# CONTRA PRE*GENERALIZED CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES 

M. Jeyachitra ${ }^{1}$, K. Bageerathi ${ }^{2}$<br>${ }^{1}$ Department of Mathematics, Sri Muthukumaran Arts and Science College, Chennai.<br>Research Scholar in Manonmaniam Sundaranar University, Tirunelveli,<br>${ }^{2}$ Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur, Tamilnadu, India.


#### Abstract

The purpose of this paper is to introduce various functions associated with $\mathrm{p} * \mathrm{~g}$-open sets. Here contra $\mathrm{p} * \mathrm{~g}$-continuous and contra $\mathrm{p}^{* g}$-irresolute functions are defined. Characterizations for these functions are given. Many other functions associated with $\mathrm{p} * \mathrm{~g}$-open sets and their contra versions are introduced and their properties are studied. In addition contra strongly $\mathrm{p} * \mathrm{~g}$ continuous functions are discussed. We also study the relationships among themselves. Further, we discuss about their composition functions with other existing functions. Some interesting properties are investigated in addition to giving some examples.


Keywords: $\mathrm{p}^{* g}$-continuous, contra $\mathrm{p} * \mathrm{~g}$-continuous, contra $\mathrm{p} * \mathrm{~g}$-irresolute functions.
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## 1. INTRODUCTION

Levine [11] introduced the class of generalized closed sets in topological spaces. Contra continuous and gsp-continuous functions have been introduced and investigated by Dontchev [2, 3]. Also the contra pre semi continuous function was given by Veerakumar in 2005 [16]. Z. Li and W. Zhu [12] studied contra continuity on generalized topological spaces in 2013. Mashhour [13], Arokiarani, I. et. al. [1], Gnanambal, Y., and Balachandran, K [6], Patil, P. G. Rayanagoudar, T. D. and Mahesh, K. Bhat [14] put forward the concepts of pre open sets and pre continuity, gp-irresolute and gp-continuous maps, gpr-continuous and $\mathrm{g}^{*} \mathrm{p}$-continuous functions in topological spaces respectively.

The authors $[7,8,9,10]$ brings out the $\mathrm{p}^{*} \mathrm{~g}$-closed sets, $\mathrm{p} * \mathrm{~g}$-open sets, $\mathrm{p} * \mathrm{~g}$-continuous functions, $\mathrm{p}^{* g}$-irresolute, strongly $\mathrm{p}^{*} \mathrm{~g}$-continuous function and perfectly $\mathrm{p}^{*} \mathrm{~g}$-continuous functions in topological spaces and established their relationships with some generalized sets in topological spaces. In this paper, our aim is to introduce the concept of contra $\mathrm{p}^{*} \mathrm{~g}$-continuous in topological spaces and investigate their basic properties. Also we discuss their relationship with already existing concepts. Together we define the notions of contra $\mathrm{p} * \mathrm{~g}$-irresolute functions and contra strongly $\mathrm{p} * \mathrm{~g}$-continuous functions and study some of their properties.

This paper is arranged as follows, section 2, we recall some notions that will be used throughout this paper. In section 3, we mention some notions in order to present contra $p^{*} g$-continuous functions and investigate its basic properties. Also found out the composition of contra $\mathrm{p}^{*} \mathrm{~g}$-continuous functions with other existing functions in section 3 . Finally, we study contra $\mathrm{p} * \mathrm{~g}$-irresolute and contra strongly p*g-continuous functions. Finally, we gives a brief summary of work done in the last section.

## 2. Preliminaries

Throughout this paper $(\mathrm{X}, \tau),(\mathrm{Y}, \sigma)$ and $(\mathrm{Z}, \eta)$ represents topological spaces on which no separation axioms are assumed unless otherwise mentioned. $(\mathrm{X}, \tau),(\mathrm{Y}, \sigma)$ and $(\mathrm{Z}, \eta)$ will be replaced by $\mathrm{X}, \mathrm{Y}$ and Z if there is no changes of confusion. For a subset A of a topological space $\mathrm{X}, \mathrm{cl}(\mathrm{A}), \operatorname{int}(\mathrm{A})$ and $\mathrm{X} \backslash \mathrm{A}$ denote the closure of A , the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.
Definition 2.1. Let $(X, \tau)$ be a topological space and $A \subseteq X$. Then
i. $\quad \mathrm{A}$ is pre open if $\mathrm{A} \subseteq \operatorname{int}(\operatorname{cl}(\mathrm{A}))$ and pre closed if $\operatorname{cl}(\operatorname{int}(\mathrm{A})) \subseteq \mathrm{A}$ [13].
ii. $\quad \mathrm{A}$ is pre*open if $\mathrm{A} \subseteq \operatorname{int} *(\operatorname{cl}(\mathrm{~A}))$ and pre*closed if $\operatorname{cl} *(\operatorname{int}(\mathrm{~A})) \subseteq \mathrm{A}[15]$.

Definition 2.2. [11] Let $(X, \tau)$ be a topological space. A subset $A$ of $X$ is said to be generalized closed (briefly g-closed) if $\mathrm{cl}(\mathrm{A}) \subseteq \mathrm{U}$ whenever $\mathrm{A} \subseteq \mathrm{U}$ and U is an open in $(\mathrm{X}, \tau)$.

Definition 2.3. [13] Let $(X, \tau)$ be a topological space and $A \subseteq X$. The pre closure of $A$ denoted by $p c l(A)$ and is defined by the intersection of all pre closed sets containing A.

Definition 2.4. Let $(X, \tau)$ be a topological space and $A \subseteq X$. The generalized closure of $A[4]$, denoted by $c l^{*}(A)$ and is defined by the intersection of all g-closed sets containing A and generalized interior of A [5], denoted by int*(A) and is defined by union of all g-open sets contained in A.

Definition 2.5. [7] A subset A of a topological space ( $X, \tau$ ) is called pre*generalized closed (briefly p*g-closed) if pcl(A) $\subseteq U$ whenever $A \subseteq U$ and $U$ is pre*open in ( $X, \tau$ ). The collection of all $\mathrm{p} * \mathrm{~g}$-closed sets of X is denoted by $\mathrm{p} * \mathrm{~g}-\mathrm{C}(\mathrm{X})$.

Results 2.6. From [7] Let (X, $\tau$ ) be a topological space. Then
i. Every closed set is p *g-closed.
ii. Every $\mathrm{p}^{*}$ g-closed set is gp-closed, gpr-closed, wg-closed, rwg-closed, $\pi$ gp-closed, gsp-closed, $\pi$ gsp-closed, pre semi closed, $\mathrm{g}^{*} \mathrm{p}$-closed.

Definition 2.7. [8] A subset A of a topological space ( $X, \tau$ ) is called $p^{*}$ g-open if $X \backslash A$ is $p^{*} g$-closed. Let $p^{*} g-O(X)$ denote the collection of all p *g-open sets in X .

Lemma 2.8. [8] Let (X, $\tau$ ) be a topological space. Then every open set is $p^{*} g$-open.
Definition 2.9. [9] A function $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is called $\mathrm{p}^{*} \mathrm{~g}$-continuous if $f^{-1}(\mathrm{~F})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in $(\mathrm{X}, \tau)$ for every closed set $F$ in (Y, $\sigma$ ).

Definition 2.10. [9] A function $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is called $\mathrm{p} * \mathrm{~g}$-irresolute if $f^{-1}(\mathrm{~F})$ is $\mathrm{p} * \mathrm{~g}$-closed in $(\mathrm{X}, \tau)$ for every $\mathrm{p} * \mathrm{~g}$-closed set $F$ in $(Y, \sigma)$.

Definition 2.11. [10] A function $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is called strongly $\mathrm{p} * \mathrm{~g}$-continuous if $f^{-1}(\mathrm{~F})$ is closed in (X, $\left.\tau\right)$ for every $\mathrm{p}^{*} \mathrm{~g}$-closed set F in $(\mathrm{Y}, \sigma)$.

Definition 2.12. A function $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is called
i. continuous [13] if $f^{-1}(\mathrm{~F})$ is closed in X for every closed set F in Y .
ii. contra continuous [3] if $f^{-1}(\mathrm{~V})$ is closed set in X for each open set V in Y .
iii. contra pre semi continuous [16] if $f^{-1}(\mathrm{~V})$ is pre semi-closed set in X for each open set V in Y .
iv. contra gp-continuous [1] if $f^{-1}(\mathrm{~V})$ is gp-closed set in X for each open set V in Y .
v. contra gpr-continuous [6] if $f^{-1}(\mathrm{~V})$ is gpr-closed set in X for each open set V in Y .
vi. contra gsp-continuous if [2] if $f^{-1}(\mathrm{~V})$ is gsp-closed set in X for each open set V in Y .
vii. contra $\mathrm{g}^{*} \mathrm{p}$-continuous [14] if $f^{-1}(\mathrm{~V})$ is $\mathrm{g}^{*} \mathrm{p}$-closed set in X for each open set V in Y .

## 3. Contra pre*generalized continuous functions

In this section, we introduce the notions of contra p *g-continuous and study their properties.
 V in Y .

The following example supports the above definition.
Example 3.2. Let $\mathrm{X}=\mathrm{Y}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ with topologies $\tau=\{\phi,\{\mathrm{a}\},\{\mathrm{b}\},\{\mathrm{a}, \mathrm{b}\}, \mathrm{X}\}$ and $\sigma=\{\phi,\{\mathrm{c}\},\{\mathrm{b}, \mathrm{c}\}, \mathrm{Y}\}$. Here $\mathrm{p}^{*} \mathrm{~g}-\mathrm{C}(\mathrm{X})=\{\phi,\{\mathrm{c}\},\{\mathrm{b}, \mathrm{c}\},\{\mathrm{a}, \mathrm{c}\}, \mathrm{X}\}$. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{b}, f(\mathrm{~b})=\mathrm{a}, f(\mathrm{c})=\mathrm{c}$. Then $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-continuous.

The following theorems show that the relationship of contra $p^{*} g$-continuous functions with other existing functions.
Theorem 3.3. Every contra continuous function is a contra $p^{*} g$-continuous function.
Proof: Let V be an open set in Y. Since $f$ is a contra continuous function, $f^{-1}(\mathrm{~V})$ is closed in X. By Result 2.6 (i), $f^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . Thus $f^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-closed in X for each V is open in Y . Therefore $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-continuous.

The converse of the above theorem is not true. It is shown in the following example.
Example 3.4. Let $X=Y=\{a, b, c\}$ with topologies $\tau=\{\phi,\{c\},\{b, c\}, X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a, b\}$, $Y\}$. Here $\mathrm{p}^{*} \mathrm{~g}-\mathrm{C}(\mathrm{X})=\{\phi,\{\mathrm{a}\},\{\mathrm{b}\},\{\mathrm{a}, \mathrm{b}\}, \mathrm{X}\}$. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{b}, f(\mathrm{~b})=\mathrm{a}, f(\mathrm{c})=\mathrm{c}$. Then $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-continuous but not contra continuous, since $\{a\}$ is an open set in Y but $f^{-1}(\{a\})=\{b\}$ is not closed in $X$.
Theorem 3.5. Every contra p *g-continuous function is a contra pre semi continuous function.

Proof: Let V be an open set in Y. Since $f$ is a contra $\mathrm{p} * \mathrm{~g}$-continuous function, $f^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-closed in X . By Result 2.6 (ii), $f^{-1}(\mathrm{~V})$ is pre semi closed in X . Thus $f^{-1}(\mathrm{~V})$ is pre semi closed in X for each V is open in Y. Therefore $f$ is contra pre semi continuous.

The following example shows that the converse of the above theorem is not true.
Example 3.6. Let $\mathrm{X}=\mathrm{Y}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ with topologies $\tau=\{\phi,\{\mathrm{a}\},\{\mathrm{a}, \mathrm{b}\}, \mathrm{X}\}$ and $\sigma=\{\phi,\{\mathrm{c}\}$, $\{\mathrm{b}, \mathrm{c}\}$, Y\}. Here $\mathrm{p}^{*} \mathrm{~g}-\mathrm{C}(\mathrm{X})=\{\phi,\{\mathrm{b}\},\{\mathrm{c}\},\{\mathrm{b}, \mathrm{c}\}, \mathrm{X}\}$. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{b}, f(\mathrm{~b})=\mathrm{a}, f(\mathrm{c})=\mathrm{c}$. Then $f$ is contra pre semi continuous but not contra $\mathrm{p}^{*} \mathrm{~g}$-continuous, since $\{\mathrm{b}, \mathrm{c}\}$ is an open set in Y but $f^{-1}(\{\mathrm{~b}, \mathrm{c}\})=\{\mathrm{a}, \mathrm{c}\}$ is not $\mathrm{p} * \mathrm{~g}$-closed in X .

Theorem 3.7. Every contra $\mathrm{p} * \mathrm{~g}$-continuous function is a contra gp-continuous function.
Proof: Let V be an open set in Y. Since $f$ is a contra $\mathrm{p} * \mathrm{~g}$-continuous function, $f^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-closed in X . By Result 2.6 (ii), $f^{-1}(\mathrm{~V})$ is gp-closed in X . Thus $f^{-1}(\mathrm{~V})$ is gp-closed in X for each V is open in Y . Therefore $f$ is contra gp-continuous.

The converse of the above theorem need not be true as shown in the following example.
Example 3.8. Let $\mathrm{X}=\mathrm{Y}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ with topologies $\tau=\{\phi,\{\mathrm{a}\},\{\mathrm{a}, \mathrm{b}\}, \mathrm{X}\}$ and $\sigma=\{\phi,\{\mathrm{c}\}$, $\{\mathrm{b}, \mathrm{c}\}$, Y $\}$. Here $\mathrm{p}^{*} \mathrm{~g}-\mathrm{C}(\mathrm{X})=\{\phi,\{\mathrm{b}\},\{\mathrm{c}\},\{\mathrm{b}, \mathrm{c}\}, \mathrm{X}\}$. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{b}, f(\mathrm{~b})=\mathrm{a}, f(\mathrm{c})=\mathrm{c}$. Then $f$ is contra gp-continuous but not contra $\mathrm{p}^{*} \mathrm{~g}$-continuous, since $\{\mathrm{b}, \mathrm{c}\}$ is an open set in Y but $f^{-1}(\{\mathrm{~b}, \mathrm{c}\})=\{\mathrm{a}, \mathrm{c}\}$ is not $\mathrm{p}^{*} \mathrm{~g}$-closed in X .

Theorem 3.9. Every contra $\mathrm{p} * \mathrm{~g}$-continuous function is a contra gpr-continuous function.
Proof: Let V be an open set in Y. Since $f$ is a contra $\mathrm{p}^{*} \mathrm{~g}$-continuous function, $f^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . By Result 2.6 (ii), $f^{-1}(\mathrm{~V})$ is gpr-closed in X . Thus $f^{-1}(\mathrm{~V})$ is gpr-closed in X for each V is open in Y. Therefore $f$ is contra gpr-continuous.

The converse of the above theorem need not be true as seen from the following example.
Example 3.10. Let $X=Y=\{a, b, c\}$ with topologies $\tau=\{\phi,\{a\},\{a, b\}, X\}$ and $\sigma=\{\phi,\{c\}$, $\{b, c\}$, Y $\}$. Here $\mathrm{p}^{*} \mathrm{~g}-\mathrm{C}(\mathrm{X})=\{\phi,\{\mathrm{b}\},\{\mathrm{c}\},\{\mathrm{b}, \mathrm{c}\}, \mathrm{X}\}$. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{b}, f(\mathrm{~b})=\mathrm{c}, f(\mathrm{c})=\mathrm{a}$. Then $f$ is contra gpr-continuous but not contra $\mathrm{p}^{*} \mathrm{~g}$-continuous, since $\{\mathrm{b}, \mathrm{c}\}$ is an open set in Y but $f^{-1}(\{\mathrm{~b}, \mathrm{c}\})=\{\mathrm{a}, \mathrm{b}\}$ is not $\mathrm{p}^{*} \mathrm{~g}$-closed in X .

Theorem 3.11. Every contra $\mathrm{p}^{*} \mathrm{~g}$-continuous function is a contra $\mathrm{g}^{*} \mathrm{p}$-continuous function.
Proof: Let V be an open set in Y. Since $f$ is a contra $\mathrm{p}^{*} \mathrm{~g}$-continuous function, $f^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X. By Result 2.6 (ii), $f^{-1}(\mathrm{~V})$ is $\mathrm{g}^{*} \mathrm{p}$-closed in X . Thus $f^{-1}(\mathrm{~V})$ is $\mathrm{g}^{*} \mathrm{p}$-closed in X for each V is open in Y . Therefore $f$ is contra $\mathrm{g}^{*} \mathrm{p}$-continuous.

The following example shows that the converse of the above theorem is not true.
Example 3.12. Let $\mathrm{X}=\mathrm{Y}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ with topologies $\tau=\{\phi,\{\mathrm{a}\},\{\mathrm{a}, \mathrm{b}\}, \mathrm{X}\}$ and $\sigma=\{\phi,\{\mathrm{c}\}$, $\{\mathrm{b}, \mathrm{c}\}$, Y$\}$. Here $\mathrm{p}^{*} \mathrm{~g}-\mathrm{C}(\mathrm{X})=\{\phi,\{\mathrm{b}\},\{\mathrm{c}\},\{\mathrm{b}, \mathrm{c}\}, \mathrm{X}\}$. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{b}, f(\mathrm{~b})=\mathrm{a}, f(\mathrm{c})=\mathrm{c}$. Then $f$ is contra $\mathrm{g}^{*} \mathrm{p}$-continuous but not contra $\mathrm{p}^{*} \mathrm{~g}$-continuous, since $\{\mathrm{b}, \mathrm{c}\}$ is an open set in Y but $f^{-1}(\{\mathrm{~b}, \mathrm{c}\})=\{\mathrm{a}, \mathrm{c}\}$ is not $\mathrm{p} * \mathrm{~g}$-closed in X .

Theorem 3.13. Every contra $\mathrm{p} * \mathrm{~g}$-continuous function is a contra gsp-continuous function.
Proof: Let V be an open set in Y. Since $f$ is a contra $\mathrm{p}^{*} \mathrm{~g}$-continuous function, $f^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . By Result 2.6 (ii), $f^{-1}(\mathrm{~V})$ is gsp-closed in X . Thus $f^{-1}(\mathrm{~V})$ is gsp-closed in X for each V is open in Y . Therefore $f$ is contra gsp-continuous.

The converse of the above theorem need not be true as seen from the following example.
Example 3.14. Let $X=Y=\{a, b, c\}$ with topologies $\tau=\{\phi,\{a\},\{b\},\{a, b\}, X\}$ and $\sigma=\{\phi,\{a\},\{a, b\}, Y\}$. Here $\mathrm{p}^{*} \mathrm{~g}-\mathrm{C}(\mathrm{X})=\{\phi,\{\mathrm{b}\},\{\mathrm{c}\},\{\mathrm{b}, \mathrm{c}\}, \mathrm{X}\}$. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{a}, f(\mathrm{~b})=\mathrm{c}, f(\mathrm{c})=\mathrm{b}$. Then $f$ is contra gsp-continuous but not contra $\mathrm{p}^{* g}$-continuous, since $\{\mathrm{a}\}$ is an open set in Y but $f^{-1}(\{\mathrm{a}\})=\{\mathrm{a}\}$ is not $\mathrm{p}^{*}$ g-closed in X .

The above discussions are summarized in the following implications.

## Diagram 3.15.



It is shown that this implication of class of contra $\mathrm{p}^{*} \mathrm{~g}$-continuous properly lies among contra continuous and contra gsp-continuous, contra pre semi continuous, contra gp-continuous, contra gpr-continuous, and contra $\mathrm{g}^{*} \mathrm{p}$-continuous respectively.

The next example shows that the composition of two contra $\mathrm{p}^{*} \mathrm{~g}$-continuous functions need not be a contra $\mathrm{p} * \mathrm{~g}$-continuous functions.

Example 3.16. Let $X=Y=Z=\{a, b, c\}, \tau=\{\phi,\{a\},\{b\},\{a, b\}, X\}, \sigma=\{\phi,\{a, b\}, Y\}$ and $\eta=\{\phi,\{c\},\{a, b\}, Z\}$. Then $(X, \tau)$, $(\mathrm{Y}, \sigma)$ and $(\mathrm{Z}, \eta)$ are topological spaces. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{b}, f(\mathrm{~b})=\mathrm{c}$ and $f(\mathrm{c})=\mathrm{a}$. Define $\mathrm{g}:(\mathrm{Y}, \sigma) \rightarrow(\mathrm{Z}, \eta)$ by $g(a)=c, g(b)=a$ and $g(c)=b$. Then $f$ and $g$ are contra $p^{*} g$-continuous. Now $\{a, b\}$ is open in $(Z, \eta)$. It can be verified that, $(\mathrm{g} \circ f)^{-1}(\{\mathrm{a}, \mathrm{b}\})=f^{-1}\left(\mathrm{~g}^{-1}(\{\mathrm{a}, \mathrm{b}\})\right)=f^{-1}(\{\mathrm{~b}, \mathrm{c}\})=\{\mathrm{a}, \mathrm{b}\}$ which is not $\mathrm{p} * \mathrm{~g}$-closed in $(\mathrm{X}, \tau)$. Hence $\mathrm{g} \circ f$ is not contra $\mathrm{p} * \mathrm{~g}$-continuous.

Now, it is proved that the composition of contra $\mathrm{p} * \mathrm{~g}$-continuous function and continuous function is again a contra $\mathrm{p}^{*} \mathrm{~g}$-continuous function.

Theorem 3.17. If $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is a contra $\mathrm{p}^{*} \mathrm{~g}$-continuous function and $\mathrm{g}:(\mathrm{Y}, \sigma) \rightarrow(\mathrm{Z}, \eta)$ is a continuous function then $\mathrm{g} \circ f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Z}, \eta)$ is contra $\mathrm{p}^{*} \mathrm{~g}$-continuous.

Proof: Let V be an open set in Z. Since g is a continuous function, $\mathrm{g}^{-1}(\mathrm{~V})$ is open in Y. Since $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-continuous, $f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X. Therefore, $f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)=(\mathrm{g} \circ f)^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-closed in X . Thus $(\mathrm{g} \circ f)^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X for each V is open in Z . Hence $\mathrm{g} \circ f$ is contra $\mathrm{p} * \mathrm{~g}$-continuous.

Definition 3.18. A function $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is called contra $\mathrm{p} * \mathrm{~g}$-irresolute if $f^{-1}(\mathrm{~F})$ is $\mathrm{p} * \mathrm{~g}$-closed set in $(\mathrm{X}, \tau)$ for every $p^{*}$ g-open $F$ in $(Y, \sigma)$.

Definition 3.19. A function $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is called contra strongly $\mathrm{p} * \mathrm{~g}$-continuous if $f^{-1}(\mathrm{~F})$ is open in $(\mathrm{X}, \tau)$ for every $\mathrm{p}^{*} \mathrm{~g}$-closed set F in ( $\mathrm{Y}, \sigma$ ).

Theorem 3.20. A function $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute if and only if the inverse image $f^{-1}(\mathrm{~F})$ is $\mathrm{p}^{*} \mathrm{~g}$-open in X for every $\mathrm{p}^{*} \mathrm{~g}$-closed set F in Y.

Proof: Let F be $\mathrm{p}^{*} \mathrm{~g}$-closed in Y. By Definition 2.7, Y YF is $\mathrm{p}^{*} \mathrm{~g}$-open in Y. Since $f$ is contra $\mathrm{p}^{* g}$-irresolute, $f^{-1}(\mathrm{Y} \backslash \mathrm{F})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . But $f^{-1}(\mathrm{Y} \backslash \mathrm{F})=\mathrm{X} \backslash f^{-1}(\mathrm{~F})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . Therefore, $f^{-1}(\mathrm{~F})$ is $\mathrm{p}^{*} \mathrm{~g}$-open in X .
Conversely, assume that the inverse image of every $\mathrm{p} * \mathrm{~g}$-closed set in Y is $\mathrm{p}^{*} \mathrm{~g}$-open in X . Let V be $\mathrm{p} * \mathrm{~g}$-open in Y . By Definition 2.7, $\mathrm{Y} \backslash \mathrm{V}$ is $\mathrm{p} * \mathrm{~g}$-closed in Y . By assumption, $f^{-1}(\mathrm{Y} \backslash \mathrm{V})$ is $\mathrm{p}^{*} \mathrm{~g}$-open in X . But $f^{-1}(\mathrm{Y} \backslash \mathrm{V})=\mathrm{X} \backslash f^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-open in X . It follows that, $f^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . Thus $f^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X for every $\mathrm{p}^{*} \mathrm{~g}$-open set V in Y. Hence $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute.

Theorem 3.21. A function $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is contra strongly $\mathrm{p} * \mathrm{~g}$-continuous if and only if the inverse image of every $\mathrm{p} * \mathrm{~g}$-open in Y is closed set in Y .

Proof: Let V be $\mathrm{p} * \mathrm{~g}$-open in Y. By Definition 2.7, $\mathrm{Y} \backslash \mathrm{V}$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in Y . Since $f$ is contra strongly $\mathrm{p}^{*} \mathrm{~g}$-continuous, $f^{-1}(\mathrm{Y} \backslash \mathrm{V})$ is open in X . But $f^{-1}(\mathrm{Y} \backslash \mathrm{V})=\mathrm{X} \backslash f^{-1}(\mathrm{~V})$ is open in X . It follows that, $f^{-1}(\mathrm{~V})$ is closed in X .
Conversely, assume that the inverse image of every $\mathrm{p} * \mathrm{~g}$-open set in Y is closed in X . Let F be $\mathrm{p} * \mathrm{~g}$-closed in Y . By Definition 2.7, $\mathrm{Y} \backslash \mathrm{F}$ is $\mathrm{p} * \mathrm{~g}$-open in Y. By assumption, $f^{-1}(\mathrm{Y} \backslash \mathrm{F})$ is closed in X. But $f^{-1}(\mathrm{Y} \backslash \mathrm{F})=\mathrm{X} f^{-1}(\mathrm{~F})$ is closed in X . It follows that, $f^{-1}(\mathrm{~F})$ is open in X . Thus $f^{-1}(\mathrm{~F})$ is open in X for every $\mathrm{p}^{*} \mathrm{~g}$-closed set F in Y . Hence $f$ is contra strongly $\mathrm{p}^{*} \mathrm{~g}$-continuous.

The following theorem shows that the composition of contra $\mathrm{p}^{*} \mathrm{~g}$-continuous function and strongly $\mathrm{p} * \mathrm{~g}$-continuous function is a contra p *g-irresolute function.

Theorem 3.22. If $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is contra $\mathrm{p}^{*} \mathrm{~g}$-continuous and $\mathrm{g}:(\mathrm{Y}, \sigma) \rightarrow(\mathrm{Z}, \eta)$ is strongly $\mathrm{p}^{*} \mathrm{~g}$-continuous, then $\mathrm{g} \circ f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Z}, \eta)$ is contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute.

Proof: Let V be $\mathrm{p}^{*} \mathrm{~g}$-open in Z. Since g is strongly $\mathrm{p}^{*} \mathrm{~g}$-continuous, $\mathrm{g}^{-1}(\mathrm{~V})$ is open in Y. Since $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-continuous, $f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . Therefore $(\mathrm{g} \circ f)^{-1}(\mathrm{~V})=f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . Thus $(\mathrm{g} \circ f)^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-closed in X for each V is $\mathrm{p}^{*} \mathrm{~g}$-open in Z . Hence $\mathrm{g} \circ f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute.

The next theorem shows that the composition of contra strongly $\mathrm{p} * \mathrm{~g}$-continuous function and $\mathrm{p} * \mathrm{~g}$-continuous function is a contra continuous function.

Theorem 3.23. If $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ is contra strongly $\mathrm{p}^{*} \mathrm{~g}$-continuous and $\mathrm{g}:(\mathrm{Y}, \sigma) \rightarrow(\mathrm{Z}, \eta)$ is $\mathrm{p}^{*} \mathrm{~g}$-continuous, then $\mathrm{g} \circ f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Z}, \eta)$ is contra continuous.

Proof: Let V be open in Z. Since g is $\mathrm{p}^{*} \mathrm{~g}$-continuous, $\mathrm{g}^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-open in Y. Since $f$ is contra strongly p *g-continuous, $f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)$ is closed in X. Therefore $(\mathrm{g} \circ f)^{-1}(\mathrm{~V})=f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)$ is closed in X . Thus $(\mathrm{g} \circ f)^{-1}(\mathrm{~V})$ is closed in X for each V is open in Z . Hence $g \circ f$ is contra continuous.

Further, it is shown that the contra $\mathrm{p} * \mathrm{~g}$-irresolute function is contra $\mathrm{p} * \mathrm{~g}$-continuous.
Theorem 3.24. Let $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ be a contra $\mathrm{p} * \mathrm{~g}$-irresolute function. Then $f$ is contra $\mathrm{p} * \mathrm{~g}$-continuous.
Proof: Let V be open in Y. Then V is $\mathrm{p}^{*} \mathrm{~g}$-open in Y. Since $f$ is contra $\mathrm{p} * \mathrm{~g}$-irresolute, $f^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-closed in X . Thus $f^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X for each V is open in Y . Therefore $f$ is contra $\mathrm{p}^{* g}$-continuous.

The converse of the above theorem need not be true as shown in the following example.
Example 3.25. Consider $\mathrm{X}=\mathrm{Y}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}, \tau=\{\phi,\{\mathrm{a}, \mathrm{b}\}, \mathrm{X}\}$ and $\sigma=\{\phi,\{\mathrm{a}\},\{\mathrm{a}, \mathrm{b}\}, \mathrm{Y}\}$. Then $(\mathrm{X}, \tau)$ and $(\mathrm{Y}, \sigma)$ are topological spaces. Define $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ by $f(\mathrm{a})=\mathrm{c}, f(\mathrm{~b})=\mathrm{a}$ and $f(\mathrm{c})=\mathrm{b}$. Then $f$ is contra $\mathrm{p} * \mathrm{~g}$-continuous. It can be found that $f^{-1}(\{\mathrm{a}, \mathrm{c}\})=\{\mathrm{b}, \mathrm{a}\}$ is not $\mathrm{p} * \mathrm{~g}$-closed. So $f$ is not contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute.

The following theorem shows that the composition of contra $\mathrm{p} * \mathrm{~g}$-irresolute function with other existing functions.
Theorem 3.26. Let $f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Y}, \sigma)$ and $\mathrm{g}:(\mathrm{Y}, \sigma) \rightarrow(\mathrm{Z}, \eta)$ two maps such that $\mathrm{g} \circ f:(\mathrm{X}, \tau) \rightarrow(\mathrm{Z}, \eta)$.
i. If g is $\mathrm{p}^{*} \mathrm{~g}$-continuous and $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute then $\mathrm{g} \circ f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-continuous.
ii. If g is $\mathrm{p}^{*} \mathrm{~g}$-irresolute and $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute, then $\mathrm{g} \circ f$ is contra $\mathrm{p}^{*} \mathrm{~g}$ - irresolute.

## Proof:

i. Let V be an open set in Z . Since g is $\mathrm{p} * \mathrm{~g}$-continuous, $\mathrm{g}^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-open in Y . Since $f$ is contra $\mathrm{p} * \mathrm{~g}$-irresolute, $f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)$ is $\mathrm{p} * \mathrm{~g}$-closed in X. Therefore $f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)=(\mathrm{g} \circ f)^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X. Thus $(\mathrm{g} \circ f)^{-1}(\mathrm{~V})$ is $\mathrm{p} * \mathrm{~g}$-closed in $X$ for each $V$ is open in $Z$. Hence $g \circ f: X \rightarrow Z$ is contra $p * g$-continuous.
ii. Let V be a $\mathrm{p}^{* g}$-open set in Z . Since g is $\mathrm{p}^{*} \mathrm{~g}$-irresolute, $\mathrm{g}^{-1}(\mathrm{~V})$ is $\mathrm{p}^{* g}$-open in Y. Since $f$ is contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute, $f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)$ is $\mathrm{p} * \mathrm{~g}$-closed in X. Therefore $f^{-1}\left(\mathrm{~g}^{-1}(\mathrm{~V})\right)=(\mathrm{g} \circ f)^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X . Thus $(\mathrm{g} \circ f)^{-1}(\mathrm{~V})$ is $\mathrm{p}^{*} \mathrm{~g}$-closed in X for each V is $\mathrm{p}^{*} \mathrm{~g}$-open in Z . Hence $\mathrm{g} \circ f: \mathrm{X} \rightarrow \mathrm{Z}$ is contra $\mathrm{p}^{*} \mathrm{~g}$-irresolute.

## 4. Conclusion

The properties of the functions namely contra $\mathrm{p}^{*} \mathrm{~g}$-continuous is properly placed in between contra continuous and contra pre
 their implications are investigated.

## 5. References

[1] Arokiarani, I, Balachandran, K, and Dontchev, J, 1999, Some Characterizations of gp-irresolute and gp-continuous Maps Between Topological Spaces, Kochi University Faculty of Science Memoirs Mathematics, vol. 20, pp. 93-104.
[2] Dontchev, J, 1995, On Generalizing Semi Pre Open Sets, Kochi University Faculty of Science Memoirs Mathematics, vol. 16, pp. 35-48.
[3] Dontchev, J, 1996, Contra Continuous Functions and Strongly S-closed Spaces, International Journal of Mathematics and Mathematical Science, vol. 19(2), pp. 303-310.
[4] Dunham, S, 1977, T1/2-spaces, Kyungpook Mathematical Journal, vol. 17, pp. 161-169.
[5] Dunham, W, 1982, A New Closure Operator for Non-T Topologies, Kyungpook Mathematical Journal, vol. 22, pp. 55-60.
[6] Gnanambal, Y, Balachandran, K, 1999, On gpr-continuous Functions in Topological Spaces, Indian Journal of Pure and

Applied Mathematics, vol. 30(6), pp. 581-593.
[7] Jeyachitra, M, Bageerathi, K, 2017, On Pre*generalized Closed sets in Topological Spaces, International Journal of Mathematical Archive, vol. 8(1), pp. 65-72.
[8] Jeyachitra, M, Bageerathi, K, 2018, Pre*generalized Open Sets and Pre*generalized Neighbourhood in Topological Spaces, Proceedings of National Conference on Recent Advancements, ISBN: 978-81-936440-7-2.
[9] Jeyachitra, M, Bageerathi, K, 2019, Pre*generalized Continuous and Pre*generalized irresolute Functions in Topological Spaces. (Submitted).
[10] Jeyachitra, M, Bageerathi, K, 2019, Some Functions via p*g-closed sets. (Submitted).
[11] Levine, N, 1970, Generalized Closed Sets in Topology, Rendiconti del Circolo Matematico di Palermo vol. 19(2), pp. 89-96.
[12] Li, Z, Zhu, W, 2013, Contra Continuity on Generalized Topological Spaces, Acta Mathematica Hungar, vol. 138, pp.34-43.
[13] Mashhour, AS. Abd, El, Monsef, ME, and El, Deeb, SN, 1982, On Pre Continuous Weak Pre Continuous Mappings, Proceedings of the Mathematical and Physical Society of Egypt, vol. 53, pp. 47-53.
[14] Patil, PG, Rayanagoudar, TD, and Mahesh, K, Bhat, 2011, On Some New Functions of g*p-Continuity, International Journal of Contemporary Mathematical Sciences, vol. 6, pp. 991-998.
[15] Selvi, T, Punitha Dharani, A, 2012, Some New Class of Nearly Closed and Open Sets, Asian Journal of Current Engineering and Maths, vol. 1(5), pp.305-307.
[16] Veerakumar, MKRS, 2005, Contra pre semi continuous functions, Bulletin of the Malaysian Mathematical Sciences Society, (2) vol. 28(1), pp. 67-71.

