# Stresses in a non-homogeneous thermoelastic medium

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Abstract

Present research is concerned with the study of two-dimensional disturbances in a non-homogeneous, isotropic, thermoelastic medium. The formulation is subjected to a thermal shock. All the mechanical and thermal properties of the solid are supposed to vary exponentially with distance. Normal mode technique is employed to obtain the exact expressions for the displacement components, stresses, change in volume fraction field and temperature field. These are also calculated numerically for a magnesium crystal-like material and depicted through graphs to observe the variations of the considered physical quantities.

**Keywords**: Lord-Shulman theory; Functionally graded materials; Normal mode analysis.

## 1 Introduction

Functionally graded materials (FGMs) are non-homogeneous materials in which the composition changes gradually with a corresponding change in the properties and the volume fractions of different constituents of composite. High temperature thermal shock generates a high magnitude of thermal stresses when sudden heating and cooling happens. In this case, FGMs have thermophysical properties to permit high magnitude thermal stresses due to large temperature dif- ference. These types of materials are massively used in important structures such as nuclear reactors, pressure vessels, pipes and chemical plants.

The coupled thermoelastic problem of a functionally graded disk based on the classical coupled theory of thermoelasticity was considered by Bakhshi et al. [1]. The electro-magneto-thermoelastic response of an infinite functionally graded cylinder was studied by Abbas and Zenkour [2], by using finite element method. By using fractional order theory of thermoelasticity, Abbas [3] scruti- nized the impacts of non-homogeneity, fractional order parameter and time on functionally graded material subjected to a thermal field. The forced vibrations of non-homogeneous, isotropic thermoelastic thin annular disk under periodic and exponential types of axi-symmetric dynamic pressures were discussed by Mishra et al. [4]. Mehditabar et al. [5] surveyed a three-dimensional asymmetric thermoe- lastic response of a rotating functionally graded truncated conical shell subjected to thermal field and inertia force.

In the current research, we consider two-dimensional disturbances in an infinite, isotropic, non-homogeneous thermoelastic half-space. All the thermomechanical properties of the FGM under consideration are assumed to vary as an exponential power of the spatial co-ordinate. The normal mode technique is used to obtain the exact expressions of all the physical variables.

## 2 Fundamental equations

The field equations and stress-strain-temperature relations in a generalized thermoelastic medium are:

### (i) Kinematic relation:

$e$ $\frac{1}{}$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{1}$$

**(ii) Constitutive relations:**

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \beta\theta). \tag{2}$$

**(iii) Stress equation of motion:**

$$\sigma_{ji,j} = \rho \ddot{u}_i. \tag{3}$$

**(iv) Equation of heat conduction:**

$$K\nabla^2\theta = \left(1 + \tau_0\frac{\partial}{\partial t}\right)(\rho C_e\dot{\theta} + \beta T_0\dot{e}), \tag{4}$$

where all the symbols have their usual meaning.

Here, the superposed dot indicates derivative with respect to time and the comma represents derivative with respect to space variable.

# 3 Problem description and exponential variation of non-homogeneity

Consider a non-homogeneous, isotropic thermoelastic half-space with voids and gravity field under the purview of LS model. Choose a fixed cartesian co- ordinate system $(x, y, z)$ having the surface of the half-space as the plane $x = 0$, with $x$-axis pointing vertically downwards into the medium. The current study is restricted to $xy$-plane and thus all the field quantities are independent of the spatial variable $z$. So, the displacement components are taken as:

$$u_1 = u_1(x, y, t),\quad u_2 = u_2(x, y, t),\quad u_3 = 0. \tag{5}$$

It is also assumed that material properties are graded only in $x$-direction. Hence we take $f(\dot{x})$ as $f(x)$. Hence

$$\sigma_{xx} = f(x)\left[(\lambda_0 + 2\mu_0)\frac{\partial u_1}{\partial x} + \lambda_0\frac{\partial u_2}{\partial y} - \beta_0\theta\right], \tag{6}$$

$$\sigma_{yy} = f(x)\left[(\lambda_0 + 2\mu_0)\frac{\partial u_2}{\partial y} + \lambda_0\frac{\partial u_1}{\partial x} - \beta_0\theta\right], \tag{7}$$

$$\sigma_{xy} = f(x)\mu_0\left[\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x}\right]. \tag{8}$$

# 4 Solution procedure

In the current section, normal mode technique is adopted which has the advantage of finding the exact solutions without any assumed constraints on the field variables. So, the physical variables under consideration and the stresses can be decomposed in terms of normal modes in the following form:

$$[u_1, u_2, \theta, \sigma_{ij}](x, y, t) = [u_1^*, u_2^*, \theta^*, \varphi^*, \sigma_{ij}^*](x)e^{(\omega t + \iota my)}, \quad (9)$$

where $u_1^*$, $u_2^*$, $\theta^*$, and $\sigma_{ij}^*$ are the corresponding amplitudes, $\omega$ is the angular fre- quency, $\iota$ is the imaginary unit and $m$ is the wave number in $y$-direction.

By virtue of (9), one can easily obtain the behavior of thermoelastic non-homogeneous medium.

## 5 Concluding remarks

The above study is of fundamental importance and finds its applications for ex- perimental researchers/engineers working in the field of geophysics, earthquake engineering and material science and also for seismologists working in the field of mining tremors and drilling into the crust of the earth.

## 6 Disclosure statement

No potential conflict of interest was reported by the authors.

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