 hours	21	6.5	7	7.5	6.7
	12 hours	14	6.3	14	7.5	6.9

5. Solution of the IPP

Since nurses usually work an eight-hour shift, it may be possible to schedule operators' working hours so that a single shift covers two or more "peak periods" of demand. By devising intelligent schedules, the productivity of the operators is increased, resulting in a smaller staff and, thus, a reduction in payroll costs. Other examples in which employee scheduling models have proved useful include bus drivers, air traffic controllers, and nurses. We now give an example problem and develop an integer programming model for scheduling nurses' working hours. Hospitals routinely face the problem of scheduling nurses' working hours. A scheduling model is an integer programming problem of minimizing the total number of workers subject to the specified number of nurses during each period of the day.

Since each nurse works eight hours, he/she can start working at the beginning of any one of the first five shifts: 8:00, 10:00, 12:00, 2:00 or 4:00. In this application, we do not consider any shift starting at 9:00, 11:00, etc. Also, there is no need to have any nurse begin working after 4:00, since his/her shift would then run past midnight when no nurses are needed. Each period is two hours long, so each nurse who reports for work in period t will also work during periods eight and 12 consecutive hours.

During time period 1, at least 10 agents must be on duty, so we must have

$$X_1 \ge 10.$$

Similarly, the requirements in time period 2 can only be met by

$$X_1 + X_2 \ge 8.$$

In this manner, we write the requirements for the remaining periods. These are:

$$X_1 + X_2 + X_3 \ge 9,$$

$$X_1 + X_2 + X_3 + X_4 \ge 11,$$

$$X_2 + X_3 + X_4 + X_5 \geq 13,$$
$$X_3 + X_4 + X_5 \geq 8,$$
$$X_4 + X_5 \geq 5,$$
$$X_5 \geq 3,$$
$$X_1, X_2, X_3, X_4, X_5 \text{ are all integers.}$$

Notice that X_1 is not included in the constraint for time period 5, since workers beginning in period 1 are no longer on the job by time period 5. Also, observe that it may be necessary to have more than the required number of agents working in some periods.

6.1 Solution by using Spreadsheet

There are of course numerous software packages which are dedicated to solving linear programs (and other types of mathematical program), of which possibly LINDO, GAMS and XPRESS-MP are the most popular. All these packages tend to be DOS based and are intended for a specialist market which requires tools dedicated to solving LP and IPPs . In recent years, however, several standard business packages, such as spreadsheets, have started to include an LP and IPP solving option, and Microsoft Excel is no exception.

Solution Table for 8 Hour Shift

K8 23

INPUT DATA

NUMBER OF NURSES (8hrs shift)

Period	Objective	X1	X2	X3	X4	X5	Totals		Limits
		1	1	1	1	1	23		
1	8.00 - 10.00	1	0	0	0	0	10	>=	10
2	10.00-12.00	1	1	0	0	0	15	>=	8
3	12.00 - 2.00	1	1	1	0	0	18	>=	9
4	2.00 - 4.00	1	1	1	1	0	18	>=	11
5	4.00- 6.00	0	1	1	1	1	13	>=	13
6	6.00 - 8.00	0	0	1	1	1	8	>=	8
7	8.00 - 10.00	0	0	0	1	1	5	>=	5
8	10.00 -12.00	0	0	0	0	1	5	>=	3

OUTPUT RESULT

	X1	X2	X3	X4	X5	Z
Solution	10	5	3	0	5	23

Total nurses required = 23

Solution Table for12 Hour Shift

N1

NURSE SCHEDULLING (12 hour shift)

INPUT DATA

NUMBER OF NURSES

Period	Objective	X1	X2	X3	X4	X5	Totals		Limits
		1	1	1	1	1	14		
1	8.00 - 10.00	1	0	0	1	1	10	>=	10
2	10.00-12.00	1	1	0	0	1	14	>=	8
3	12.00 - 2.00	1	1	1	0	0	11	>=	9
4	2.00 - 4.00	1	1	1	1	0	11	>=	11
5	4.00- 6.00	1	1	1	1	1	14	>=	13
6	6.00 - 8.00	1	1	1	1	1	14	>=	8
7	8.00 - 10.00	0	1	1	1	1	7	>=	5
8	10.00 -12.00	0	0	1	1	1	3	>=	3

OUTPUT RESULT

	X1	X2	X3	X4	X5	Z
Solution	7	4	0	0	3	14

Total nurses required = 14

In conclusion, Excel Solver provides a simple, yet effective, medium for allowing users to explore Integer linear programs. It can be used for large problems containing hundreds of variables and constraints, and does these relatively quickly, but as a teaching tool using small illustrative problems it is very potent, the structure of a ILP when entering it into the spreadsheet.

This is particularly noticeable as the model formulation is easily accessible at the same time as the model results, these being simply placed in adjacent worksheets, accessible with a simple mouse click. Overall, using Excel, provides a rich location for thinking Integer linear programming and it explore their models in a structured, however adaptable way.

6. Conclusion

The conclusion to be drawn from this paper is that the linear and Integer programming model allows for simple implementation in mathematical programming program easily. Due to the formulation of the problem, the linear nature allows for the application of solvers which are extremely efficient at solving such problems. Given that the nurse scheduling problem is considered by many to be hard problem, the ability to solve it using a linear and integer mathematical solver makes it easier to implement an automated approach. In this regard, the model makes it a practical computerized tool. The model is also generalized to a degree where it is possible to extend it to incorporate various new constraints and preferences.

In all, with no doubt, nurse scheduling process is playing important role inhealthcare institutions around the world. Hence, they are few decades of studies in hospitals, which applying various techniques or algorithms to develop effective nurse schedule which from exploring optimization methods to search methods. Thus far, hybrid search techniques are the most liked approaches for nurse scheduling arena.

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