Proof of Twin Prime Conjecture

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Author's Biography

The author of this research paper is K.H.K. Geerasee Wijesuriya . And this proof of twin prime conjecture is completely K.H.K. Geerasee Wijesuriya's proof.

Geerasee is now 30 years old and she studied before at Faculty of Science, University of Colombo Sri Lanka. And she graduated with BSc (Hons) in Physics and Mathematics from the University of Colombo, Sri Lanka in 2014 June. And in March 2018, she completed her Doctorate Degree in Physics with first class recognition. Now she is following her second PhD in Astrophysics with Belarusian National Technical University.

Geerasee has been invited by several Astronomy/Physics institutions and organizations world-wide, asking to get involve with them. Also, She has received several invitations from some private researchers around the world asking to contribute to their researches. She worked as Mathematics tutor/Instructor at Mathematics department, Faculty of Engineering, University of Moratuwa, Sri Lanka. Furthermore she has achieved several other scientific achievements already.

Acknowledgement

I would be thankful to my parents who gave me the strength to go forward with mathematics and Physics knowledge and achieve my scientific goals.

List of abbreviations

Faculty of Science, University of Colombo, Sri Lanka , Belarusian National Technical University Belarus

Abstract

A twin prime is a prime number that is either 2 less or 2 more than another prime number. In other words, a twin prime is a prime that has a prime gap of two. Sometimes the term twin prime is used for a pair of twin primes; an alternative name for this is prime twin or prime pair. Up to date there is no any valid proof of twin prime conjecture. Through this research paper my attempt is to provide a valid proof for twin prime conjecture.

Literature Review

The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that p + 2 is also prime. In 1849, de Polignac made the more general conjecture that for every natural number k, there are infinitely many primes psuch that p + 2k is also prime. The case k = 1 of de Polignac's conjecture is the twin prime conjecture.

A stronger form of the twin prime conjecture, the Hardy–Littlewood conjecture (see below), postulates a distribution law for twin primes akin to the prime number theorem. On April 17, 2013, Yitang Zhang announced a proof that for some integer N that is less than 70 million, there are infinitely many pairs of primes that differ by N. Zhang's paper was accepted by Annals of Mathematics in early May 2013. Terence Tao subsequently proposed a Polymath Project collaborative effort to optimize Zhang's bound. As of April 14, 2014, one year after Zhang's announcement, the bound has been reduced to 246. Further, assuming the Elliott–Halberstam conjecture and its generalized form, the Polymath project wiki states that the bound has been reduced to 12 and 6, respectively. These improved bounds were discovered using a different approach that was simpler than Zhang's and was discovered independently by James Maynard and Terence Tao.

Assumption

Let's assume that there are finite number of twin prime numbers. Let the highest twin prime numbers are and P_{n-1} and $P_{n-1}+2$. Then for all prime numbers P_n greater than P_{n-1}, $(P_n + 2)$ is not a prime number.

Methodology

With this mathematical proof, I use the contradiction method to proof the twin prime conjecture.

Let P_n is an arbitrary prime number greater than P_{n-1} (because there are infinite number of prime numbers). Then according to our assumption, (P_n +2) is not a prime number. Since $P_n > 2$ and since P_n is a prime number and since P_n is an odd number, for all prime numbers P_j (< P_n / 2): $P_n / P_j = r$ ....(01)

But since P_n is a prime number, r is not a natural number.

But according to our assumption, P_n +2 is not a prime number. Also since P_n is a prime number greater than 2, (P_n + 2) is an odd number. Thus for some prime number P_i (< [(P_n + 2) / 2]) ; (P_n + 2) / P_i = x . Where we choose P_i such that x is a natural number. But since previously chose P_j is an arbitrary prime number less than P_n / 2; now we consider $P_j = P_i$.

Then (P_n + 2) / P_j = x...........(02)

By 01 and 02, r. P_j + 2 = x. P_j .

Thus ,

P_j (x - [P_n / P_j]) = 2 …………..(03)

Let's consider another Prime number $P_N > P_n$

Then according to our initial assumption, (P_N + 2) is not a prime number.

Then there is a number P_k (>1) such that (P_N + 2) / P_k = x' ; where x' is a natural number and also we should responsible to choose P_k such that x' = (P_N + 2) / P_k = P_j . Here P_k (> 1) may or may not a prime number. Also P_k may or may not be a natural number.

But P_k > 1. And x and x' are natural numbers.

Then P_k (x' – [P_N / P_k]) = 2…………(04)

Thus by 03 and 04 : (P_k .x' - P_N) * (P_j .x - P_n) = 4

[x.x' – 4] = ([P_n . P_k .x' + P_N . P_j .x] - P_N . P_n) / (P_k . P_j)

Since x, x' are natural numbers:

([P_n . P_k .x' + P_N . P_j .x] - P_N . P_n) / (P_k . P_j) = C

Where C is a natural number.

$[P_n . P_k .x' + P_N . (P_j .x - . P_n)] / (P_k . P_j) = C$

$[P_n . P_k .x' + 2.P_N] / (P_k . P_j) = C$

But $P_k .x' = P_N + 2$

Thus $[P_n . (P_N + 2) + 2.P_N] / (P_k . P_j) = C$

$[P_n . x' / P_j] + [2. P_N / P_k . P_j] = C$ …………..(05)

But $x' = P_j$

By 05 : $[P_n . P_j / P_j] + [2. P_N / P_k . P_j] = C$

$2.[P_N / P_k . P_j] = (C - P_n) = d$; where d is a natural number………(06)

P_k , P_j are not even numbers. And P_N is a prime number.

And $(P_N + 2) / x' = P_k = (P_N + 2) / P_j$. Because $x' = P_j$

Thus $(P_N + 2) / x' = P_k = (P_N + 2) / P_j > (P_N / P_j)$

Thus P_k not equals to (P_N / P_j) ………..(07). And $P_N > P_j$.

Since P_N is a natural number, although if P_k is a prime number :

P_N not equals to $P_k . P_j$ (Since P_N is a prime number)…..(08)

By 06, 07, 08 : # we get a contradiction…………..(*)

Discussion

We know that according to our initial assumption, For all P_n primes (> P_{n-1}) : $(P_n + 2)$ is not prime. Therefore there exist prime number P_j (= x') such that $(P_n + 2) / P_j = x$; x is a natural number. Thus for that fact, I have used the fact of $(P_n + 2)$ is not a prime number.

Also I have used that P_j is a prime in the statement of " $(P_n + 2) / P_j = x$; x is a natural number ". Because for all non prime $(P_n + 2)$, there exist definitely a prime number P_j such that $(P_n + 2) / P_j = x$; x is a natural number. Thus for that fact, I have used that P_j is prime.

Also I have used that " P_n " is a prime number, with the main steps of the proof as described below.

If P_n (> P_j) is not a prime number, we can't have the below mentioned equation 09 for any random prime number P_j (< P_n). Thus for our main equation 09 (Which is in the proof), I have used that P_n is a prime number.

Because (refer below statements):

We choose ($P_n + 2$) non prime randomly (which is greater than ($P_{n-1} + 2$)). Then the prime number P_j (which divides the random ($P_n + 2$)) is also a random prime number.

But we know that $x - [P_n / P_j] = 2 / P_j$ ………09 (by the expression $(P_n + 2) / P_j = x$)

But $P_j > 1$. Since x is a natural number, by 09;

For any random prime number P_j : [P_n / P_j] is not a natural number. And $P_n > P_j$. Therefore it is obvious that I have considered that P_n as a prime number in equation 09 also, as indicated above. Therefore it is obvious that I have used that P_n is a prime number, with the initial steps of this mathematical proof.

Therefore I have used P_n as an arbitrary prime number greater than P_{n-1} and thus I have used (P_n + 2) not as prime number.

Conclusion

Therefore I have used all my assumptions to get the contradiction (*). Therefore it is possible to conclude that our main assumption is false.

Thus there are infinitely many twin prime numbers.

Appendix

Prime number: A natural number which divides by 1 and itself only.

Twin Prime Numbers: Two prime numbers which have the difference exactly 2.

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