DETERIORATING ITEMS SUPPLY CHAIN INVENTORY MODEL FOR MULTIPLE BUYERS AND SINGLE VENDOR UNDER EXPONENTIAL DEMAND

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Abstract: An optimal strategy under multiple buyers and single vendor are investigated when units of buyers' inventory are based on deteriorations and demand is exponential function of time. The models are derived for single vendor and multiple buyers as profit maximization to determine optimal cycle time without collaboration. We also determine the total profit of buyers and vendor jointly under the strategy of supply chain inventory. The models are illustrated with numerical examples and observed that both buyers and vendor have significant profits in supply chain inventory system with collaboration as compare to without collaboration.

***Keywords*: Supply Chain, Optimal Strategy, Deterioration, Multiple Buyers, Exponential Demand**

I. INTRODUCTION

Supply chain is the strategy of collaboration between suppliers, vendors, distributers, buyers etc. To satisfy the buyers' demand in current competitive business, considerable information requires to be shared through the supply chain. Collaboration between vendors and buyers execution is necessary for current business scenario. In supply chain inventory buyers deal with vendor to purchase products for gaining more profit. The buyers may adopt the policy at regular basis and purchase goods from vendor as long-term agreements. The association between buyers and vendor creates greater commitment for quality maintenance, its develop trusts between the buyers and vendor over time. The following literatures are develop collaborative supply chain inventory model for vendor and buyers with various assumptions for demand pattern like price-dependent, time dependent demand, etc.

Yang and Wee (2001) derived the joint inventory model by considering deterioration of units' after they received inventory for multiple buyers and single vendor. Woo et al. (2001) considered supply chain inventory models when multiple buyers and single vendor adopted applications of information technologies and reduction efforts for ordering cost is to be updated and results in highest coordination and mechanization between associated business parties. Zavanella and Zanoni (2009) introduced an analytical model when single vendor and multiple buyers' referred the industrial case for combined situation. Singh and Chandramouli (2011) introduced collaborative production inventory policy where single vendor multiple buyers considered deterioration for variable demand rate and constant production rate when shortages are allowed for buyers. Shah et al. (2011) derived joint inventory model under supply chain system when multiple buyers and single vendor considered quadratic demand. Giri and Roy (2015) derived supply chain inventory model by assuming the collaboration between multiple buyers and single manufacturer when lead-time demand was normally distributed and demand is price dependent. Ghiami and Williams (2015) delivered two levels production inventory models with multiple buyers and one manufacturer when deteriorating items has fixed production rate and the order quantities are dispatched by the manufacturer to the consumers for definite period and the surplus inventory supplies for successive deliveries. Gani and Dharik (2018) developed single vendor with multiple buyers supply chain model based on a consignment stock and vendor's inventory policy was derived when demand and production rates were depended on trapezoidal fuzzy numbers.

Here we have considered exponential demand and derived supply chain when the system considered single vendor and multiple buyers and deteriorations are considered for buyers and time varying holding cost for the vendor and buyers.

II. NOTATIONS

$D(t) = a_i e^{b_i t}$, Demand is exponential function of time, where $a_i > 0$, $0 < b_i < 1$

$I_{bi}(t)$ = i^{th} buyer inventory level at any instant of time t

$I_v(t)$ = i^{th} vendor inventory level at any instant of time t

A_i = Ordering cost of i^{th} buyer per order

A_v = Ordering cost of vendor per order

C_b = Purchase cost of i^{th} buyer per unit

θ_i = i^{th} buyer's deterioration rates

x_{bi} = i^{th} buyer's fixed holding cost

y_{bi} = Varying holding cost of i^{th} buyer

x_v = Vendor's fixed holding cost

y_v = Vendor's varying holding cost

p_i = i^{th} buyer's selling price per unit

n_i = Number of times order placed by i^{th} buyer's during cycle time.

N = Number of buyers

TP_b = Total profits of buyers

TP_v = Total profit of vendor

TP = Integrated total profit for both vendor and buyers

$t_1 = v_1 * T/n_i$

T = Decision variable of vendor's cycle time

III. ASSUMPTIONS

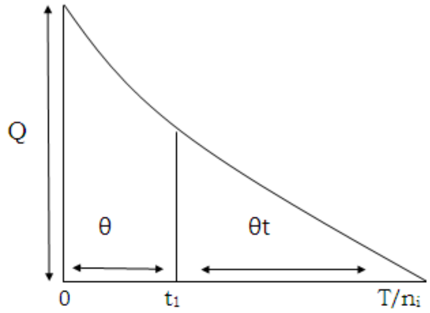
Under the following assumptions the inventory models are developed:

1. Product's demand is decreasing function of exponential distribution depending on time.
2. Single vendor and multiple buyers are considered.
3. Shortages are not allowed.
4. Lead time is zero.
5. Deteriorated units cannot be repaired or replaced throughout the cycle time and deterioration is dependent on time for buyer's inventory.
6. Time varying holding cost is considered for buyers and vendor.

IV. MATHEMATICAL MODEL

Let inventory level be $I_{bi}(t)$ at time t ($0 \le t \le T/n_i$) as shown in figure.

Buyer's Inventory



Two situations are discussed. The first situation does not consider the vendor and buyers collaboration, while the second situation considers vendor and buyers collaboration. Inventory level for both vendor and buyers is depleted by exponential demand. The differential equations of buyer's and vendor's inventory are given by:

$$\frac{dI_{b_i}(t)}{dt} + \theta_i I_{b_i}(t) = -a_i e^{b_i t} \quad ; \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_{b_i}(t)}{dt} + \theta_i t I_{b_i}(t) = -a_i e^{b_i t} \quad ; \quad t_1 \le t \le \frac{T}{n_i} \tag{2}$$

$$\frac{dI_v(t)}{dt} = -\sum_{i=1}^{N} a_i e^{b_i t} \quad ; \quad 0 \le t \le T \tag{3}$$

When the boundary conditions are consider as:

$$I_{b_i}(0) = Q, \quad I_{b_i}\left(\frac{T}{n_i}\right) = 0, \quad and \quad I_v(T) = 0$$

Their solutions are given by

$$I_{b_i}(t) = Q(1-\theta_i t) - a_i\left(t + \frac{b_i}{2}t^2 + \frac{\theta_i}{2}t^2 + \frac{b_i \theta_i}{3}t^3\right) + a_i \theta_i t\left(t + \frac{b_i}{2}t^2\right) \tag{4}$$

$$I_{b_i}(t) = a_i \left\{ \begin{bmatrix} \left(\frac{T}{n_i} - t\right) + \frac{b_i}{2}\left(\frac{T^2}{n_i^2} - t^2\right) + \frac{\theta_i}{6}\left(\frac{T^3}{n_i^3} - t^3\right) \\ + \frac{b_i \theta_i}{8}\left(\frac{T^4}{n_i^4} - t^4\right) - \frac{\theta_i t^2}{2}\left(\frac{T}{n_i} - t\right) - \frac{b_i \theta_i t^2}{4}\left(\frac{T^2}{n_i^2} - t^2\right) \end{bmatrix} \right\} \tag{5}$$

$$I_v(t) = \sum_{i=1}^{N} a_i \left[(T - t) + \frac{b_i}{2}\left(T^2 - t^2\right) \right] \tag{6}$$

(By neglecting higher power of θ)

Substituting $t = t_1$ in equations (4) and (5) and simplifying, we get

$$Q = \frac{1}{(1-\theta_i t_1)} \left\{ \begin{array}{l} a_i\left(\frac{\theta_i}{2}t_1^2 + \frac{b_i \theta_i}{3}t_1^3\right) - a_i \theta_i t_1\left(t_1 + \frac{b_i}{2}t_1^2\right) \\ + a_i \begin{bmatrix} \frac{T}{n_i} + \frac{b_i}{2}\frac{T^2}{n_i^2} + \frac{\theta_i}{6}\left(\frac{T^3}{n_i^3} - t_1^3\right) \\ + \frac{b_i \theta_i}{8}\left(\frac{T^4}{n_i^4} - t_1^4\right) - \frac{\theta_i}{2}t_1^2\left(\frac{T}{n_i} - t_1\right) \\ - \frac{b_i \theta_i}{4}t_1^2\left(\frac{T^2}{n_i^2} - t_1^2\right) \end{bmatrix} \end{array} \right\} \tag{7}$$

Putting the value of Q in equation (4) we get

$$
I_{b_i}(t) = \frac{(1-\theta_i\, t)}{(1-\theta_i\, t_1)}\left\{
\begin{array}{l}
a_i\left(\dfrac{\theta_i}{2}t_1^2 + \dfrac{b_i\,\theta_i}{3}t_1^3\right) - a_i\,\theta_i\,t_1\left(t_1 + \dfrac{b}{2}t_1^2\right) \\
+\,a_i\left[\begin{array}{l}
\left(\dfrac{T}{n_i} + \dfrac{b}{2}\dfrac{T^2}{n_i^2} + \dfrac{\theta_i}{6}\left(\dfrac{T^3}{n_i^3} - t_1^3\right)\right. \\
\left. + \dfrac{b_i\,\theta_i}{8}\left(\dfrac{T^4}{n_i^4} - t_1^4\right) - \dfrac{\theta_i}{2}t_1^2\left(\dfrac{T}{n_i} - t_1\right)\right. \\
\left. - \dfrac{b_i\,\theta_i}{4}t_1^2\left(\dfrac{T^2}{n_i^2} - t_1^2\right)\right)
\end{array}\right]
\end{array}\right\}
$$
$$
- a_i\left(t + \frac{b_i}{2}t^2 + \frac{\theta_i}{2}t^2 + \frac{b_i\,\theta_i}{3}t^3\right) + a_i\,\theta_i\; t\left(t + \frac{b_i}{2}t^2\right) \tag{8}
$$

V. BUYER'S RELEVANT COSTS:

Holding cost

$$
HC_b = \sum_{i=1}^{N} n_i \left\{ x_{b_i}\left(\int_0^{t_1} I_{b_i}(t)\,dt + \int_{t_1}^{\frac{T}{n_i}} I_{b_i}(t)dt\right) + y_{b_i}\left(\int_0^{t_1} t\, I_{b_i}(t)dt + \int_{t_1}^{\frac{T}{n_i}} t\, I_{b_i}(t)dt\right)\right\} \tag{9}
$$

Deterioration cost

$$
DC_b = \sum_{i=1}^{N} n_i\, C_b \theta_i \left\{ \int_0^{t_1} I_{b_i}(t)dt + \int_{t_1}^{\frac{T}{n_i}} t I_{b_i}(t)dt \right\} \tag{10}
$$

Ordering cost:

$$
OC_b = \sum_{i=1}^{N} n_i\, A_i \tag{11}
$$

Sales revenue:

$$
SR_b = \sum_{i=1}^{N} n_i\;\; p_i \int_0^{\frac{T}{n_i}} D(t)dt
$$
$$
= \sum_{i=1}^{N} n_i\;\; p_i \int_0^{\frac{T}{n_i}} a_i\;(1 + b_i\; t)dt
$$
$$
= \sum_{i=1}^{N} n_i\; p_i\, a_i \left(\frac{T}{n_i} + \frac{b_i}{2}\frac{T^2}{n_i^2}\right) \tag{12}
$$

Total profit:

$$
TP_b = \frac{1}{T}\left[SR_b - HC_b - DC_b - OC_b\right] \tag{13}
$$

VI. VENDOR'S RELEVANT COSTS:

Holding cost:

$$
\begin{aligned}
HC_v = x_v &\left\{ \int_0^{T} I_v(t)\,dt - \sum_{i=1}^{N} n_i \left(\int_0^{t_1} I_{b_i}(t)\,dt + \int_{t_1}^{\frac{T}{n_i}} I_{b_i}(t)\,dt \right)\right\} \\
+\, y_v &\left\{ \int_0^{T} t\, I_v(t)\,dt - \sum_{i=1}^{N} n_i \left(\int_0^{t_1} t\, I_{b_i}(t)\,dt + \int_{t_1}^{\frac{T}{n_i}} t\, I_{b_i}(t)\;dt \right)\right\}
\end{aligned} \tag{14}
$$

Ordering cost:

$$
OC_v = A_v \tag{15}
$$

Sales revenue: (16)

$$SR_v = C_b \sum_{i=1}^{N} \int_0^T D(t)dt$$

$$= C_b \sum_{i=1}^{N} \int_0^T a_i e^{b_i t} dt$$

$$= C_b \sum_{i=1}^{N} a_i \left(T + \frac{b_i}{2} T^2 \right)$$

Total profit:

$$TP_v = \frac{1}{T}\left[SR_v - HC_v - OC_v\right] \quad (17)$$

VII. SITUATION-I: BUYERS AND VENDOR TAKE INDEPENDENT DECISION

Here the buyers and vendor make decision independently.

Buyer's maximum profit TP_b can be determined by following conditions:

$$\frac{dTP_b}{dT_b} = 0, \text{ where } T_b = \frac{T}{n_i} \quad (18)$$

provided it satisfies the condition

$$\frac{d^2 TP_b}{dT_b^2} < 0 \quad (19)$$

This solution (n, T) maximizes TP_v.

Then the total profit without collaboration is given by;

$$TP = \max(TP_b + TP_v) \quad (20)$$

VIII. SITUATION-II: BUYERS AND VENDOR TAKE JOINT DECISION

Here the buyers and the vendor jointly make decision:

Moreover, the optimum value of T must satisfy the following conditions, which maximize total profit (TP) when buyer and vendor take joint decision:

$\frac{dTP}{dT} = 0$ for T provided it satisfies the condition. (21)

$$\frac{d^2 TP}{dT^2} < 0 \quad (22)$$

where total profit (TP) with collaboration is given by;

$$TP = TP_b + TP_v. \quad (23)$$

IX. NUMERICAL EXAMPLE:

Numerical analysis considers the values of various parameters in appropriate units, $a_1 = 450$, $a_2 = 550$, $b_1 = 0.045$, $b_2 = 0.055$, $\theta_1=0.06$, $\theta_2=0.04$, $x_{b1} = 9.5$, $y_{b1} = 0.025$, $x_{b2} = 10.5$, $y_{b2} = 0.035$, $x_v = 8$, $y_v = 0.01$, $A_1 = 60$, $A_2 = 90$, $A_v = 1500$, $C_b = 35$, $p_1 = 43$, $p_2 = 47$, $v_1 = 0.4$, $N = 2$. The optimal values of T and profits for buyer and vendor are given in Table-1.

The optimal total profit TP = Rs. 74886.37at $n_1=4$ and $n_2=4$ for buyers' profit TP_b^* = Rs. 43556.57, T* = 0.7242and TP_v= 31329.80 when buyers and vendor take independent decision. While when buyers and vendor take joint decision then the optimal total profit TP^* = Rs. 75208.79at $n_1=2$ and $n_2=2$ and $T^* = 0.6671$with buyers' profit TP_b = Rs. 42478.16and TP_v= 32730.63.

The second order conditions given in equation (19) and equation (22) are also satisfied. The graphical representations of the concavity of the profits of independent and joint decisions are also given.

Table-1

The optimal solutions for without collaboration and with collaboration

	without collaboration	with collaboration
n	n_1=4, n_2=4	n_1=2, n_2=2
T	0.7238	0.7057
Buyer's Profit	43556.17	43159.52
Vendor's Profit	31329.82	32039.84
Total Profit	74885.99	75199.36

Table-2

Sensitivity Analysis

		without collaboration			With collaboration		
%	Para -meters	TPb	TPv	TP	TPb	TPv	TP
20%	a_1, a_2	52199.50	37979.80	90179.30	51028.87	39504.49	90533.36
10%		47775.60	34647.90	82423.40	46654.69	36105.44	82760.13
-10%		38938.90	28008.70	66947.60	37924.75	29322.32	67247.06
-20%		34447.20	24807.60	59254.80	33571.03	25939.93	59510.95
20%	A_1, A_2	43200.10	31402.40	74602.50	42199.94	32736.08	74936.02
10%		43277.40	31362.90	74640.30	42242.90	32723.72	74966.70
-10%		43440.20	31297.60	74737.90	43012.70	32021.47	75034.20
-20%		43530.10	31256.00	74786.10	43066.40	32010.94	75077.30
20%	x_{b1}, x_{b2}	43182.00	31265.10	74447.20	42700.20	31945.54	74645.78
10%		43266.50	31297.70	74564.20	42827.00	31988.85	74815.82
-10%		43451.80	31373.40	74825.20	41917.80	33400.61	75318.40
-20%		43551.20	31428.20	74979.40	42074.20	33606.64	75680.85
20%	θ_1, θ_2	43333.50	31318.00	74651.50	42926.30	32020.32	74946.65
10%		43344.30	31321.00	74665.30	42943.00	32025.98	74969.01
-10%		43366.10	31326.50	74692.60	42305.40	32727.80	75033.20
-20%		43380.20	31327.90	74708.10	41801.50	33270.40	75071.95
20%	x_v	43358.20	30959.00	74317.30	41763.70	33219.91	74983.60
10%		43358.20	31179.40	74537.60	41766.00	33215.86	74981.89
-10%		43355.20	31491.40	74846.60	42932.40	32169.49	75101.90
-20%		43355.20	31770.90	75126.10	42881.20	32410.45	75291.69
20%	A_v	43355.20	30909.20	74264.30	42861.70	31719.89	74581.60
10%		43355.20	31116.50	74471.70	42910.40	31872.34	74782.70
-10%		43358.20	31557.50	74915.70	42380.80	32974.81	75228.60
-20%		43358.20	31803.40	75161.60	42478.30	32992.23	75470.60

Sensitive analysis is carry out by changing the values of one parameter at a time from given parameters a_i , A_{bi}, A_v, x_{bi}, x_v, and θ_i respectively, and kept reaming parameters constant. From Table-2 we observed that total profits increase when vendor and buyers take decision with collaboration instead of without collaboration. When a increases/decreases then total profit will increase/decrease, while if A_{bi}, x_{bi}, x_v, A_v, and θ_i increase/decrease then total profit will decrease/increase in independent and joint decision.

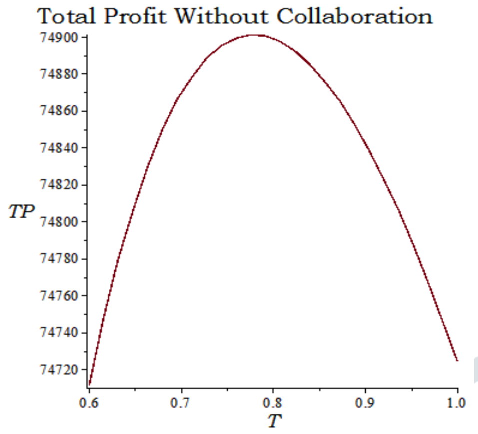


Figure-2

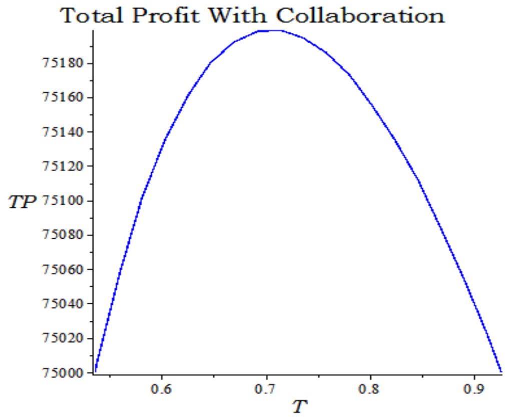


Figure-3

X. CONCLUSION:

The result shows that the optimal cycle time is significantly decreased and total profit significantly increased when buyers and vendor take joint decision as compared to independent decision taken by buyers and vendor. We also observe that the vendor's profit is increased and number of times order placed by buyer during cycle time is decreased when buyers and vendor take joint decision.

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