METRO DOMINATION NUMBER OF FRIENDSHIP GRAPH AND CROWN GRAPH

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Abstract : A Set $D \subseteq V(G)$ is called dominating set if every vertex $v \in V - D$ is adjacent to atleast one vertex in D. The dominating set with minimum cardinality is called domination number of G & it is denoted by $\gamma(G)$. Let $G(V,E)$ be a graph. A set $S \subseteq V$ is a resolving set if for every $u,v \in V(G)$ there exists $w \in V$ such that $d(u,w) \neq d(v,w)$.The resolving set with minimum cardinality is called metric basis and its cardinality is called metric dimension and is denoted by $\beta(G)$. A set D which is both resolving set as well as dominating set is called metro dominating set. The minimum cardinality of a metro dominating set is called domination number of G and is denoted by $\gamma_\beta(G)$. In this paper we determine metro domination of friendship graph, Crown graph and Dragon Graph.

Key Words : Domination, Metric Dimension, Locating Domination, Metro Domination.

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1. Introduction

All the graphs considered are simple, finite and connected. A set of vertices S resolves a graph G if every vertex of G is uniquely determined by its vector of distance to the vertices in S. This work undertakes a general study of resolving sets in a friendship graph and Crown Graph. Given a graph $G(V,E)$ and $u,v \in V(G)$, $d(u,v)$ denoted the distance between u & v in G that is the length of a shortest $u-v$ path. On 1976, Harary and Melter [4] introduce the notation of metric dimension. The vertex set and edge set of a graph G denoted by $V(G)$ and $E(G)$.The distance between vertices $u,v \in V(G)$ is denoted by $d(u,v)$. A vertex $x \in V(G)$ resolves a pair of vertices $v,w \in V(G)$ if $d(v,x) \neq d(w,x)$. A set of vertices $S \subseteq V(G)$ resolves G, & S is a resolving set S of G with minimum cardinality is a metric basis of G & its cardinality is the metric dimension of G denoted by $\beta(G)$.

2. Definitions.

2.1 Metric Dimension

A vertex $x \in V(G)$ resolves a pair of vertices $v,w \in V(G)$ if $d(v,x) \neq g(w,x)$. A set of vertices $S \subseteq V(G)$ resolves G, and S is a resolving set S of G with minimum cardinality is a metric basis of G, and its cardinality is the metric dimension of G, denoted by $\beta(G)$.

2.2 Domination

Let $G(V,E)$ be a graph. A subset of vertices $D \subseteq V$ is called a dominating set (γ-set) if every vertex in $V-D$ adjacent to at least one vertex in D.The minimum cardinality of a dominating set is called the domination number of the graph G and is denoted by $\gamma(G)$.

2.3 Locating Number

A subset D of $V(G)$ is called a dominating set, if even vertex $V-D$ is adjacent to at least one vertex in D. The minimum cardinality of a dominating set is called the domination number of graph G. The metric dimension of a graph $G(V,E)$ is the cardinality of a minimal subset S of V such that for each pair of vertices u, v of V there is a vertex w in S such that the length of the shortest path from w to u is different from the length of a shortest path from w to v.

The metric dimension of a graph G is also called as a locating number of G and studied its dominating property independently by Slater. A dominating set D is called a locating dominating set or simply LD-set if for each pair of vertices $u,v \in V-D$, $ND(u) \neq ND(v)$,where $ND(u) = N(u) \cap D$. The minimum cardinality of an LD-set of the graph G is called the locating domination number of G denoted by $\gamma_L(G)$.

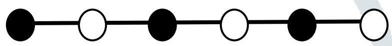

Figure :1 The graph G and its LD-set

Figure :2 The graph G and its MD-set

2.4 Metro Domination

A dominating set D of $V(G)$ having the property that for each pair of vertices u,v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$ is called the metro dominating set of G or simply an MD-set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and it is denoted by $\gamma_\beta(G)$.

2.5 Friendship Graph

A friendship Graph F_n is a graph. F_n is a graph that can be constructed by coalescence n copies of the cycle Graph C_3 of length 3 with a common vertex. The friendship graph F_n is also planar graph with 2n+1 vertices & $3n$ edges. The vertex set is $V(F_n)=\{c, v_1, v_2, \dots v_{2n}\}$ and the edge set is

$E(F_n)=\{cv_1, cv_2, cv_3, .. cv_{2n}\} \cup \{v_1v_2, v_3v_4, \dots . v_{2i-1}v_{2i}, \dots \dots . v_{2n-1}v_{2n}\}$ for $n \geq 2$ [1]

2.6 Crown graph

An crown, $C_n \square P_2$ is a graph in which pathP_2 is attached at each vertex of the cycle C_n.
Let $v_0, v_1, v_2, \dots v_{n-1}$ be the vertices of the cycle, $C_n \square P_2$ such that v_i is adjacent to v_{i+1}, $0 \leq i \leq n-2$ and v_0 is adjacent to v_{n-1}. Also let u_0, u_1 be the vertices of the path which is attached to every vertices of C_n such that u_0 adjacent to u_1. We denote the vertices of the path adjacent to the vertex v_i, as u_{i0}, u_{i1},……. u_{im-1}.where u_{i0} adjacent to v_i, $0 \leq i \leq n-1$.

2.7 Dragran Graph

Let $v_0\ v_1, v_2, \dots v_{n-1}$, be the vertices of the cycle of $D_n m$ such that v_i is adjacent to v_{i+1}, $0 \leq i \leq n-2$ and v_0 is adjacent to v_{n-1}. Also let $u_0, u_1, u_2, \dots u_{m-1}$ be the vertices of the path or tail of $D_n m$. Such that u_j is adjacent to u_{i+1}, $0 \leq j \leq m-2$ and u_0 is adjacent to v_0 .

2.8 Resolving Set [4]

The subset S is a resolving set if $r\left(\frac{v}{s}\right)$ for every two vertices of G have distinct representation. A resolving set of minimum cardinality for G is called a minimum resolving set or a basis for G. If $S=\{s_1, s_2, \ldots. s_k\}$then $r\left(\frac{v}{s}\right) = (d(v,s_1), d(v,s_2), \ldots. d(v,s_k))$.

3. Some known results

In this section we mention some of the known results on metric dimension, domination and metro domination which we use in the subsequent section.

Theorem 3.1. (Harary & Melter [4])
The metric dimension of a non trivial complete graph of order n is $n-1$.

Theorem 3.2.(Khuller, Raghavachari, Rosenfeld [8])
The metric dimension of a graph G is 1 if and only if G is a path.

Theorem 3.3. (Harary and Melter [4])
The metric dimension of a complete bipartite graph K_{mn} is $m+n-2$.

Theorem 3.4. [9] if G is a graph with no isolated vertices and S is a minimal dominating set of G then $V(G)-S$ is a dominating set of G.

Theorem 3.5. (Sooryanarayana and Raghunath [7])
The metro domination number of a graph G is $\left\lceil\frac{n}{5}\right\rceil$ if and only if G is a cycle.

Theorem 3.6. (Sooryanarayana and Raghunath [7])
Let G be a graph on n vertices. Thus $\gamma_\beta(G) = n-1$ if and only G is K_n or $K_{1,n-1}$ fo r n≥1

Theorem 3.7.(Sooryanarayana and Raghunath [7]) For any integer n, $\gamma_\beta(P_4)$ is=$\left\lceil\frac{n}{3}\right\rceil$.

Remark 3.8. For any connected graph G, $\gamma_\beta(G) \geq$ max $\{\gamma(G), \beta(G)\}$. [6]

Theorem 3.9. For all integer $n \geq 2$, dim $(F_n) = n$.

4. Our results :

Theorem 4.1 For any integer $n \geq 3$, $\gamma_\beta(F_n) = n$.
Proof: It is obvious that $\gamma(F_n) = 1$. (Central vertex ‘c’ dominates all the vertices of F_n). Also by theorem 3.9 $\dim(F_n) = n$. By remark 3.8.

$$\gamma_\beta(F_n) \geq \max(\beta,(F_n), \gamma(F_n))$$
$$\gamma_\beta(F_n) \geq \max(n,1).$$
$$\gamma_\beta(F_n) \geq n. \qquad (1)$$

To prove the reserve inequality we define a dominating set. $D = \{ v_{2k-1} / k \geq 1,\ 1 \leq k \leq 2n-1\}$
he above set serves as a domination set, which is also a resolving set by theorem 3.9.

$\therefore \gamma_\beta(F_n) \leq n$ (2)

Thus from (1) & (2)
$\gamma_\beta(F_n) = n$. □

Lemma 4.2 For any integer $n \geq 3, \gamma(C_n \square P_2) = n.$
Proof: Since the crown graph $C_n \square P_2$ contains n-pendent vertices, to dominates these pendent vertices we require minimum n vertices of $C_n \square P_2$ thus $\gamma(C_n \square P_2) \geq n.$ (1)
To prove the reverse inequality we define a set $D = \{V_k / 0 \leq k \leq n-1\}$.
The above set serves as a dominating set of $C_n \square P_2$ thus $\gamma(C_n \square P_2) \leq n.$ (2)
From (1) and (2) $\gamma(C_n \square P_2) = n.$ □

Theorem 4.3. For any integer $n \geq 3, \beta(C_n \square P_2)$=3
Proof : Case-1 for odd n
We choose a subset $S = \{u_{00}, u_{10}, u_{\lfloor \frac{n}{2} \rfloor 0}\}$
and we must show that $\dim(C_n \square P_2)$=3
For $n \geq 3$. By definition 2.7, we got the representations of vertices of graph $C_n \square P_2$ with respect to S are
$r\,(v_0 \backslash s) = (1,2, \lceil \frac{n}{2} \rceil)$
$r\,(v_1 \backslash s) = (2,1, \lfloor \frac{n}{2} \rfloor)$
$r\,(v_i \backslash s) = (i+1, i, \lfloor \frac{n}{2} \rfloor - (i-1))$ $\quad 2 \leq i \leq \lfloor n/2 \rfloor$
$r\left(v_{\lceil \frac{n}{2} \rceil} \backslash s\right) = \left(\lceil \frac{n}{2} \rceil, \lceil \frac{n}{2} \rceil, 2\right)$
$r\,(v_i \backslash s) = (2\lceil \frac{n}{2} \rceil - i, 2\lceil \frac{n}{2} \rceil - (i-1), i - (\lceil \frac{n}{2} \rceil - 1))$ $\quad \lceil \frac{n}{2} \rceil + 1 \leq i \leq n-1.$
$r\,(u_{00} \backslash s) = (4,3, \lfloor \frac{n}{2} \rfloor)$

Case – 2 for even n
We choose a set $S = \left\{u_{00}, u_{10}, u_{\left(\frac{n-2}{2}\right)0}\right\}$
By definition 2.7, we got the representations of vertices of graph $C_n \square P_2$ with respect to S are
$r\,(v_0 \backslash s) = (1, 2, \frac{n}{2})$
$r\,(v_1 \backslash s) = (2, 1, \frac{n-2}{2})$
$r\,(v_i \backslash s) = (i+1, i, \frac{n}{2} - i)$ $\quad 2 \leq i \leq \frac{n-2}{2}$
$r\,(v_{\frac{n}{2}} \backslash s) = (\frac{n+2}{2}, \frac{n}{2}, 2)$
$r\,(v_i \backslash s) = (n-(i-1), n-(i-2), i - \left(\frac{n-4}{2}\right))$ $\quad \frac{n+2}{2} \leq i \leq n-1$

From the above (case 1 and case 2), the representations of vertices of cycle C_n in the graph $C_n \square P_2$ are distinct. Also we can observe that the representation of the vertices $u_{i0} (0 \leq i \leq n-1)$are also distinct. This implies that S is resolving set, but it is not necessarily the lower bound. Thus the upper bound is dim $(C_n \square P_2) \leq 3$.
Now we show that $\dim(C_n \square P_2) \geq 3$.

Let $S = \{u_{00}, u_{10}, u_{(\frac{n-2}{2})0}\}$.Is a resolving set which is$|S| = 3$. Assume that S_1 is another minimum resolving set or we can denote $|S_1| < 3$. If we choose an ordered set $S_1 \subseteq S - \{u_{i0}\}, i = 0,1,(\frac{n-2}{2})$ so that there are two vertices which are of same representation. S_1 is not a resolving set is a contradiction with assumptions.
Thus the lower bond is $\dim(C_n \square P_2) \geq 3$.
From above proving we conclude that $\dim(C_n \square P_2) = 3$.

Theorem 4.4. For any integer $n \geq 3, \gamma_\beta(C_n \square P_2) = n$.
Proof: By lemma 4.3 $\dim(C_n \square P_2) = 3$, also by lemma 4.2 $\gamma(C_n \square P_2) = n$
By remark 3.8 $\gamma_\beta(C_n \square P_2) \geq \max\{\beta(C_n \square P_2), \gamma(C_n \square P_2)\}$.
$\geq \max\{3, n\}$

$$\gamma_\beta(C_n \square P_2) \geq n \quad (1)$$

To prove the reverse inequality we defined a dominating set $D = \{\frac{u_{i0}}{0} \leq i \leq n-1\}$.
We note that the above set serves as a dominating set which also a resolving set

$$\therefore \gamma_\beta(C_n \square P_2) \leq n \quad (2)$$

Thus $\gamma_\beta(C_n \square P_2) = n$.

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