Regular Semi Irresolute on Intuitionistic Fuzzy Topological Spaces in Ŝostak's Sense

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Abstract

In this paper, we introduce the concepts of fuzzy (r,s)-regular semi irresolute on intuitionistic fuzzy topological spaces in $\hat{S}$ostak's sense and then we investigate some of their characteristic properties.

Keywords and phrases: intuitionistic fuzzy topology in $\hat{S}$ostka's sense, fuzzy (r,s)-regular semi irresolute.

AMS (2000) subject classification: 54A40.

1.Introduction

The concept of fuzzy sets was introduced by Zadeh [13]. Change [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by $\hat{S}$ostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra and Samanta [3], and by Ramadan [10]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker and his collegues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined intuitionistic fuzzy topological spaces in $\hat{S}$ostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces. In this paper, we introduce the concepts of fuzzy (r,s)-regular semi irresolute on intuitionistic fuzzy topological spaces in $\hat{S}$ostak's sense and then we investigate some of their characteristic properties.

2.Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. For any $\mu \in I^X$, μ^c denotes the complement of $\tilde{1}-\mu$. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An intuitionistic fuzzy set A is an ordered pair

$A=(\mu_A,\gamma_A)$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and degree of non-membership, respectively, and $\mu_A+\gamma_A \le \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu,\tilde{1}-\mu)$.

Definition 2.1 *[1] Let $A=(\mu_A,\gamma_A)$ and $B=(\mu_B,\gamma_B)$ be intuitionistic fuzzy sets on X. Then*

1. $A \subseteq B$ iff $\mu_A \le \mu_B$ and $\gamma_A \ge \gamma_B$,
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$,
3. $A^c = (\gamma_A,\mu_A)$,
4. $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$
5. $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$,

6. $0_{\sim} = (\tilde{0},\tilde{1})$ and $1_{\sim} = (\tilde{1},\tilde{0})$.

Definition 2.2 *[1] Let f be a map from a set X to a set Y. Let $A=(\mu_A,\gamma_A)$ be a intuitionistic fuzzy set of X and $B=(\mu_B,\gamma_B)$ an intuitionistic fuzzy set of Y. Then:*

1. The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by
$f(A)=(f(\mu_A),\tilde{1}-f(\tilde{1}-\gamma_A))$.

2. The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by
$f^{-1}(B)=(f^{-1}(\mu_B),f^{-1}(\gamma_B))$.

Definition 2.3 *[10] A smooth fuzzy topology on X is a map $T:I^X\to I$ which satisfies the following properties:*

1. $T(\tilde{0})=T(\tilde{1})=1$,
2. $T(\mu_1\wedge\mu_2)\geq T(\mu_1)\wedge T(\mu_2)$,
3. $T(\vee\mu_i)\geq\wedge T(\mu_i)$.

The pair (X,T) is called a smooth fuzzy topological space.

Definition 2.4 *[5] An intuitionistic fuzzy topology on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:*

1. $0_{\sim},1_{\sim}\in T$,
2. If $A_1,A_2\in T$, then $A_1\cap A_2\in T$,
3. If $A_i\in T$ for all i, then $\cup A_i\in T$.

The pair (X,T) is called an intuitionistic fuzzy topological space.

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I\otimes I$ be the set of the pair (r,s) such that $r,s\in I$ and $r+s\leq 1$

Definition 2.5 *[6] Let X be a nonempty set. An intuitionistic fuzzy topology in $\hat{S}$ostak's sense (SoIFT for short) $T=(T_1,T_2)$ on X is a map $T:I(X)\to I\otimes I$ which satisfied the following properties:*

1. $T_1(0_{\sim})=T_1(1_{\sim})=1$ and $T_2(0_{\sim})=T_2(1_{\sim})=1$,
2. $T_1(A\cap B)\geq T_1(A)\wedge T_1(B)$ and $T_2(A\cap B)\leq T_2(A)\vee T_2(B)$,
3. $T_1(\cup A_i)\geq\wedge T_1(A_i)$ and $T_2(\cup A_i)\leq\vee T_2(A_i)$

The $(X,T)=(X,T_1,T_2)$ is said to be an intuitionistic fuzzy topological space in $\hat{S}$ostak's sense (SoIFTS for short). Also, we call $T_1(A)$ a gradation of openness of A and $T_2(A)$ a gradation of nonopenness of A

Definition 2.6 *[8] Let A be an intuitionistic fuzzy set in a SoIFTS (X,T_1,T_2) and $(r,s)\in I\otimes I$. Then A is said to be*

1. fuzzy (r,s)-open if $T_1(A)\geq r$ and $T_2(A)\leq s$,
2. fuzzy (r,s)-closed if $T_1(A^c)\geq r$ and $T_2(A^c)\leq s$.

Definition 2.7 *[8] Let (X,T_1,T_2) be a SoIFTS. For each $(r,s)\in I\otimes I$ and for each $A\in I(X)$, the fuzzy (r,s)-interior is defined by*

$int_{T_1,T_2}(A,r,s)=\cup\{B\in I(X)\mid A\supseteq B, B\,\text{is fuzzy}(r,s)-\text{open}\}$.

and the fuzzy (r,s)-closure is defined by
$cl_{T_1,T_2}(A,r,s) = \cap\{B \in I(X) \mid A \subseteq B, B \text{ is fuzzy}(r,s)-\text{closed}\}$.

The operators $int : I(X)\times I\otimes I \to I(X)$ and $cl : I(X)\times I\otimes I \to I(X)$ are callled the fuzzy interior operotar and fuzzy closure operator in (X,T_1,T_2), respectively.

Lemma 2.1 *[8] For an intuitionistic fuzzy set* A *in a SoIFTS* (X,T_1,T_2) *and* $(r,s)\in I\otimes I$

1. $int_{T_1,T_2}(A,r,s)^c = cl_{T_1,T_2}(A^c,r,s)$,
2. $cl_{T_1,T_2}(A,r,s)^c = int_{T_1,T_2}(A^c,r,s)$.

Definition 2.8 *[8] Let* (X,T) *be an intuitionistic fuzzy topological space in* $\hat{S}$*ostak's sense. Then it is easy to see that for each* $(r,s)\in I\otimes I$*, the family* $T_{(r,s)}$ *defined by*

$T_{(r,s)} = \{A \in I(X) \mid T_1(A) \ge r \text{ and } T_2(A) \le s\}$

is an intuitionistic fuzzy topology on X.

Definition 2.9 *[8] Let* (X,T) *be an intuitionistic fuzzy topological space* $(r,s)\in I\otimes I$*. Then the map* $T^{(r,s)} : I(X) \to I\otimes I$ *defined by*

$$T^{(r,s)}(A) = \begin{cases} (1,0), & \text{if } A = 0,1 \\ (r,s), & \text{if } A \in T-\{0,1\} \\ (0,1), & \text{otherwise,} \end{cases}$$

becomes an intuitionistic fuzzy topology in $\hat{S}$ostak's sense on X.

Definition 2.10 *[9] Let* A *be an intuitionistic fuzzy set in a SoIFTS* (X,T_1,T_2) *and* $(r,s)\in I\otimes I$*. Then* A *is said to be*

1. fuzzy (r,s)-semi open if there is a fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq cl_{T_1,T_2}(B,r,s)$,
2. fuzzy (r,s)-semi closed if there is a fuzzy (r,s)-closed set B in X such that $int_{T_1,T_2}(B,r,s) \subseteq A \subseteq B$.

Definition 2.11 *[11] Let* A *be an intuitionistic fuzzy set in a SoIFTS* (X,T_1,T_2) *and* $(r,s)\in I\otimes I$*. Then* A *is said to be*

1. fuzzy (r,s)-regular open if $A = int_{T_1,T_2}(cl_{T_1,T_2}(A,r,s),r,s)$,
2. fuzzy (r,s)-regular closed if $A = cl_{T_1,T_2}(int_{T_1,T_2}(A,r,s),r,s)$,
3. fuzzy (r,s)-α open if $A \subseteq int_{T_1,T_2}(cl_{T_1,T_2}(int_{T_1,T_2}(A,r,s),r,s)r,s)$,
4. fuzzy (r,s)-α closed if $A \supseteq cl_{T_1,T_2}(int_{T_1,T_2}(cl_{T_1,T_2}(A,r,s),r,s)r,s)$,
5. fuzzy (r,s)-pre open if $A \subseteq int_{T_1,T_2}(cl_{T_1,T_2}(A,r,s),r,s)$,
6. fuzzy (r,s)-pre closed if $A \supseteq cl_{T_1,T_2}(int_{T_1,T_2}(A,r,s),r,s)$,
7. fuzzy (r,s)-β open if $A \subseteq cl_{T_1,T_2}(int_{T_1,T_2}(cl_{T_1,T_2}(A,r,s),r,s)r,s)$,
8. fuzzy (r,s)-β closed if $A \supseteq int_{T_1,T_2}(cl_{T_1,T_2}(int_{T_1,T_2}(A,r,s),r,s)r,s)$,
9. fuzzy (r,s)-regular semi open if there is a fuzzy (r,s)-regular open set B in X such that $B \subseteq A \subseteq cl_{T_1,T_2}(B,r,s)$,
10. fuzzy (r,s)-regular semi closed if there is a fuzzy (r,s)-regular closed set B in X such that $int_{T_1,T_2}(B,r,s) \subseteq A \subseteq B$.

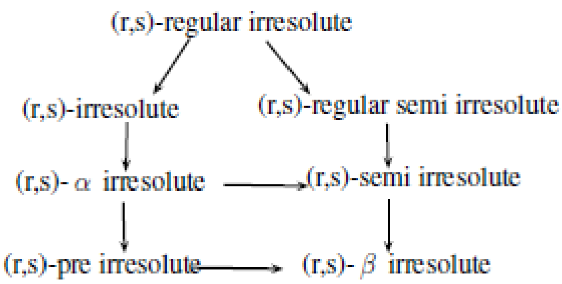
Definition 2.12 *[11] Let* (X, T_1, T_2) *be a SoIFTS. For each* $(r,s) \in I \otimes I$ *and for each* $A \in I(X)$ *, the fuzzy* (r,s)*-regular semi (resp.* (r,s)*-*α*,* (r,s)*-pre and* (r,s)*-*β*)-interior is defined by* $rsint_{T_1,T_2}(A,r,s)$ (resp. $\alpha int_{T_1,T_2}(A,r,s)$, $pint_{T_1,T_2}(A,r,s)$ and $\beta int_{T_1,T_2}(A,r,s)$)
$= \bigcup \{B \in I(X) \mid A \supseteq B, B$ is fuzzy $(r,s)-$regular semi (resp. α, pre and β) $-$ open$\}$ *and the fuzzy* (r,s) *-regular semi (resp.* (r,s) *-* α *,* (r,s) *-pre and* (r,s) *-* β *) - closure is defined by* $rscl_{T_1,T_2}(A,r,s)$ (resp. $\alpha cl_{T_1,T_2}(A,r,s)$, $pcl_{T_1,T_2}(A,r,s)$ and $\beta cl_{T_1,T_2}(A,r,s)) = \bigcap \{B \in I(X) \mid A \subseteq B,$
B is fuzzy $(r,s)-$regular semi(resp. α, pre and β) $-$ closed$\}$

3.Fuzzy (r,s)-regular semi(resp. α, pre and β) irresolute functions

Definition 3.1 *Let* $f:(X,T_1,T_2) \to (Y,W_1,W_2)$ *be a mapping from a SoIFTS* X *to another SoIFTS* Y *and* $(r,s) \in I \otimes I$*. Then* f *is said to be*

1. fuzzy (r,s)-regular semi (resp. α, pre and β) irresolute if $f^{-1}(B)$ is a fuzzy (r,s)-regular semi (resp. α, pre and β) open set of X for each fuzzy (r,s)-regular semi (resp. α, pre and β) open set B of Y,
2. fuzzy (r,s)-regular semi irresolute open if $f(A)$ is a fuzzy (r,s)-regular semi (resp. α, pre and β) open set of Y for each fuzzy (r,s)-regular semi (resp. α, pre and β) open set A of X,
3. fuzzy (r,s)-regular semi (resp. α, pre and β) irresolute closed if $f(A)$ is a fuzzy (r,s)-regular semi (resp. α, pre and β) closed set of Y for each fuzzy (r,s)-regular semi (resp. α ,pre and β) closed set A of X,
4. fuzzy (r,s)-regular semi (resp. α, pre and β) irresolute homeomorphism iff f is bijective, f and f^{-1} are fuzzy (r,s)-regular semi (resp. α, pre and β) irresolute.

From the above definitions it is clear that the following implications are true for $(r,s) \in I \otimes I$



The converses of the above implications are not true as the following examples show:

Example 3.1 *Let* $X = \{a,b\} = Y$ *and let* $A_1, A_2 \in I(X)$ *,* $B_1 \in I(Y)$ *be defined by* $A_1(x) = B_1(x) = (0.1, 0.8), A_1(y) = B_1(y) = (0.1, 0.5)$ *,* $A_2(x) = (0.3, 0.6), A_2(y) = (0.3, 0.5)$ *. Define* $T: I(X) \to I \otimes I$ *and* $W: I(Y) \to I \otimes I$ *by*

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = \underset{\sim}{0}, \underset{\sim}{1}, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, A_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

$$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1,0) & \text{if } B = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } B = B_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

For $r = \frac{1}{2}, s = \frac{1}{2}$. Then the identity mapping $f : (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(\frac{1}{2}, \frac{1}{2})$-irresolute which is not fuzzy $(\frac{1}{2}, \frac{1}{2})$-regular irresolute. Since B_1 is fuzzy $(\frac{1}{2}, \frac{1}{2})$-regular open in W but $f^{-1}(B_1)$ is not fuzzy $(\frac{1}{2}, \frac{1}{2})$-regular open in T.

Example 3.2 *Let* $X = \{a, b\} = Y$ *, and let* $A_1, A_2 \in I(X)$ *,* $B_1 \in I(Y)$ *be defined as* $A_1(x) = B_1(x) = (0.1, 0.8), A_1(y) = B_1(y) = (0.1, 0.5)$ *,* $B_2(x) = (0.3, 0.6), B_2(y) = (0.3, 0.5)$ *. Define* $T : I(X) \to I \otimes I$ *and* $W : I(Y) \to I \otimes I$ *by*

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

$$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1,0) & \text{if } B = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } B = B_1, B_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

For $r = \frac{1}{2}, s = \frac{1}{2}$. Then the identity mapping $f : (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(\frac{1}{2}, \frac{1}{2})$-regular semi irresolute which is not fuzzy $(\frac{1}{2}, \frac{1}{2})$-regular irresolute. Since B_2 is fuzzy $(\frac{1}{2}, \frac{1}{2})$-regular open in W but $f^{-1}(B_2)$ is not fuzzy $(\frac{1}{2}, \frac{1}{2})$-regular open in T.

Example 3.3 *Let* $X = \{a, b\} = Y$*, and lett* $A_1 \in I(X), B_1, B_2, B_1 \cup B_2, B_1 \cap B_2 \in I(Y)$ *be defined as* $A_1(x) = B_1(x) = (0.3, 0.1), A_1(y) = B_1(y) = (0.3, 0.1)$ *,* $B_2(x) = (0.4, 0.3), B_2(y) = (0.4, 0.3)$ *,* $B_1(x) \cup B_2(x) = (0.4, 0.1), B_1(y) \cup B_2(y) = (0.4, 0.1)$ *,* $B_1(x) \cap B_2(x) = (0.3, 0.3), B_1(y) \cap B_2(y) = (0.3, 0.3)$ *Define* $T : I(X) \to I \otimes I$ *and* $W : I(Y) \to I \otimes I$ *by*

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

$$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1,0) & \text{if } B = 0, 1, \\ (\frac{1}{2}, \frac{1}{2}) & \text{if } B = B_1, B_2, B_1 \cup B_2, B_1 \cap B_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

For $r = \frac{1}{2}, s = \frac{1}{2}$. Then the identity mapping $f : (X, T_1, T_2) \to (Y, W_1, W_2)$ is fuzzy $(\frac{1}{2}, \frac{1}{2})$- α irresolute which is not fuzzy $(\frac{1}{2}, \frac{1}{2})$-irresolute. Since B_2 is fuzzy $(\frac{1}{2}, \frac{1}{2})$-open in W but $f^{-1}(B_2)$ is not fuzzy $(\frac{1}{2},\frac{1}{2})$-open in T.

Example 3.4 *Let* $X=\{a,b\}=Y$, *and let* $A_1 \in I(X)$, $B_1, B_2 \in I(Y)$ *be defined as* $A_1(x)=B_1(x)=(0.3,0.6), A_1(y)=B_1(y)=(0.3,0.5)$, $B_2(x)=(0.4,0.3), B_2(y)=(0.4,0.3)$ *Define* $T: I(X)\to I\otimes I$ *and* $W: I(Y)\to I\otimes I$ *by*

$$T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_{:},1_{:}, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1, \\ (0,1) & \text{otherwise.}\end{cases}$$

$$W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_{:},1_{:}, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1,B_2, \\ (0,1) & \text{otherwise.}\end{cases}$$

For $r=\frac{1}{2}, s=\frac{1}{2}$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(\frac{1}{2},\frac{1}{2})$-semi irresolute, β-irresolute which is not fuzzy $(\frac{1}{2},\frac{1}{2})$-$\alpha$ irresolute, pre irresolute. Since B_2 is fuzzy $(\frac{1}{2},\frac{1}{2})$-pre open, α open in W but $f^{-1}(B_2)$ is not fuzzy $(\frac{1}{2},\frac{1}{2})$-pre open, α open in T.

Example 3.5 *Let* $X=\{a,b\}=Y$, *and let* $A_1 \in I(X)$, $B_1, B_2 \in I(Y)$ *be defined as* $A_1(x)=(0.2,0.2), A_1(y)=(0.1,0.2)$, $B_1(x)=(0.1,0.1), B_1(y)=(0.1,0.1)$. *Define* $T: I(X)\to I\otimes I$ *and* $W: I(Y)\to I\otimes I$ *by*

$$T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_{:},1_{:}, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1, \\ (0,1) & \text{otherwise.}\end{cases}$$

$$W(B)=(W_1(B),W_2(B))=\begin{cases}(1,0) & \text{if } B=0_{:},1_{:}, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } B=B_1, \\ (0,1) & \text{otherwise.}\end{cases}$$

For $r=\frac{1}{2}, s=\frac{1}{2}$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(\frac{1}{2},\frac{1}{2})$-semi irresolute which is not fuzzy $(\frac{1}{2},\frac{1}{2})$-regular semi irresolute. Since B_1 is fuzzy $(\frac{1}{2},\frac{1}{2})$-regular semi open in W but $f^{-1}(B_1)$ is not fuzzy $(\frac{1}{2},\frac{1}{2})$-regular semi open in T.

Example 3.6 *Let* $X=\{a,b\}=Y$, *and let* $A_1 \in I(X)$, $B_1, B_2 \in I(Y)$ *be defined as* $A_1(x)=(0.1,0.1), A_1(y)=(0.1,0.1)$, $B_1(x)=(0.2,0.2), B_1(y)=(0.1,0.2)$. *Define* $T: I(X)\to I\otimes I$ *and* $W: I(Y)\to I\otimes I$ *by*

$$T(A)=(T_1(A),T_2(A))=\begin{cases}(1,0) & \text{if } A=0_{:},1_{:}, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } A=A_1, \\ (0,1) & \text{otherwise.}\end{cases}$$

$$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1,0) & \text{if } B = 0, 1, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } B = B_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

For $r=\frac{1}{2}, s=\frac{1}{2}$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(\frac{1}{2},\frac{1}{2})$-semi irresolute which is not fuzzy $(\frac{1}{2},\frac{1}{2})$-$\beta$ irresolute. Since B_1 is fuzzy $(\frac{1}{2},\frac{1}{2})$-$\beta$ open in W but $f^{-1}(B_1)$ is not fuzzy $(\frac{1}{2},\frac{1}{2})$-$\beta$ open in T.

Example 3.7 *Let* $X=\{a,b\}=Y$, *and let* $A_1 \in I(X)$, $B_1, B_2, B_1\cup B_2, B_1\cap B_2 \in I(Y)$ *be defined as* $A_1(x)=B_2(x)=(0.1,0.3)$, $A_1(y)=B_2(y)=(0.1,0.3)$ $B_1(x)=(0.3,0.6)$, $B_1(y)=(0.3,0.5)$, $B_1(x)\cup B_2(x)=(0.3,0.3)$, $B_1(y)\cup B_2(y)=(0.3,0.3)$, $B_1(x)\cap B_2(x)=(0.1,0.6)$, $B_1(y)\cap B_2(y)=(0.1,0.5)$ *Define* $T: I(X)\to I\otimes I$ *and* $W: I(Y)\to I\otimes I$ *by*

$$T(A) = (T_1(A), T_2(A)) = \begin{cases} (1,0) & \text{if } A = 0, 1, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

$$W(B) = (W_1(B), W_2(B)) = \begin{cases} (1,0) & \text{if } B = 0, 1, \\ (\frac{1}{2},\frac{1}{2}) & \text{if } B = B_1, B_2, B_1\cup B_2, B_1\cap B_2, \\ (0,1) & \text{otherwise.} \end{cases}$$

For $r=\frac{1}{2}, s=\frac{1}{2}$. Then the identity mapping $f:(X,T_1,T_2)\to(Y,W_1,W_2)$ is fuzzy $(\frac{1}{2},\frac{1}{2})$-$\alpha$ irresolute which is not fuzzy $(\frac{1}{2},\frac{1}{2})$-pre irresolute. Since B_2 is fuzzy $(\frac{1}{2},\frac{1}{2})$-pre open in W but $f^{-1}(B_1)$ is not fuzzy $(\frac{1}{2},\frac{1}{2})$-pre open in T.

Theorem 3.1 *Let* $(X,T_1,T_2),(Y,W_1,W_2)$ *be SoIFTSs and let* $f: X\to Y$ *be mappings and* $(r,s)\in I\otimes I$. *Then the following statements are equivalent:*

1. f is fuzzy (r,s)-regular semi irresolute.
2. For each (r,s)-frsc set $B\in I^Y$, $f^{-1}(B)$ is (r,s)-frsc set in X.
3. $f(rscl_{T_1,T_2}(A,r,s)) \subseteq rscl_{W_1,W_2}(f(A),r,s)$, for each $A\in I^X$ and $(r,s)\in I\otimes I$.
4. $rscl_{T_1,T_2}(f^{-1}(B),r,s)\subseteq f^{-1}(rscl_{W_1,W_2}(B,r,s))$, for each $B\in I^Y$ and $(r,s)\in I\otimes I$.
5. $f^{-1}(rsint_{W_1,W_2}(B,r,s))\subseteq rsint_{T_1,T_2}(f^{-1}(B),r,s)$, for each $B\in I^Y$ and $(r,s)\in I\otimes I$.

Proof. (i) $\Leftrightarrow$ (ii) It is easily proved from Definition 10(`)@

(ii) $\Rightarrow$ (iii) Let (ii) holds and let $f(rscl_{T_1,T_2}(A,r,s))$ Ú $rscl_{W_1,W_2}(f(A),r,s)$ for some $A\in I^X$ and $(r,s)\in I\otimes I$. So there exists $y\in Y, t\in(0,1]$ such that:

$$f(rscl_{T_1,T_2}(A,r,s))(y) > t > rscl_{W_1,W_2}(f(A),r,s)(y)$$

If $f^{-1}(y)=\phi$, it is a contradiction, because $f(rscl_{T_1,T_2}(A,r,s))(y)=0$. So, if $f^{-1}(y)\neq\phi$, there exists $x\in f^{-1}(y)$ such that

$$f(rscl_{T_1,T_2}(A,r,s))(y) \supseteq rscl_{T_1,T_2}(A,r,)(x) > t > rscl_{W_1,W_2}(f(A),r,s)(f(x)) \quad (1)$$

Also, $rscl_{W_1,W_2}(f(A),r,s)(f(x)) < t$, implies that there exists r-frsc set B with $f(A) = B$ such that $rscl_{W_1,W_2}(f(A),r,s)(f(x)) \subseteq B(f(x)) < t$. Moreover, $f(A) \subseteq B$ implies $A \subseteq f^{-1}(B)$. From (2), $f^{-1}(B)$ is (r,s)-frsc. Thus $rscl_{T_1,T_2}(A,r) \subseteq f^{-1}(B)$ and this implies:

$$rscl_{T_1,T_2}(A,r,s)(x) \subseteq f^{-1}(B)(x) = B(f(x)) < t \quad (2)$$

From relations (1) and (2) we see that:
$rscl_{T_1,T_2}(A,r,s)(x) > t$ and $rscl_{T_1,T_2}(A,r,s)(x) \le t$ which is a contradiction. Hence the result.

(iii)$\Rightarrow$(iv) Put $A = f^{-1}(B)(B \in I^Y)$ and apply (iii) we have

$$f(rscl_{T_1,T_2}(f^{-1}(B),r,s)) \subseteq rscl_{W_1,W_2}(f(f^{-1}(B),r,s)) \subseteq rscl_{W_1,W_2}(B,r,s).$$

Hence, $rscl_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(rscl_{W_1,W_2}(B,r,s))$.

(iv)$\Rightarrow$(v) It follows immediately by taking the complement of (iv).

(v)$\Rightarrow$(i) Let (v) holds and let B be (r,s)-frso in Y. By (v) $f^{-1}(rsint_{W_1,W_2}(B,,s,r)) \subseteq rsint_{T_1,T_2}(f^{-1}(B),r,s)$. Then, $f^{-1}(B) \subseteq rsint_{T_1,T_2}(f^{-1}(B),r,s)$. But $rsint_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(B)$. Hence $rsint_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(B)$. So, $f^{-1}(B)$ is (r,s)-frso and there by f is (r,s)-fuzzy regular semi irresolute.

Theorem 3.2 *Let $(X,T_1,T_2),(Y,W_1,W_2)$ be SoIFTSs and let $f: X \to Y$ be mappings and $(r,s) \in I \otimes I$. Then the following statements are equivalent:*

1. f is fuzzy (r,s)-α irresolute.
2. For each fuzzy (r,s)-α closed set $B \in I^Y$, $f^{-1}(B)$ is fuzzy (r,s)-α closed set in X.
3. $f(\alpha cl_{T_1,T_2}(A,r,s)) \subseteq \alpha cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$ and $(r,s) \in I \otimes I$.
4. $\alpha cl_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(\alpha cl_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.
5. $f^{-1}(\alpha int_{W_1,W_2}(B,r,s)) \subseteq \alpha int_{T_1,T_2}(f^{-1}(B),r,s)$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.

Proof. It follows from Theorem 4(`)@

Theorem 3.3 *Let $(X,T_1,T_2),(Y,W_1,W_2)$ be SoIFTSs and let $f: X \to Y$ be mappings and $(r,s) \in I \otimes I$. Then the following statements are equivalent:*

1. f is fuzzy (r,s)-pre irresolute.
2. For each fuzzy (r,s)-pre closed set $B \in I^Y$, $f^{-1}(B)$ is fuzzy (r,s)-pre closed set in X.
3. $f(pcl_{T_1,T_2}(A,r,s)) \subseteq pcl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$ and $(r,s) \in I \otimes I$.
4. $pcl_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(pcl_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.
5. $f^{-1}(pint_{W_1,W_2}(B,r,s)) \subseteq pint_{T_1,T_2}(f^{-1}(B),r,s)$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.

Proof. It follows from Theorem 4(`)@

Theorem 3.4 *Let $(X,T_1,T_2),(Y,W_1,W_2)$ be SoIFTSs and let $f: X \to Y$ be mappings and $(r,s) \in I \otimes I$. Then the following statements are equivalent:*

1. f is fuzzy (r,s)-β irresolute.
2. For each fuzzy (r,s)-β closed set $B \in I^Y$, $f^{-1}(B)$ is fuzzy (r,s)-β closed set in X.

3. $f(\beta cl_{T_1,T_2}(A,r,s)) \subseteq \beta cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$ and $(r,s) \in I \otimes I$.
4. $\beta cl_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(\beta cl_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.
5. $f^{-1}(\beta int_{W_1,W_2}(B,r,s)) \subseteq \beta int_{T_1,T_2}(f^{-1}(B),r,s)$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.

Proof. It follows from Theorem 4(`)@

Theorem 3.5 *Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f : X \to Y$ be a mapping. The following statements are equivalent:*

1. A map f is (r,s)-fuzzy regular semi irresolute open.
2. $f(rsint_{T_1,T_2}(A,r,s)) \subseteq rsint_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$ and $(r,s) \in I \otimes I$.
3. $rsint_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(rsint_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.
4. For any $A \in I^Y$ and any (r,s)-frso set $B \in I^X$ such that $B \supseteq f^{-1}(A)$, there exists a (r,s)-frscl set $C \in I^Y$ with $A \subseteq C$ such that $f^{-1}(C) \subseteq A$.

Proof. (i)$\Rightarrow$(ii) Since $rsint_{T_1,T_2}(A,r,s) \subseteq A$, then $rsint_{T_1,T_2}(A,r,s) \subseteq f(A)$. By (i),

$$f(rsint_{T_1,T_2}(A,r,s)) \subseteq rsint_{W_1,W_2}(f(A),r,s).$$

(ii)$\Rightarrow$(iii) Let (ii) holds. Take $A = f^{-1}(B), A \in I^Y$ and apply part (ii).

(iii) $\Rightarrow$ (iv) Let $B \in I^X$ be a (r,s) -frsc set. Since $f^{-1}(A) \subseteq B$, then $1 - B \subseteq 1 - f^{-1}(A) = f^{-1}(1 - A)$. It follows, $rsint_{T_1,T_2}(1 - B,r,s) \subseteq 1 - B \subseteq rsint_{T_1,T_2}(f^{-1}(1 - A),r,s)$ By (iii),

$$1 - B \subseteq rsint_{T_1,T_2}(f^{-1}(1 - A),r,s) \subseteq f^{-1}(rsint_{W_1,W_2}(1 - A,r,s)).$$

It implies $B \supseteq 1 - f^{-1}(rsint_{W_1,W_2}(1 - A,r,s)) = f^{-1}(1 - rsint_{W_1,W_2}(1 - A,r,s)) = f^{-1}(rscl_{W_1,W_2}(A,r,s))$. So, $B \supseteq f^{-1}(rscl_{T_1,T_2}(A,r,s))$. Take $C = rscl_{T_1,T_2}(A,r,s))$. Then C is (r,s)-frsc such that $B \supseteq f^{-1}(C)$ and $C \supseteq A$. Hence the result.

(iv)$\Rightarrow$(i) Let D be (r,s)-frso in X. Put $A = 1 - f(D)$ and $B = 1 - D$. It is easy to see that $B \supseteq f^{-1}(A)$. By part (iv), there exists (r,s)-frsc set $C \in I^Y$ such that $C \supseteq A$ and $B \supseteq f^{-1}(C)$ or $1 - D \supseteq f^{-1}(C)$. It implies $D \subseteq 1 - f^{-1}(C) = f^{-1}(1 - C)$. Thus $f(w) \subseteq ff^{-1}(1 - C) \subseteq 1 - C$. On the other hand, $A \subseteq C$, $f(D) = 1 - A$ implies $f(D) = 1 - A \supseteq 1 - C$. Finally, we have $f(D) = 1 - C$ and therefore $f(D)$ is (r,s)-frso. Hence f is fuzzy (r,s) regular semi irresolute open.

Theorem 3.6 *Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f : X \to Y$ be a mapping. The following statements are equivalent:*

1. A map f is fuzzy (r,s)-α irresolute open.
2. $f(\alpha int_{T_1,T_2}(A,r,s)) \subseteq \alpha int_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$ and $(r,s) \in I \otimes I$.
3. $\alpha int_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(\alpha int_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.
4. For any $A \in I^Y$ and any fuzzy (r,s)-α open set $B \in I^X$ such that $B \supseteq f^{-1}(A)$, there exists a fuzzy (r,s)-α closed set $C \in I^Y$ with $A \subseteq C$ such that $f^{-1}(C) \subseteq A$.

Proof. It follows from Theorem 5(`)@

Theorem 3.7 *Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f : X \to Y$ be a mapping. The* *following statements are equivalent:*

1. A map f is fuzzy (r,s)-pre irresolute open.
2. $f(pint_{T_1,T_2}(A,r,s)) \subseteq pint_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$ and $(r,s) \in I \otimes I$.
3. $pint_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(pint_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.
4. For any $A \in I^Y$ and any fuzzy (r,s)-pre open set $B \in I^X$ such that $B \supseteq f^{-1}(A)$, there exists a fuzzy (r,s)-pre closed set $C \in I^Y$ with $A \subseteq C$ such that $f^{-1}(C) \subseteq A$.

Proof. It follows from Theorem 5(`)@

Theorem 3.8 *Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f: X \to Y$ be a mapping. The following statements are equivalent:*

1. A map f is fuzzy (r,s)-β irresolute open.
2. $f(\beta int_{T_1,T_2}(A,r,s)) \subseteq \beta int_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$ and $(r,s) \in I \otimes I$.
3. $\beta int_{T_1,T_2}(f^{-1}(B),r,s) \subseteq f^{-1}(\beta int_{W_1,W_2}(B,r,s))$, for each $B \in I^Y$ and $(r,s) \in I \otimes I$.
4. For any $A \in I^Y$ and any fuzzy (r,s)-β open set $B \in I^X$ such that $B \supseteq f^{-1}(A)$, there exists a fuzzy (r,s)-β closed set $C \in I^Y$ with $A \subseteq C$ such that $f^{-1}(C) \subseteq A$.

Proof. It follows from Theorem 5(`)@

Theorem 3.9 *Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f: X \to Y$ be a mapping. The following statements are equivalent:*

1. f is fuzzy (r,s)-regular semi irresolute closed.
2. $f(rscl_{T_1,T_2}(A,r,s)) \supseteq rscl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$.

Proof. (i)$\Rightarrow$(ii) Let $A \in I^X$. Since $A \subseteq rscl_{T_1,T_2}(A,r,s)$, then $f(A) \subseteq f(rscl_{T_1,T_2}(A,r,s))$. It implies $rscl_{W_1,W_2}(f(A),r,s) \subseteq f(rscl_{T_1,T_2}(A,r,s))$.

(ii)$\Rightarrow$(i) Let (ii) holds and $A \in I^X$ such that A is (r,s)-frsc. Then $rscl_{W_1,W_2}(f(A),r,s) \subseteq f(A)$. But $f(A) \subseteq rscl_{W_1,W_2}(f(A),r,s)$. Hence $f(A)$ is (r,s)-frsc and therefore f is fuzzy (r,s)-regular semi irresolute closed.

Theorem 3.10 *Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f: X \to Y$ be a mapping. The following statements are equivalent:*

1. f is fuzzy (r,s)-α irresolute closed.
2. $f(\alpha cl_{T_1,T_2}(A,r,s)) \supseteq \alpha cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$.

Proof. It follows from Theorem 4(`)@

Theorem 3.11 *Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f: X \to Y$ be a mapping. The following statements are equivalent:*

1. f is fuzzy (r,s)-pre irresolute closed.
2. $f(pcl_{T_1,T_2}(A,r,s)) \supseteq pcl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$.

Proof. It follows from Theorem 4(`)@

Theorem 3.12 *Let (X,T_1,T_2) and (Y,W_1,W_2) be SoIFTSs. Let $f: X \to Y$ be a mapping. The following statements are equivalent:*

1. f is fuzzy (r,s)-β irresolute closed.

2. $f(\beta cl_{T_1,T_2}(A,r,s)) \supseteq \beta cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X$.

Proof. It follows from Theorem 4(`)@

Theorem 3.13 *Let* (X,T_1,T_2) *and* (Y,W_1,W_2) *be SoIFTS's. Let* $f: X \to Y$ *is bijective. Then :*

1. f is fuzzy (r,s)-regular semi irresolute closed iff $f^{-1}(rscl_{W_1,W_2}(B,r,s)) \subseteq rscl_{T_1,T_2}(f^{-1}(B),r,s)$ for each $B \in I^Y$ and $(r,s) \in I \otimes I$.

2. f is fuzzy (r,s)-regular semi irresolute closed iff f is fuzzy (r,s)-regular semi irresolute open.

Proof. (1)($\Rightarrow$): Let f be fuzzy (r,s)-regular semi irresolute closed. From Theorem 4(`)@ we have:

$f(rscl_{T_1,T_2}(A,r,s)) \supseteq rscl_{W_1,W_2}(f(A),r,s))$, $A \in I^X$.

Let $B \in I^Y$ and put $A = f^{-1}(B)$, we have

$f(rscl_{T_1,T_2}(f^{-1}(B),r,s)) \supseteq rscl_{W_1,W_2}(ff^{-1}(B),r,s) = rscl_{W_1,W_2}(B,r,s)$.

It implies $rscl_{T_1,T_2}(f^{-1}(B),r,s) \supseteq f^{-1}(rscl_{W_1,W_2}(B,r,s))$.

($\Leftarrow$): On the other hand let the condition is satisfied and let $B \in I^X$ such that B is (r,s)-frsc. Then $f(B) \in I^Y$. Apply the condition we have:

$$rscl_{T_1,T_2}(f^{-1}f(B,r,s)) \supseteq f^{-1}(rscl_{W_1,W_2}(f(B),r,s),r,s).$$

It implies that $rscl_{T_1,T_2}(B,r,s) \supseteq f^{-1}(rscl_{W_1,W_2}(f(B),r,s),r,s)$. Then, $f(rscl_{T_1,T_2}(B,r,s)) \supseteq rscl_{W_1,W_2}(f(B),r,s)$. So by Theorem 4(`)@ f is fuzzy (r,s)-regular semi irresolute closed.

(ii) Apply Theorem 4(`)@ and taking the complement we have the required result.

From Theorems 4(`)@ 5(`)@ 4(`)@ 2(`)@ we obtain the following Theorem.

Theorem 3.14 *Let* (X,T_1,T_2) *and* (Y,W_1,W_2) *be SoIFTS's. Let* $f: X \to Y$ *is bijective. Then :*

1. f is fuzzy (r,s)-α irresolute closed iff $f^{-1}(\alpha cl_{W_1,W_2}(B,r,s)) \subseteq \alpha cl_{T_1,T_2}(f^{-1}(B),r,s)$ for each $B \in I^Y$ and $(r,s) \in I \otimes I$.

2. f is fuzzy (r,s)-α irresolute closed iff f is fuzzy (r,s)-α irresolute open.

Proof. It follows from Theorem 2(`)@

Theorem 3.15 *Let* (X,T_1,T_2) *and* (Y,W_1,W_2) *be SoIFTS's. Let* $f: X \to Y$ *is bijective. Then :*

1. f is fuzzy (r,s)-pre irresolute closed iff $f^{-1}(pcl_{W_1,W_2}(B,r,s)) \subseteq pcl_{T_1,T_2}(f^{-1}(B),r,s)$ for each $B \in I^Y$ and $(r,s) \in I \otimes I$.

2. f is fuzzy (r,s)-pre irresolute closed iff f is fuzzy (r,s)-pre irresolute open.

Proof. It follows from Theorem 2(`)@

Theorem 3.16 *Let* (X,T_1,T_2) *and* (Y,W_1,W_2) *be SoIFTS's. Let* $f: X \to Y$ *is bijective. Then :*

1. f is fuzzy (r,s)-β irresolute closed iff $f^{-1}(\beta cl_{W_1,W_2}(B,r,s)) \subseteq \beta cl_{T_1,T_2}(f^{-1}(B),r,s)$ for each $B \in I^Y$ and $(r,s) \in I \otimes I$.

2. f is fuzzy (r,s)-β irresolute closed iff f is fuzzy (r,s)-β irresolute open.

Proof. It follows from Theorem 2(`)@

Theorem 3.17 *Let* (X,T_1,T_2) *and* (Y,W_1,W_2) *be SoIFTSs. Let* $f: X \to Y$ *be a mapping. The* *following statements are equivalent:*

1. f is fuzzy (r,s)-regular semi irresolute homeomorphism,
2. f is fuzzy (r,s)-regular semi irresolute and fuzzy (r,s)-regular semi irresolute open,
3. f is fuzzy (r,s)-regular semi irresolute and fuzzy (r,s)-regular semi irresolute closed,
4. $f(rsint_{T_1,T_2}(A,r,s)) = rsint_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X, (r,s) \in I \otimes I$,
5. $f(rscl_{T_1,T_2}(A,r,s)) = rscl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X, (r,s) \in I \otimes I$,
6. $rsint_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(rsint_{W_1,W_2}(B,r,s))$,
7. $rscl_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(rscl_{W_1,W_2}(B,r,s))$, $\mu \in I^Y, (r,s) \in I \otimes I$.

Theorem 3.18 *Let* (X,T_1,T_2) *and* (Y,W_1,W_2) *be SoIFTSs. Let* $f: X \to Y$ *be a mapping. The following statements are equivalent:*

1. f is fuzzy (r,s)-α irresolute homeomorphism,
2. f is fuzzy (r,s)-α irresolute and fuzzy (r,s)-α irresolute open,
3. f is fuzzy (r,s)-α irresolute and fuzzy (r,s)-α irresolute closed,
4. $f(\alpha int_{T_1,T_2}(A,r,s)) = \alpha int_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X, (r,s) \in I \otimes I$,
5. $f(\alpha cl_{T_1,T_2}(A,r,s)) = \alpha cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X, (r,s) \in I \otimes I$,
6. $\alpha int_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(\alpha int_{W_1,W_2}(B,r,s))$,
7. $\alpha cl_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(\alpha cl_{W_1,W_2}(B,r,s))$, $\mu \in I^Y, (r,s) \in I \otimes I$,

Theorem 3.19 *Let* (X,T_1,T_2) *and* (Y,W_1,W_2) *be SoIFTSs. Let* $f: X \to Y$ *be a mapping. The following statements are equivalent:*

1. f is fuzzy (r,s)-pre irresolute homeomorphism,
2. f is fuzzy (r,s)-pre irresolute and fuzzy (r,s)-pre irresolute open,
3. f is fuzzy (r,s)-pre irresolute and fuzzy (r,s)-pre irresolute closed,
4. $f(pint_{T_1,T_2}(A,r,s)) = pint_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X, (r,s) \in I \otimes I$,
5. $f(pcl_{T_1,T_2}(A,r,s)) = pcl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X, (r,s) \in I \otimes I$,
6. $pint_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(pint_{W_1,W_2}(B,r,s))$,
7. $pcl_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(pcl_{W_1,W_2}(B,r,s))$, $\mu \in I^Y, (,r,s) \in I \otimes I$.

Theorem 3.20 *Let* (X,T_1,T_2) *and* (Y,W_1,W_2) *be SoIFTSs. Let* $f: X \to Y$ *be a mapping. The following statements are equivalent:*

1. f is fuzzy (r,s)-β irresolute homeomorphism,
2. f is fuzzy (r,s)-β irresolute and fuzzy (r,s)-β irresolute open,
3. f is fuzzy (r,s)-β irresolute and fuzzy (r,s)-β irresolute closed,
4. $f(\beta int_{T_1,T_2}(A,r,s)) = \beta int_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X, (r,s) \in I \otimes I$,
5. $f(\beta cl_{T_1,T_2}(A,r,s)) = \beta cl_{W_1,W_2}(f(A),r,s)$, for each $A \in I^X, (r,s) \in I \otimes I$,
6. $\beta int_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(\beta int_{W_1,W_2}(B,r,s))$,
7. $\beta cl_{T_1,T_2}(f^{-1}(B),r,s) = f^{-1}(\beta cl_{W_1,W_2}(B,r,s))$, $\mu \in I^Y, (r,s) \in I \otimes I$.

Note that the composition of two fuzzy regular semi(resp. α ,pre and β) irresolute mappings is fuzzy regular semi(resp. α ,pre and β) irresolute. In general, the composition of two fuzzy regular semi (resp. α ,pre and β) continuous mappings is not fuzzy regular semi(resp. α ,pre, β) continuous.

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