A NEW APPROACH FOR SOLVING FUZZY GAME PROBLEM

[1]E.SARAVANABHAVAN, [2]A.MAGESWARI, [3]M.V.SURESH,
[1]Reasearch Scholar, Department of Mathematics, SPIHER, Avadi, Chennai-54
[2] Asst. Prof, Department of Mathematics, SPIHER, Avadi, Chennai-54
[3] Asst. Prof, Department of Mathematics, SPIHER, Avadi, Chennai-54

Abstract: Game theory problems occur frequently in Economics, Business Administration, Sociology, Political sciences, Military operations and so on. To tackle the uncertainty in Games, fuzzy set theory is an excellent basis for studying the kind of game in which the Payoffs are represented by fuzzy Numbers. In this paper we consider two people zero sum Game whose imprecise values are triangular fuzzy umbers and we have solved it without converting to crisp valued Game problem using ranking criteria. Also we have proved that the optimal value of the fuzzy valued game problem without converting to the crisp valued game problem gives much more optimal, when it is solved after converting to crisp valued game problem using any ranking methods

Key Words: **Triangular fuzzy number, Fuzzy matrix, Determinant of fuzzy matrix.**

1 Introduction

In 1965, fuzzy sets were introduced by Zadeh L.A provides natural way of dealing with problems in which the source of imprecision and vagueness occurs. A fuzzy number is a quantity whose values are precise rather than exact as in the case with single valued numbers. To deal imprecise in real life situation many researchers used triangular and trapezoidal fuzzy numbers.

Fuzzy numbers and their fuzzy operations are the seeds of fuzzy number theory, and are used to modeling expert systems, cognitive computational models, measurement knowledge. When we apply the Game theory to model some practical problems which we encounter in real life situation, we have to know the values of payoffs exactly. However it is difficult to know the exact values of payoffs and we could only know the values of payoffs approximately.

In such situation, it is useful to mode the problems as games with fuzzy payoffs. Ragab et al given some properties on determinant and adjoint of square fuzzy matrix. Kim and Monoranjan B et al. presented some important results on determinant of square fuzzy matrices. Pal and Shyamal first time introduced triangular fuzzy matrices.

In many research articles, the payoff matrix is fuzzy with triangular, trapezoidal fuzzy numbers etc. But the values of the game for each player are assumed to be crisp numbers. But from the logical point of view, it seems natural that if the payoff matrix is fuzzy, the value of the game for player I and player II should also be fuzzy.

Li solved fuzzy matrix games with payoffs of triangular fuzzy number by solving auxiliary multi-objective programming models using lexicographic method in which the value of the game for each player obtained as triangular fuzzy number.

Nagoor Gani. A et al solved fully fuzzy linear programming with triangular fuzzy number using simplex algorithm without using ranking function, optimal value is also triangular fuzzy number.

2. Preliminaries

Definition 2.1 (Fuzzy set)

A Fuzzy set $\tilde{A}$ in X (Set of realnumbers) is a set of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x \in X\},$$

$\mu_{\tilde{A}}$ is called membership function of x in $\tilde{A}$ which maps X to $[0,1]$.

Definition 2.2 (α-cut of Fuzzy Set)

The α -cut of α -level setof fuzzy set $\tilde{A}$is a set consisting of those elements of the universe$X$ whose membership values exceed the threshold levelα. That is

$$\tilde{A}_{\alpha} = \{x/\mu_{\tilde{A}}(x) \geq \alpha\}.$$

Definition 2.3 (Support of Fuzzy Set)

The support of fuzzy set$\tilde{A}$ is the set of all points x in X such that $\mu_{\tilde{A}}(x) > 0$.

That is

Support$(\tilde{A}) = \{x/\mu_{\tilde{A}}(x) > 0\}$.

Definition 2.4 (Convex Fuzzy Set)

A fuzzy set $\tilde{A}$ is a convexfuzzy set if any and only if each of its α-cut $A_{\alpha}$is a convex set.

Definition 2.5 (α-cut of a triangular fuzzy number)

We get a crisp interval by α-cut operation, interval A_{α}shall be obtainedas follows, for every $\alpha \in [0,1]$.Thus

$$A_{\alpha} = [a_1^{\alpha}, a_2^{\alpha}] \quad = [(a_2 - a_1)\alpha + a_1, a_3 - (a_3 - a_2)\alpha].$$

Definition 2.6 (Fuzzy Number)

A fuzzy set $\tilde{A}$ is definedon the set of real numbers R is said to be a fuzzy numberof its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the followingcharacteristics.

i) $\tilde{A}$is normal. It means that there exists an $x \in R$ such that$\mu_{\tilde{A}}(x) = 1$.

ii) $\tilde{A}$is convex. It means that for every $x_1, x_2 \in R$,

$$\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq min\,\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\};\ \ \lambda \in [0,1].$$

iii) $\mu_{\tilde{A}}$is upper semi-continuous.

iv) $Supp(\tilde{A})$ is bounded in R.

Definition 2.7 (Arithmetic Operations on Triangular Fuzzy Number)

The following are the four operations that can be performed on triangular fuzzy numbers: Let $\tilde{A} = (a_1, a_2, a_3)$ and$\tilde{B} = (b_1, b_2, b_3)$ then,

(i) Addition

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3).$$

(ii) Subtraction

$$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1).$$

(iii) Multiplication

$\tilde{A} \times \tilde{B} = \left(\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3)\,, a_2b_2, max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)\right)$.

(iv) Division

$$\tilde{A} \div \tilde{B} = \left(\min\left(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}\right), \frac{a_2}{b_2}, max\left(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}\right)\right)$$

Definition 2.8 (Triangular Fuzzy Number)

It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$.

This representation is interpreted as membership functions and holds the following conditions

(i) a_1 to a_2 is increasing function.

(ii) a_2 to a_3 is decreasing function.

(iii) $a_1 \leq a_2 \leq a_3$

Its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for\ x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ 1 & for\ x = a_2 \\ \dfrac{a_3-x}{a_3-a_2} & for\ a_2 \le x \le a_3 \\ 0 & for\ x > a_3 \end{cases}$$

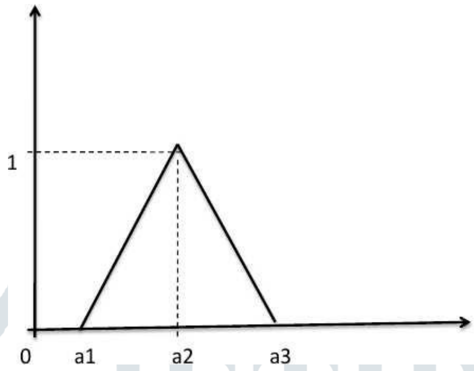

Figure 2.1: Triangular Fuzzy Number

Definition2. 9 (Ranking of Triangular Fuzzy Number)

Let$\tilde{A} = \left(\underline{A}(r), \overline{A}(r)\right)$, $0 \le r \le 1$ be a fuzzy number. The measure of$\tilde{A}$, $M_0^{Tri}(\tilde{A})$is calculated as follows

$M_0^{Tri}(\tilde{A}) = \frac{1}{2}\int_0^1 \left(\underline{A}(r), \overline{A}(r)\right) dr$, Where $0 \le r \le 1$,

$$M_0^{Tri}(\tilde{A}) = \frac{1}{4}[2a_2 + a_1 + a_3]$$

Definition 2.10 (Determinant of Triangular Fuzzy number)

The triangular fuzzy determinant of a Triangular fuzzy number matrix (TFNM) A of order $n \times n$ is denoted by $|A|$ or$det(A)$ is defined as,

$$|A| = \sum_{\sigma \in S_n} sgn\ \sigma \left(\mathrm{m}_{1\sigma(1)}, \alpha_{1\sigma(1)}, \beta_{1\sigma(1)}\right), \dots, \left(\mathrm{m}_{n\sigma(n)}, \alpha_{n\sigma(n)}, \beta_{n\sigma(n)}\right)$$

$$= \sum_{\sigma \in S_n} sgn\ \sigma \prod_{i=1}^{n} a_{i\sigma(i)}$$

Where$a_{i\sigma(i)} = \left(\mathrm{m}_{i\sigma(i)}, \alpha_{i\sigma(i)}, \beta_{i\sigma(i)}\right)$areTFNMs and S_n denotes the symmetric group of all permutationsof the indices $\{1, 2, \dots, n\}$ and $Sgn\ \sigma =\ 1$ or -1 according as thepermutation

$\sigma = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \cdots & \sigma(n) \end{pmatrix}$ is even or odd respectively.

The computation of $det(A)$ involves several products of TFNs. Since the product of two or more TFNs is an approximate TFN, the value of $det(A)$ is also an approximate TFN.

Remark 2.11

If $\tilde{A} = (a_1, a_2, a_3)$is a triangular fuzzy numberthen

(i) Modules of $\tilde{A}\ = a_2$

(ii) $div(\tilde{A}) = a_3 - a_1$

3 Mathematical Formulation of Fuzzy Game Problem

Consider 2 competitors (called players) in the market. Let Player$A$ has mStrategies $A_1, A_2, \dots, A_m$ and player B has n strategies$B_1, B_2, \dots, B_n$ Here it is assumed that each player A has his choicesfrom amongst the pure strategies. Also it is assumed that player $A$is always gainer and player B is always looser. That is all pay offsare assumed in terms of player$A$. If player A chooses strategy $A_i$and player B chooses strategy B_j then pay off matrix to player Ais

$$\begin{array}{ll} \text{Player B} & \quad B_1 \quad B_2 \quad B_3 \quad \cdots \quad B_n \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{pmatrix} \end{array}$$

3.1 Procedure for Solving Fuzzy Game Problem Using Matrix Oddment Method

Step 1:

Let $A = (a_{ij})$ be a $n \times n$ fuzzy matrix Obtain a new matrix$C$, whose first column is obtained from A by subtracting 2nd columnfrom 1st;

Second column is obtained by subtracting$A$'s 3rd columnfrom 2nd and so on till the last column of A is taken care of.

Thus$C$ is a $n \times (n - 1)$ matrix.

Step 2:

Obtain a new matrix R from$A$, by subtracting itssuccessive rows from the preceding ones, in exactly the same manner was done for the columns in step 1.

Thus R is a $(n-1) \times n$ matrix.

Step 3:

Determine the magnitude of oddments corresponding to each row and each column of A. The oddment corresponding to ith row of A is defined as the determinant $|C_i|$ where C_i is obtainedfrom C by deleting ith row.

Similarly oddment corresponding jth column of $A = |R_j|$, defined as determinant where R_j is obtainedfrom R by deleting its jth column.

Step 4:

Write the magnitude of oddments (after ignoring negativesigns, if any) against their respective rows and columns.

Step 5:

Check whether the sum of row oddments is equal to the sum of column oddments. If so, the oddments expressed as fractions of the grand total yields the optimum strategies. If not, the method fails.

Step 6:

Calculate the expected value of the game corresponding to the optimum mixed strategy determined above for the row player (against any move of the column player).

4 Numerical Example

Consider the fuzzy payoff matrix with triangular fuzzy numbers as elements

$$\begin{array}{lcc} & \text{Player B} & \text{Row min} \\ \text{Player A} & \begin{pmatrix} (-6,-5,4) & (-4,2,8) & (2,3,4) \\ (3,4,5) & (-8,-2,4) & (-2,1,8) \\ (-2,1,4) & (-2,1,4) & (-4,-1,2) \end{pmatrix} & \begin{array}{c} (-6,-5,4) \\ (-8,-2,4) \\ (-4,-1,2) \end{array} \\ \text{Col max} & (3,4,5) \quad (-4,\ 2,\ 8) \quad (2,3,4) & \end{array}$$

Find the measure for each a_{ij} of the fuzzy game which is given inthe following table for the purpose of comparison only.

Triangular Fuzzy Number	Measure of Triangular Fuzzy Number
	$\frac{1}{4}[2a_2 + a_1 + a_3]$

$\tilde{a}_{11} = (-6,-5,4)$,	$M_0^{Tri}(\tilde{a}_{11}) = -3$
$\tilde{a}_{12} = (-4,2,8)$,	$M_0^{Tri}(\tilde{a}_{12}) = 2$
$\tilde{a}_{13} = (2,3,4)$,	$M_0^{Tri}(\tilde{a}_{13}) = 3$
$\tilde{a}_{21} = (3,4,5)$,	$M_0^{Tri}(\tilde{a}_{21}) = 4$
$\tilde{a}_{22} = (-8,-2,4)$,	$M_0^{Tri}(\tilde{a}_{22}) = -2$
$\tilde{a}_{23} = (-2,1,8)$,	$M_0^{Tri}(\tilde{a}_{23}) = 2$
$\tilde{a}_{31} = (-2,1,4)$,	$M_0^{Tri}(\tilde{a}_{31}) = 1$
$\tilde{a}_{32} = (-2,1,4)$,	$M_0^{Tri}(\tilde{a}_{32}) = 1$
$\tilde{a}_{13} = (-4,-1,2)$,	$M_0^{Tri}(\tilde{a}_{33}) = -1$

Maxi Min= $(-4,-1,2)$

Mini Max= $(-4,2,8)$. Maxi Min ≠ Mini Max

So, it has no saddle point.So, we can use matrix oddment method to solve the problem.

Compute the matrices C and R by subtracting the successivecolumns from the preceding columns and subtracting the successive rows from the preceding rows.

By the definition of Arithmetic Operations on Triangular Fuzzy Number:

The following are the four operations that can be performed on triangular fuzzy numbers: Let $\tilde{A} = (a_1, a_2, a_3)$ and$\tilde{B} = (b_1, b_2, b_3)$ then, using the formula

$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

Column $C = \begin{pmatrix} (-14,-7,8) & (-8,-1,6) \\ (-1,6,13) & (-16,-3,6) \\ (-6,0,6) & (-4,2,8) \end{pmatrix}$ Thus C is a $n \times (n-1)$ matrix.

Row $R = \begin{pmatrix} (-11,-9,1) & (-8,4,16) & (-6,2,6) \\ (-1,3,7) & (-12,-3,6) & (-4,2,12) \end{pmatrix}$ Thus R is a $(n-1) \times n$ matrix.

To find the magnitude of oddments corresponding to each row and each column of A.

The oddment corresponding to 1st rowof A is defined as $|C_1|$, whereC_1 is obtained from C by deleting 1st row. Similarly $|R_1|$ is obtained from R.

By definition of Arithmetic Operations on Triangular Fuzzy Number

Using the formula

$$\tilde{A} \times \tilde{B} = \left(\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)\right)$$

$$|C_1| = \begin{vmatrix} (-1,6,13) & (-16,-3,6) \\ (-6,0,6) & (-4,2,8) \end{vmatrix} = (-148,12,200).$$

$$|C_2| = \begin{vmatrix} (-14,-7,8) & (-8,-1,6) \\ (-6,0,6) & (-4,2,8) \end{vmatrix} = (-160,-14,112).$$

$$|C_3| = \begin{vmatrix} (-14,-7,8) & (-8,-1,6) \\ (-1,6,13) & (-16,-3,6) \end{vmatrix} = (-206,27,328).$$

$$|R_1| = \begin{vmatrix} (-8,4,16) & (-6,2,6) \\ (-12,-3,6) & (-4,2,12) \end{vmatrix} = (-168,14,264).$$

$$|R_2| = \begin{vmatrix} (-11,-9,1) & (-6,2,6) \\ (-1,3,7) & (-4,2,12) \end{vmatrix} = (-174,-24,86).$$

$$|R_3| = \begin{vmatrix} (-11,-9,1) & (-8,4,16) \\ (-1,3,7) & (-12,-3,6) \end{vmatrix} = (-178,15,188).$$

So, augmented payoff matrix is

$$\text{Player A} \quad \overset{\text{Player B}}{\begin{pmatrix} (-6,-5,4) & (-4,2,8) & (2,3,4) \\ (3,4,5) & (-8,-2,4) & (-2,1,8) \\ (-2,1,4) & (-2,1,4) & (-4,-1,2) \end{pmatrix}} \quad \overset{\text{Row oddments}}{\begin{matrix} (-148,12,200) \\ (-160,-14,112) \\ (-206,27,328) \end{matrix}}$$

Column oddments: $(-168,14,264)$ $(-174,-24,86)$ $(-178,15,188)$

By using the remark, if $\tilde{A} = (a_1, a_2, a_3)$is a triangular fuzzy number then

Modules of $\tilde{A} = a_2$

Sum of the modulus values of the row oddments= $12 + 14 + 27 = 53$,

Sum of the modulus values of the column oddments = $14 + 24 + 15 = 53$

Strategy for Player $A = \left(\frac{12}{53}, \frac{14}{53}, \frac{27}{53}\right)$

Strategy for Player $B = \left(\frac{14}{53}, \frac{24}{53}, \frac{15}{53}\right)$

To find value of the game

$$\frac{12}{53}(a_{11}) + \frac{14}{53}(a_{21}) + \frac{27}{53}(a_{31})$$

$$= \frac{12}{53}(-6,-5,4) + \frac{14}{53}(3,4,5) + \frac{27}{53}(-2,1,4)$$

$= \left(-\frac{84}{53}, \frac{23}{53}, \frac{226}{53}\right)$ Is a triangular fuzzy number.

Crisp value of the solution of fuzzy valued game problem without converting to crisp valued game problem using measure

$$= \frac{23}{53} \cong 0.43.$$

But in ref [1], Authors converted the same fuzzy valued game problem to crisp valued game problem using measure, value of the game

$$= \frac{12}{14} \cong 0.86.$$

5 Conclusion

In this chapter, we have considered 3×3 fuzzy payoff matrix whoseelements are triangular fuzzy numbers. We proved that optimalsolution of the fuzzy valued game problem without converting tocrisp valued game problem using matrix oddment method is also a triangular fuzzy number.

For comparison, we used the "Measure" ranking method. Crisp value of the optimal solution is much more optimal than the solution obtained for the same problem after converting to crisp valued game problem using matrix oddment method.

6 References

[1] K.Selvakumari, S.Lavanya, An approach for solving fuzzy Game problem, Indian Journal of science and Technology, 8, No. 15 (2015), 1-6.

[2] L.A.Zadeh, Fuzzy Sets, Information and Control, 8 (1965) 338-353.

[3] D.F. Li, Lexicographic method for matrix games with Payoffs of triangular fuzzy numbers, International Journal of Uncertainty, Fuzziness and knowledge based systems, 16, No, 3 (2008), 371-389.

[4] A.Nagoorgani, S.N. Mohamed Assarudeen. A new Operation on triangular fuzzy number for solving fuzzy linear Programming problem, Applied Mathematical sciences, 6; No 11 (2012), 525-532.

[5] J.B. Kim, Determinant theory for fuzzy and Boolean matrices, Congresses Numerantium (1988),273- 276.

[6] A.Z.Ragab, E.G.Emam, The determinant and adjoint of a Square fuzzy matrix, Fuzzy Sets and Systems 61 (1994), 297-307.

[7] B.Monoranjan, Madhumangal pal, Anita pal, Circulant Triangular fuzzy number matrices, Journal of physical sciences, 12 (2008), 141-154.

[8] A.K.Shyamal, M. Pal, Tiangular fuzzy matrices, Indian Journal of fuzzy systems, 4,No.1(2007),75-87.