J-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract : In this paper, a new class of sets J-closed sets is initiated in topological spaces. The properties and relationships with other g-closed sets are analysed. Some important characterizations are obtained.

IndexTerms - J-closed set, η *-open set, regular*-open set, g-closed set, gδ-closed set.

I.Introduction

In 1937, Stone [24] introduced regular open sets and used it to define the semi-regularization of a topological space.In 1968, Velicko [28] proposed δ-open sets which are stronger than open sets. Levine[14]has brought generalized closed sets in 1970. Dunham [7] has established a generalized closure using Levine's generalized closed sets as Cl*.In 2016,Annalakshmi [21] has instituted regular*-open sets using Cl*.In 2018,I have introduced a class of new sets namely η *-open sets [16] which is placed between the classes of **δ**-Open set and open set. Its basic properties are procured and the concepts of η *-cluster point,η *-adherent point and a η *-derived set are introduced and studied in the same paper. In this paper, J-closed sets are introduced using η *-open sets and their features are studied.

For this paper some basic definitions and results in topological spaces are needed which are given in section II.Throughout this paper, (Y, ζ) will always denote the topological space.

II.Preliminaries

Definition 2.1 Let (Y, ζ) be a topological space. If D is a non-empty subset of (Y, ζ) then the intersection of all closed sets containing D is called ***closure of D*** and is denoted by Cl(D).The union of all open sets contained in D is called ***interior of D*** and is denoted by int(D).

Definition 2.2 If A is a subset of a space (Y,ζ),

(i) The ***generalized closure*** of D [7] is defined as the intersection of all g-closed sets in Y containing D and is denoted by Cl*(D).

(ii) The ***generalized interior*** of D [7] is defined as the union of all g-open sets in Y contained in D and is denoted by int*(D).

Definition 2.3 Let (Y, ζ) be a topological space. A subset D of (Y, ζ) is called

1) ***regular closed set*** [24] if D = Cl(int(D))
2) ***semi-closed set*** [13] if int(Cl(D)) $\subseteq$ D
3) ***α-closed set*** [19] if Cl(int(Cl(D)) $\subseteq$ D
4) ***pre-closed set*** [15] if Cl(int(D)) $\subseteq$ D
5) ***semi pre-closed set*** [1] if int(Cl(int(D))) $\subseteq$ D
6) ***π-closed set*** [29] if it is the finite union of regular closed sets.

The complements of the above mentioned sets are called ***regular open ,semi-open, α-open, pre-open and semi pre-open, π-open sets*** respectively.

The intersection of all regular closed (resp. semi-closed,α-closed, pre-closed and semi pre-closed) subsets of (Y,) containing D is called the ***regular closure (resp.semi-closure, α-closure, pre-closure and semi pre-closure, π-closure)*** of D and is denoted by rCl(D)(resp.sCl(D),αCl(D), pCl(D) and spCl(D), Cl(D)). A subset D of (Y, ζ) is called clopen if it is both open and closed in (Y, ζ).

Definition 2.4 [28] The ***δ-interior*** of a subset D of Y is the union of all regular open sets of Y contained in D and is denoted by $int_{\delta}(D)$.The subset D is called ***δ-open*** if $D = int_{\delta}(D)$,i.e. a set is δ-open if it is the union of regular open sets, the complement of δ-open is called ***δ-closed***. Alternatively, a set D ⊆ Y is δ-closed if D = δCl(D), where ***δCl(D)*** is the intersection of all regular closed sets of (Y, ζ)containing D.

Definition 2.5 [21] Let (Y, ζ) be a topological space. A subset D of (Y, ζ) is called ***regular*-open (or r*-open)*** if D = int(Cl*(D)).The complement of regular*-open set is called ***regular*-closed set***. The union of all regular*-open sets of Y contained in D is called ***regular*-interior*** and is denoted by r*int(D). The intersection of all regular*-closed sets of Y containing D is called ***regular*-closure*** is denoted by r*Cl(D).

Definition 2.6 [16] A subset D of a topological space (Y, ζ)is called ***η *-open*** set if it is a union of regular*-open sets (r*-open sets).The complement of a η *-open set is called a ***η *-closed set***. A subset D of a topological space (Y, ζ) is called ***η *-Interior*** of D is the union of all η *-open sets of Y contained in D.We denote the symbol by η *-Int(D).The intersection of all η *-closed sets of Y containing D is called as the ***η *-closure*** of D and denoted by η *-Cl(D).

Definition 2.7 A subset A of a topological space (Y, ζ)is called

1) ***generalized closed*** (briefly ***g-closed***) [14] if Cl(D) ⊆ M whenever D ⊆ M and M is open in (Y, ζ).
2) ***generalized semi-closed*** (briefly ***gs-closed***) [2] if sCl(D) ⊆M whenever D ⊆ M and U is open in (Y, ζ).
3) ***δ-generalized closed*** (briefly ***δg-closed***) [5] if $\delta Cl(D) \subseteq M$ whenever D ⊆ M and M is open in (Y, ζ).
4) ***generalized pre regular -closed*** (briefly ***gpr-closed***) [8] if pCl(D) ⊆ M whenever D ⊆ M and M is regular open in (Y, ζ).
5) ***π-generalized closed*** (briefly ***πg-closed***) [6] if Cl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y, ζ).
6) ***ĝ -closed*** [26] if Cl(D) ⊆ M whenever D ⊆ M and M is semi-open in (Y, ζ).
7) ***generalized δ-closed*** (briefly ***gδ-closed***) [4] if Cl(D) ⊆ M whenever D ⊆ M and M is δ-open in (Y, ζ).
8) ***δg‡-closed*** [4] if δCl(D) ⊆ M whenever D ⊆ M and M is δ-open in (Y, ζ).
9) ***#gs –closed*** [27] if sCl(D) ⊆ M whenever D ⊆ M and U is *g-open in (Y, ζ).
10) ***π-generalized semi-closed*** (briefly ***πgs-closed***) [3] if sCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y, ζ).
11) ***π-generalized pre-closed*** (briefly ***πgp-closed***) [20] if pCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y, ζ).
12) ****g -closed*** [9] if Cl(D) ⊆ M whenever D ⊆ M and M is ĝ-open in (Y, ζ).
13) ***regular weakly generalized -closed*** (briefly ***rwg-closed***) [17] if Cl(int(D)) ⊆ M whenever D ⊆ M and M is regular open in (Y, ζ).
14) ***π-generalized α -closed*** (briefly ***πgα-closed***) [10] if αCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y, ζ).
15) ***π-generalized semi pre-closed*** (briefly ***πgsp-closed***) [23] if spCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y, ζ).
16) ***generalized semi pre regular -closed*** (briefly ***gspr-closed***) [18] if spCl(D) ⊆ M whenever D ⊆ M and M is regular open in (Y, ζ).
17) ***g*s-closed*** [22] if sCl(D) ⊆ M whenever D ⊆ M and M is gs-open in (Y, ζ).
18) ***δg* -closed*** [25] if δCl(D) ⊆ M whenever D ⊆ M and M is g-open in (Y, ζ).
19) The complements of the above mentioned sets are called their respective open sets.

Remark 2.8 [16] (i) π-closed (open) →regular closed (open) → δ-closed (open) → η*-closed (open) → closed (open) → semi-closed (open) → semi pre-closed(open).

(ii) π-closed (open) →regular closed (open) → δ-closed (open) → η *-closed (open) → closed (open) → closed (open) → -closed (open).

(iii) π-closed (open) → regular closed (open) → δ-closed (open) → η *-closed (open) → closed (open) →g-closed (open).

(iv) π-closed (open) →regular closed (open) → δ-closed (open) → η *-closed (open) → closed (open) →pre-closed (open).

Remark 2.9 [16] For every subset D of Y,

(i) $spCl(D) \subseteq sCl(D) \subseteq Cl(D) \subseteq \eta^*\text{-}Cl(D) \subseteq Cl(D) \subseteq rCl(D) \subseteq \pi Cl(D)$.

(ii) $Cl(D) \subseteq Cl(D)) \subseteq \eta^*\text{-}Cl(D) \subseteq Cl(D)) \subseteq rCl(D) \subseteq \pi Cl(D)$.

(iii) $gCl(D) \subseteq Cl(D)) \subseteq \eta^*\text{-}Cl(D) \subseteq Cl(D) \subseteq rCl(D) \subseteq \pi Cl(D)$.

(iv) $pCl(D) \subseteq Cl(D)) \subseteq \eta^*\text{-}Cl(D) \subseteq Cl(D) \subseteq rCl(D) \subseteq \pi Cl(D)$.

Remark 2.10 A topological space (Y,ζ) is said to be a

1. ***$T_{1/2}$ -space*** [14] if every g-closed subset of (Y,ζ) is closed in (Y,ζ).
2. ***Semi regular space*** [24] if $\zeta = \zeta_s$ .In a semi regular space every δ-open sets coincides with open sets.
3. **Definition 2.11** A subset D of a topological space (Y,ζ) is called ***clopen*** if it is both open and closed in (Y,ζ).

III.J-CLOSED SETS

In this section a new class of generalized closed sets, called J-closed sets are introduced. The relations between J-closed sets and various existing closed sets are analysed.

Definition 3.1 A subset D of a topological space (Y,ζ) is said to be ***J-closed set*** if $Cl(D) \subseteq M$ whenever $D \subseteq M$, M is η^*-open in (Y,ζ).
The class of all J-closed sets of (Y,ζ) is denoted by ***JC(Y,ζ).***

Proposition 3.2 ***Every δ-closed set is J-closed but not conversely.***

Proof: Let D be a δ- closed set and M be any η^*-open set containing D. Since D is δ-closed, δCl(D)=D. Therefore $\delta Cl(D)=D \subseteq M$.As $Cl(D) \subseteq \delta Cl(D)$ and hence D is J-closed.

Counter Example 3.3 Let Y={p,q,r}, ζ={ϕ,X, {p}, {p,q}}. In this topology the subset {r} is J-closed but not δ-closed.

Proposition 3.4 ***Every δg^*-closed set is J-closed but not conversely.***

Proof: Let D be a δg^*-closed and M be any η^*-open set containing M in Y.By Remark 2.8(iii), every η^*-open set is a g-open set and D is δg^*-closed, $\delta Cl(D) \subseteq M$. As $Cl(D) \subseteq \delta Cl(D)$ [By Remark 2.9(i)].We get $Cl(D) \subseteq M$ implies D is J-closed.

Counter Example 3.5 Let Y={p,q,r}, ζ={ϕ, Y, {p}, {p,q}, {p,r}}. Then the subset {q} is J-closed but not δg^*-closed in (Y,ζ).

Proposition 3.6 ***Every δg-closed set is J-closed but not conversely.***

Proof: Let D be a δg-closed and M be any η^*-open set containing D in Y.By Remark 2.8(i), every η^*-open set is an open set and D is δg-closed, $\delta Cl(D) \subseteq M$. As $Cl(D) \subseteq \delta Cl(D)$ [By Remark 2.9(i)].We get $Cl(D) \subseteq M$ implies D is J-closed.

Counter Example 3.7 Let Y={p,q,r}, ζ ={ϕ, Y,{p},{p,q},{p,r}}. Then the subset {q} is J-closed but not δg-closed in (Y,ζ).

Proposition 3.8 ***Every g-closed set is J-closed but not conversely.***

Proof: Let D be a g-closed set and M be any η^*-open set containing D. By Remark 2.8(i), η^*-open set is open and D is g-closed, $Cl(D) \subseteq M$. Therefore D is J-closed.

Counter Example 3.9 Let Y={p,q,r}, ζ ={ϕ, Y, {p},{p,q}}. In this topology the subset {q} is J-closed but not g-closed in (Y,ζ).

Proposition 3.10 ***Every J-closed set is gδ- closed but not conversely.***

Proof: Let D be J-closed set and M be any δ-open set containing D in Y. By Remark 2.8(i), every δ-open is η^*-open and D is J-closed, $Cl(D) \subseteq M$. Hence D is ***gδ*** -closed.

Counter Example 3.11 Let Y={p,q,r}, ζ={ϕ, Y,{p}, {p,q}}. Then the subset {p} is gδ-closed but not J-closed in (Y,ζ).

Remark 3.12

δ- closed → δg^*-closed → δg-closed → g-closed → J-closed → gδ-closed

Proposition 3.13 ***Every J-closed set is rg-closed but not conversely.***

Proof: Let D be J-closed and U be any regular open set containing D in Y. By Remark 2.8(i), every regular open is η *- open and D is J-closed, Cl(D)⊆U. Hence D is rg-closed.

Counter Example 3.14 Let Y={p,q,r},ζ={ϕ, Y, {p},{p,q}}.Then the subset {p} is rg-closed but not J-closed in (Y,ζ).

Proposition 3.15 ***Every J-closed set is gpr-closed but not conversely.***

Proof: Let D be J-closed set and M be any regular open set containing D in Y. By Remark 2.8(i), every regular open set is η *-open and D is J-closed, Cl(D)⊆M. As pCl(D)⊆Cl(D) [By Remark 2.9(iv)].We get pCl(D)⊆ M implies D is gpr- closed.

Counter Example 3.16 Let Y={p,q,r},ζ ={ϕ, Y,{p},{q},{p,q}} Then the subset {p,q} is gpr-closed but not J-closed in (Y,ζ).

Proposition 3.17 ***Every J-closed set is rwg-closed but not conversely.***

Proof : Let D be J-closed and M be any regular open set containing D in Y. By Remark 2.8(i), every regular open set is η *-open and D is J-closed, Cl(D)⊆M. As int(D)⊆D[By Definition 2.1]. We have Cl(int(D))⊆Cl(D)⊆M and hence D is rwg-closed.

Counter Example 3.18 LetY={p,q,r},ζ={ϕ, Y,{p},{q},{p,q}}. Then the subset {p,q} is rwg -closed, but not J-closed in (Y,ζ).

Proposition 3.19 ***Every J-closed set is gspr-closed but not conversely.***

Proof: Let D be J-closed and M be any regular open set containing D in Y. By Remark 2.8(i),every regular open set is η *-open and D is J-closed, Cl(D)⊆M. As spCl(D)⊆Cl(D) [By Remark 2.9(i)]. We get spCl(D)⊆M implies D is gspr –closed.

Counter Example 3.20 Let Y={p,q,r},ζ={ϕ, Y,{p},{q},{p,q}}. Then the subset {q} is gspr-closed but not J-closed in (Y,ζ).

Proposition 3.21 ***Every J-closed set is πg-closed but not conversely.***

Proof: Let D be J-closed and M be any π-open set containing D in Y. By Remark 2.8(i), every π-open set is η *-open and D is J-closed, Cl(D)⊆M and hence D is πg-closed.

Counter Example 3.22 Let Y={p,q,r},ζ={ϕ, Y,{p}, {p,q}}. Then the subset {p} is πg-closed but not J-closed in(Y,ζ).

Proposition 3.23 ***Every J-closed set is πgp-closed but not conversely.***

Proof: Let D be J-closed and M be any π-open set containing D in Y. By Remark 2.8(i), every π-open set is η *- open and D is J-closed, Cl(D)⊆M. As pCl(D)⊆Cl(D) [By Remark 2.9(iv)].We get pCl(D)⊆ M implies D is πgp-closed.

Counter Example 3.24 Let Y={p,q,r},ζ ={ϕ, Y,{p}, {p,q}}Then the subset {p} is πgp-closed but not J-closed in (Y,ζ).

Proposition 3.25 ***Every J-closed set is πgsp-closed but not conversely.***

Proof: Let D be J-closed set and M be any π-open set containing D in Y. By Remark 2.8(i), every π-open set is η *-open and D is J-closed, Cl(D)⊆M. As spCl(D)⊆Cl(D)) [By Remark 2.9(i)]. We get spCl(D) ⊆M implies D is πgsp-closed.

Counter Example 3.26 Let Y={p,q,r},ζ={ϕ, Y, {p}, {q}, {p,q}}. Then the subset {p} is πgsp-closed but not J-closed in (Y,ζ).

Proposition 3.27 ***Every J-closed set is πgs-closed but not conversely.***

Proof: Let D be J-closed set and M be any π-open set containing D in Y. By Remark 2.8(i), every π-open set is η *-open and D is J-closed, Cl(D)⊆M. As sCl(D)⊆Cl(D) [By Remark 2.9(i)]. We get sCl(D)⊆M implies D is πgs-closed.

Counter Example 3.28 Let Y={p,q,r},ζ ={ϕ, Y, {p}, {q},{p,q}}. Then the subset {p} is πgs-closed but not J-closed in (Y,ζ).

Proposition 3.29 ***Every J-closed set is πgα-closed but not conversely.***

Proof: Let D be J-closed set and M be any π-open set containing D in Y. By Remark 2.8(i), every π-open set is η *-open and D is J-closed, Cl(D)⊆M. As αCl(D)⊆Cl(D) [By Remark 2.9(ii)]. We get αCl(D)⊆M implies D is πgα-closed.

Counter Example 3.30 Let Y={p,q,r}, ζ={ϕ, Y, {p},{p,q}}. Then the subset {p} is πgα-closed but not J-closed in (Y, ζ).

Remark 3.31 ***The following counter examples show that J-closed set is independent from gs-closed, #gs-closed, g*s-closed and δg‡-closed sets.***

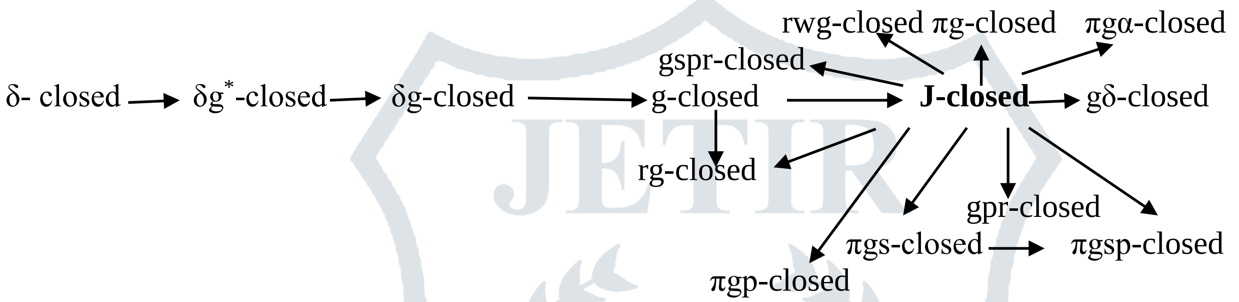
Counter Example 3.32 Let Y={p,q,r}, ζ={ϕ, Y, {p},{q},{p,q}}. In this topology the subset {p} is gs-closed, #gs-closed, g*s-closed but not J-closed.

Counter Example 3.33 Let Y={p,q,r}, ζ={ϕ, Y, {p,q}} In this topology the subset {p} is J-closed but not gs-closed, #gs-closed, g*s-closed.

Counter Example 3.34 Let Y={p,q,r,s}, ζ={ϕ, Y, {p}}. In this topology the subset {p} is not J-closed but it is δg‡-closed.

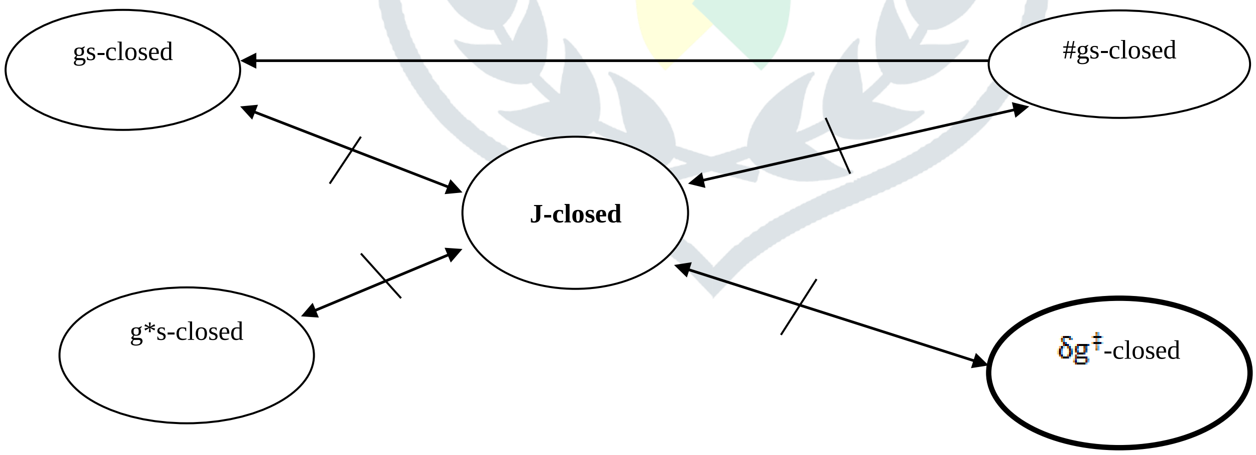
Counter Example 3.35 Let Y={p,q,r,s}, ζ ={ϕ, Y, {p},{r},{p,q},{p,r},{p,q,r},{p,r,s}} In this topology the subset {q} is J-closed but not δg‡-closed.

Remark 3.36 ***From the above discussions, we get J-closed sets is equivalent with other existing g-closed sets.***



In the above diagram, A → B represents A implies B but not reversible.

Remark 3.37 ***From the above discussions, we get J-closed sets is not equivalent with other existing g-closed sets. (In this diagram, A ↮ B represents A and B are not equivalent).***



IV.Properties of J-Closed Sets in Topological Spaces

Theorem 4.1 *The finite union of J-closed sets is J-closed.*

Proof: Let {D_i / i=1,2,3…….n} be a finite class of J-closed sets of (Y, ζ).Let $D = \bigcup_{i=1}^{n} D_i$. Let M be a η *-open set containing D. This implies $D_i \subseteq M$ for every i. By assumption Cl(Di) $\subseteq$M for every i. This implies $\bigcup_{i=1}^{n}$ Cl(Di)$\subseteq$M. Then Cl($\bigcup_{i=1}^{n} D_i$)$\subseteq$M. Thus Cl(D)$\subseteq$M. Hence finite union of J-closed sets is J-closed in (Y, ζ).

Remark 4.2 ***The following counter example shows that the countable union of J-closed sets need not be J-closed.***

Counter Example 4.3 Consider R, the real line with the usual topology . Set $D = \bigcup_{n\in N}\left\{\frac{1}{n}\right\}$, N being the set of all positive integers. Clearly D is the countable union of J-closed sets but D is not J-closed in (R,ζ), since $D \subseteq (0,2), 0 \in Cl(D) \Rightarrow Cl(D) \nsubseteq (0,2)$.

Remark 4.4 ***The following counter example shows that the difference of any two J-closed sets in Y need not be J-closed.***

Counter Example 4.5 Let Y = {p,q,r,s }, ζ = {Y, ϕ,{p,q}}. Then the set {p,r} and {p,s} are J-closed but their difference {p} is not J- closed in (Y,ζ).

Theorem 4.6 ***Let D be a J-closed set of (Y,ζ). Then Cl(D) – D does not contain a non- empty η *-closed set.***

Proof: Suppose that D is J-closed, let M be a η *-closed set contained in Cl(D) – D. Now M^c is a η *-open set in Y such that D⊆M^c.Since D is J-closed set of Y,Cl(D)⊆M^c.Thus M⊆$(Cl(D))^c$.Also M⊆Cl(D) − D. Therefore M⊆$(Cl(D))^c$∩ Cl(D) =ϕ.Hence M = ϕ.

Proposition 4.7 ***If D is a η *-open set and a J-closed set of (Y,ζ),then D is a closed set of Y.***

Proof: Since D is η *-open and J-closed, Cl(D)⊆D. Hence D is closed in (Y,ζ).

Theorem 4.8 ***If D is J-closed and η *-open and F is closed in (Y,ζ),then*** $D\cap F$ ***is closed.***

Proof: Since D is J-closed and η *-open, D is closed by Proposition 4.7.Since F is closed in Y, $D\cap F$ is closed in Y.

Theorem 4.9 ***The intersection of a J-closed set and a δ-closed set is always J-closed.***

Proof: Let D be J-closed and F be δ-closed. Let V = $D\cap F$ .Let M be η *-open such that V ⊆ M. Then $D\cap F\subseteq$ M which implies $D\subseteq M\bigcup F^c$. Here F^c is δ-open. So F^c is η *-open. Hence M⋃F^c is η *-open (By Theorem 3i from [16]) and by assumption $D\subseteq M\bigcup F^c$ which implies Cl(D)⊆M⋃F^c which implies Cl(D)∩F⊆M. Now Cl(V) = Cl(D∩ F)⊆Cl(D)∩Cl(F)⊆Cl(D)∩ δ-Cl(F)= Cl(D)∩ F⊆M. Therefore Cl(V)⊆M. Hence A∩ F is J-closed.

Proposition 4.10 ***If D is a J-closed set in a space (Y,ζ),and D⊆B⊆Cl(D) then B is also a J-closed set.***

Proof: Let M be η *-open set of Y such that B⊆M. Then D⊆M. Since D is J-closed set, Cl(D)⊆M. Since B⊆Cl(D), Cl(B)⊆Cl(Cl(D)) = Cl(D). Hence Cl(B)⊆M. Therefore B is also a J-closed set .

Theorem 4.11 ***Let D be a J-closed set of*** (Y,ζ). ***Then D is closed iff Cl(D) – D is η *-closed.***

Proof: (Necessity): Let D be a closed subset of (Y,ζ). Then Cl(D) = D and therefore Cl(D) – D=ϕ which is η *-closed.

(Suffficiency): Let Cl(D) −D be η *-closed set. Since D is J-closed, by Theorem 4.6, Cl(D)−D does not contain a non-empty η *-closed set which implies Cl(D)−D=ϕ. That is Cl(D)=D. Hence D is closed.

Definition 4.12 Let B ⊆ A ⊆ Y. Then B is **J-closed relative to A** if $Cl_A(B)$⊆M, whenever B ⊆ M, M is η *-open in A.

Theorem 4.13 ***Let B ⊆ A ⊆ Y and suppose that B is J-closed in Y, then B is J- closed relative to A. The converse is true if A is closed in Y.***

Proof: Suppose that B is J-closed in Y. Let B ⊆ M, M is η *-open in A. Since M is η *-open in A, M = V∩ A where V is η*-open in Y. Hence B⊆M⊆V. Since B is J-closed in Y, Cl(B)⊆V. Hence Cl(B)∩ A⊆V∩A which in turn implies that $Cl_A(B)$⊆V∩ A = M. Therefore B is J-closed relative to A.

Now to prove the converse, assume that B⊆A⊆Y where A is closed in Y and B is J-closed relative to A. Let B⊆M, M is η *-open in Y. Then A∩ M is η*-open in A by the definition of subspace topology. Since B⊆A and B⊆M, B ⊆A∩ M. Since B is J-closed relative to A, $Cl_A(B)$⊆A∩M.Since B⊆A, Cl(B)⊆Cl(A). Hence Cl(B)⊆A. Therefore Cl(B)∩ A = Cl(B) which implies $Cl_A(B)$ = Cl(B). Hence Cl(B)⊆A∩ M = M. Thus B is J-closed in Y.

Characterization theorems of J-closed sets

Proposition 4.14 ***In $T_{1/2}$ spaces η *-closed sets coincides with δ-closed sets.***

Proof In $T_{1/2}$ spaces g-closed sets coincides with closed sets [By Remark 2.10 (1)]**.**Therefore generalized closure of a set equals with closure of a set. Hence regular* -open sets are regular open sets in $T_{1/2}$ spaces and η *-closed sets coincides with δ-closed sets.

Theorem 4.15 ***Let D be a subset of a*** $T_{1/2}$ ***-space (Y,ζ) then***

(a) D is J-closed if and only if D is gδ-closed.

(b) If, in addition,(Y, ζ) is semi-regular space, then D is J-closed if and only if D is g-closed.

Proof:(a) Every J-closed is gδ-closed by Proposition 3.10, Conversely let D be gδ-closed. Let D $\subseteq$ M, where M is η *-open.In a $T_{1/2}$-space, every η *-open set is δ-open (By Proposition 4.14). Since D is gδ-closed, Cl(D)$\subseteq$M. Therefore D is J-closed.

(b) Every g-closed is J-closed by Proposition 3.8.Conversely let D be J-closed. Let D $\subseteq$ M, where M is open. In a semi regular space, every open is δ-open [By Remark 2.10 (2)] and (Y, ζ) is $T_{1/2}$ M is η *-open [By Proposition 4.14]. Since D is J-closed, Cl(D)$\subseteq$M. Hence D is J -closed.

The previous observation leads to the problem of finding the spaces (Y,ζ) in which the g-closed sets of (Y,ζ_s)are J-closed in (Y, ζ).While we have not been able to completely resolve this problem, we offer partial solutions. For that reason we will call the spaces with $T_{1/2}$ semi-regularization ***almost-weakly Hausdorff***. Recall that a space is called ***weakly Hausdorff*** if its semi-regularization is T_1.The point excluded topology on any infinite set gives an example of an almost weakly Hausdorff space, which is not weakly Hausdorff from [5].

Theorem 4.16 ***In an almost weakly Hausdorff space (Y, ζ),the g-closed sets of (Y,ζ_s)are δ-closed in (Y, ζ)and thus J- closed in (Y, ζ).***

Proof: Let D $\subseteq$ Y be a g-closed subset of (Y,ζ_s). Let $x \in$ δCl(D).If {x} is δ-open, then $x \in$ D.If not, then Y \{x} is δ-open, since Y is almost weakly Hausdorff.Assume that $x \notin D$.Since D is g-closed in (Y, ζ_S). then δCl(D) $\subseteq$ Y \{x},that is $x \notin$ δCl(D).By contradicition $x \in$ D.Hence D is δ-closed. Since D is δ-closed and hence by Proposition 3.2, D is J-closed in (Y, ζ).

Definition 4.17 A topological space (Y,) is called $\mathbf{R_1}$***-space*** if every two different points with distinct closures have disjoint neighbourhoods.

Remark 4.18 In general the concepts of J-closed and $\boldsymbol{\delta g^{\ddagger}}$ -closed are not equivalent. But for a compact subset D of an $T_{1/2}$ R_1**-topological space, they coincides. This can be seen in the following Theorem 4.19.**

Theorem 4.19 ***For a compact subset D of a $T_{1/2}$ and R_1-topological space (Y, ζ) the following conditions are equivalent.***

(a) D is a J-closed set.

(b) D is a $\delta g^{\ddagger}$ -closed set.

Proof:

(a) ⇒ (b) : Let D $\subseteq$ M, where M is δ-open. In a $T_{1/2}$-space, every δ-open set is η *-open [By Proposition 4.14].We get M is η *-open.Since D is J-closed,Cl(D)$\subseteq$M ——— (1). Also in R_1-spaces the concepts of closure and δ-closure coincide for compact sets (By Theorem 3.6 from[11]). Hence for a compact subset D,δCl(D) =Cl(D) ——— (2).From (1) & (2), D is a $\delta g^{\ddagger}$-closed set [by Definition 2.7(8)].

(b) ⇒ (a) : Let D $\subseteq$ M, where M is η *-open. By Proposition 4.14, M is δ-open and since D is $\delta g^{\ddagger}$-closed, δCl(D)$\subseteq$M ———(1).Also in R_1-spaces the concepts of closure and δ-closure coincide for compact sets (By Theorem 3.6 from [11]).Hence Cl(D) = δCl(D) ——— (2). From (1) & (2),D is a J -closed set.

Corollary 4.20 ***In Hausdroff spaces, a finite set is J-closed if and only if it is $\delta g^{\ddagger}$-closed.***

Theorem 4.21 ***Let D be a pre-open subset of a topological space (Y, ζ).Then the following conditions are equivalent:***

(a) D is δg-closed.

(b) D is J-closed.

(c) D is gδ-closed.

Proof:

(a) ⟹ (b) and (b) ⟹(c) are true for any subset D by Proposition 3.6 & 3.10.If D is pre-open in (Y,ζ),then by a result of [12],Cl(D) = δCl(D) and so (c) ⟹ (a).

Definition 4.22 A ***partition space*** is a topological space where every open set is closed.

Concerning partition spaces, the following characterization via J-closed sets is obtained.

Corollary 4.23 ***Let D be a subset of the partition space (Y, ζ). Then the following conditions are equivalent:***

(a) D is δg-closed.

(b) D is J-closed.

(c) D is gδ-closed.

Proof: A topological space is a partition space if and only if every subset is pre-open. Thus the claim follows straight from Theorem 4.21.

Theorem 4.24 ***If (Y, ζ) is a*** $T_{1/2}$ ***-space, then the following conditions hold good.***

(a) For a topological space (Y,ζ),if Y is a partition space, then every subset of Y is J-closed.

(b) The converse is true if Y is a semi regular space.

Proof : (a) Let D be an arbitrary subset of Y and M is η *-open containing D.Since η *-open is open and Y is a Partition space, M is clopen. Thus Cl(D) ⊆ Cl(M) = M. Hence every subset of (Y, ζ) is J-closed.

(b) Let D be an open set in (Y, ζ).By criteria ,every subset of Y is J-closed. In a semi regular space J-closed set and g-closed set coincide [By Theorem 4.15(b)].In a $T_{1/2}$ -space, every g-closed set is closed [By Remark 2.10(1)].Hence D is closed in (Y, ζ). Thus (Y, ζ) is a partition space.

V.References

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