Queuing model of general service time distribution with phases of patch up work and zeroth time work

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Abstract- This paper deals with the steady state behavior of a $M^X/G/1$ Queue with new innovative ideas of zeroth time work and stages of patch up process. The problem is solved by supplementary variable method and the corresponding performance measures are derived. The model is well defined by an application and the numerical justification is made at the end.

Keywords: Set up time, Supplementary variable technique, Revamp process.

1. *Introduction*

Queuing systems with vacations have been developed for wide range applications in production. They have many applications in machining systems working in industrial environment such as manufacturing and production systems. Server vacation models are useful for the system in which a server wants to utilize the idle time for different purposes. A few creators have examined Queuing models in sorts of services in differing charges that incorporate unpredictable service interruption and repair. Borthakur and Choudhury[1] investigated a batch arrival queue with generalized vacation .Dhanalakshmi and Maragathasundari [2] studied the queuing approach in Mobile adhoc networks. Khalaf et.al [3] derived the performance measures for the queuing problem of random breakdowns and general repair times. Madan. K.C and Abu-Dayyeh[4] made an analysis on single server queue with optional phase type server vacations. Madan. K.C. and Chodhury. G studied about the queuing problem on restricted admissibility Maragathasundari.S[6]investigated the queueing model of three stages of service.Maragathasundari.S and Srinivasan[7]made an Analysis on feedback queue Maragathasundari.S and Karthikeyan.K[8] investigated the work on bulk queueing model of optional second phase service with short and long vacations.Maragathasundari.S and Srinivasan.S[9] studied the multiple server vacation.Sowmiah. S and Maragathasundari.S[10] made a report on the performance measures on optional services.

2. *APPLICATION*

Client/server is a term used to describe a computing model for the development of computerized systems. This model is based on the distribution of functions between two types of independent and autonomous processes; servers and clients.

2.1. *Basic Components*

A client is any process that requests specific services from server processes. A server is a process that provides requested services for clients. Both clients and servers can reside in the same computer or in different computers connected by a network.

2.2. *Set up time*

Usually the server requires some setup time or processing time to reboot the system after providing each and every service. This processing time is to load the cache memory and to restore its history for further references.

2.3. *Break down process*

Server down or server crashes: In a client server architecture, a server's responsibility is always to serve its client. When a server fails to serve its client, we may say it as server crashes or server down. This means that when a server breaks down or server crashes, the client connected to that server does not get access to that server and the client will often receive messages like - "Server Not Found", "Server is not responding" and so on.

2.4. ***Repair process:*** Repair activity will be taken by means of three steps

2.4.1. ***Step1:*** As with production servers, administrators must diagnose symptoms as soon as possible when the Web server is down. Some of the key points that need to be resolved are:

Have there been any suggested power outages, generator tests or other such happenings that could affect the overall physical environment? Is access to your Web server completely shut down or can some IP address ranges still affect the server? Is management access to the server still possible? Have there been any unusual entries into the logs?

2.4.2. ***Step 2:*** To take a macro view of the problem, countless man hours and costs of finding a solution would have been saved. For example, does the host Typeset sub-subheadings in medium face and capitalize the first letter of the first word only.

2.4.3. ***Step 3:*** Now that you've checked all of your cables and other peripheral devices, try to ping the device from within the LAN. Fortunately, the ping command is universal, so this should be straightforward regardless of the platform in use. If you can ping the server from within the LAN, try pinging the server from outside of the LAN. Doing this will go a long way in determining if the problem is at the routing and switching level, rather than at the server level. Also, if your Web server is virtualized, try pinging the IP address of the physical machine itself. This will help us to further isolate the problem. If we can't ping the server at all, and you've definitely checked the network connection, it's time to go deeper.

3. *MATHEMATICAL DESCRIPTION OF THE MODEL*

Customers arrival follows a Poisson process. Let $\lambda^+ a_j$, be the first order probability that a batch of j customers arrives at the system during a short interval of time$(t, t+dt)$, where $0 \leq a_j \leq 1$ and $\sum_{j=1}^{n} a_j = 1$ and $\lambda^+ > 0$ is the mean arrival rate of batches. Here the set up time follows exponential distribution. Service time and the repair process follows a general distribution.

4. *STEADY STATE CONDITIONS OVERSEEING THE FRAMEWORK*

For set up time,

$$\frac{d}{dt}\theta_n(t) + (\xi + \lambda^+)\theta_n(t) = \lambda^+ \sum_{j=1}^{n} a_j \theta_{n-j}(t) + \lambda^+ a_n F(t)$$
Eq. (1)

For service time,

$$\frac{d}{dx}Y_n(x) + (\lambda^+ + \psi + \varphi(x))Y_n(x) = \lambda^+ \sum_{j=1}^{n} a_j Y_{n-j}(x), n \geq 1$$ Eq. (2)

For the revamp process of the three stages,

$$\frac{d}{dx}\pi_n^{(a)}(x) + (\lambda^+ + \delta_a(x))\pi_n^{(a)}(x) = \lambda^+ \sum_{j=1}^{n} a_j \pi_{n-j}^{(a)}(x), n \geq 1$$ Eq. (3)

$$\frac{d}{dx}\pi_n^{(b)}(x) + (\lambda^+ + \delta_b(x))\pi_n^{(b)}(x) = \lambda^+ \sum_{j=1}^{n} a_j \pi_{n-j}^{(b)}(x), n \geq 1$$ Eq. (4)

$$\frac{d}{dx}\pi_n^{(c)}(x) + (\lambda^+ + \delta_c(x))\pi_n^{(c)}(x) = \lambda^+ \sum_{j=1}^{n} a_j \pi_{n-j}^{(c)}(x), n \geq 1$$
Eq. (5)

$$\frac{d}{dx}\pi_0^{(i)}(x) + (\lambda^+ + \delta_i(x))\pi_0^{(i)}(x) = 0, \text{ for } i = a, b, c$$
Eq. (6)

For the idle time,

$$\frac{d}{dt}F(t) = -\lambda^+ F + \int_0^\infty \pi_0^{(c)}(x)\delta_c(x)dx$$
Eq. (7)

5. *BOUNDARY CONDITIONS*

The above equations are to be solved subject to the boundary conditions

$$Y_n(0) = \int_0^\infty \pi_{n+1}^{(c)}(x)\delta_c(x)dx + \lambda^+ a_{n+1}F + \xi\theta_n(t)$$
Eq. (8)

$$\pi_n^{(a)}(0) = \psi \int_0^\infty Y_n(x)dx$$
Eq. (9)

$$\pi_n^{(b)}(0) = \int_0^\infty \pi_n^{(a)}(x)\delta_a(x)dx$$
Eq. (10)

$$\pi_n^{(c)}(0) = \int_0^\infty \pi_n^{(b)}(x)\delta_b(x)dx$$
Eq. (11)

6. *USAGE OF SUPPLEMENTARY VARIABLE METHOD*

Multiply Eq. (2) by z^n summing from 0 to

$$(\lambda^+ - \lambda^+ A(z) + \xi)\theta(z) = \lambda^+ A(z)F$$
Eq. (12)

$$\frac{d}{dx}Y^*(x,z) + \big(\lambda^+ - \lambda^+ A(z) + \psi + \varphi(x)\big)Y(x,z) = 0$$
Eq. (13)

Similarly,

$$\frac{d}{dx}\pi^{*(a)}(x,z) + (\lambda^+ - \lambda^+ A(z) + \delta_a(x))\pi^{(a)}(x,z) = 0$$
Eq. (14)

$$\frac{d}{dx}\pi^{*(b)}(x,z) + (\lambda^+ - \lambda^+ A(z) + \delta_b(x))\pi^{(b)}(x,z) = 0$$
Eq. (15)

$$\frac{d}{dx}\pi^{*(b)}(x,z) + (\lambda^+ - \lambda^+ A(z) + \delta_b(x))\pi^{(b)}(x,z) = 0$$
Eq. (15)

$$\frac{d}{dx}\pi^{*(c)}(x,z) + (\lambda^+ - \lambda^+ A(z) + \delta_c(x))\pi^{(c)}(x,z) = 0$$
Eq. (16)

Now continuing the same with the boundary conditions, we have

$$zY^*(0,z) = \int_0^\infty \pi^{*(c)}(x,z)\,\delta_c(x)dx + \lambda^+ A(z)F + z\xi\theta(z) - \int_0^\infty \pi_0^{(c)}(x,z)\,\delta_c(x)dx$$ Eq. (17)

$$\pi^{*(a)}(0,z) = \psi z \int_0^\infty Y^*(x,z)dx = \psi z Y^*(z)$$ Eq. (18)

$$\pi^{*(b)}(0,z) = \int_0^\infty \pi^{*(a)}(x,z)\,\delta_a(x)dx$$ Eq. (19)

$$\pi^{*(c)}(0,z) = \int_0^\infty \pi^{*(b)}(x,z)\,\delta_b(x)dx$$ Eq. (20)

Integrating Eq. (13) from 0 to x yields

$$Y^*(x,z) = Y^*(0,z)e^{-(\lambda^+ - \lambda^+ A(z) + \psi)x - \int_0^x \varphi(x)dx}$$
Eq. (21)

Let $\lambda^+ - \lambda^+ A(z) + \psi = g$

Integrating Eq. (21) by parts with respect to x yields

$$Y^*(z) = Y^*(0,z)\left[\frac{1-\overline{B_1}(g)}{g}\right]$$ Eq. (22)

Where $\overline{B_1}(g) = \int_0^\infty e^{-(\lambda^+ - \lambda^+ A(z) + \psi)x}\, dB_1(x)$ is the laplace Stieltje's transform of the service time $B_1(x)$.

Substituting Eq. (22) in Eq. (18)

$$\pi^{*(a)}(0,z) = \psi z Y^*(0,z)\left[\frac{1-\overline{B_1}(g)}{g}\right]$$

Likewise Integrating Eq. (14) and applying the same procedure , we have

$$\pi^{*(a)}(z) = \pi^{*(a)}(0,z)\left[\frac{1-\overline{B_2}(h)}{h}\right]$$ Eq. (23)

where $h = \lambda^+ - \lambda^+ A(z)$ and also we have

$$\int_0^\infty \pi^{*(a)}(x,z)\delta_a(x)dx = \psi z Y^*(0,z)\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)$$

Clearly,

$$\pi^{*(b)}(0,z) = \psi z Y^*(0,z)\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)$$

Also $\pi^{*(b)}(z) = \pi^{*(b)}(0,z)\left[\frac{1-\overline{B_3}(h)}{h}\right]$ Eq. (24)

So $\int_0^\infty \pi^{*(b)}(x,z)\delta_b(x)dx = \psi z Y^*(0,z)\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)$

Also for third repair stage, we have

$$\pi^{*(c)}(0,z) = \psi z Y^*(0,z)\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)$$

Also, $\pi^{*(c)}(z) = \pi^{*(c)}(0,z)\left[\frac{1-\overline{B_4}(h)}{h}\right]$ Eq. (25)

So

$$\int_0^\infty \pi^{*(c)}(x,z)\delta_c(x)dx = \psi z Y^*(0,z)\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\overline{B_4}(h)$$

Now Eq. (17) becomes,

$$zY^*(0,z) = \psi z Y^*(0,z)\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\overline{B_4}(h) + \lambda^+ A(z)F - \lambda^+ F + z\xi\theta(z)$$

$$\therefore Y^*(0,z) = \frac{\lambda^+ F(A(z)-1) + \frac{z\xi\lambda^+ A(z)F}{\lambda^+ - \lambda^+ A(z) + \xi}}{z - \psi z\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\overline{B_4}(h)} \quad \text{Eq. (26)}$$

Substituting Eq. (26) in Eq. (22),Eq. (23),Eq. (24) and Eq. (25),we get

$$Y^*(z) = \left[\frac{\lambda^+ F(A(z)-1) + \frac{z\xi\lambda^+ A(z)F}{\lambda^+ - \lambda^+ A(z) + \xi}}{z - \psi z\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\overline{B_4}(h)}\right]\left[\frac{1-\overline{B_1}(g)}{g}\right]$$

$$\pi^{*(a)}(z) = \psi z\left[\frac{\lambda^+ F(A(z)-1) + \frac{z\xi\lambda^+ A(z)F}{\lambda^+ - \lambda^+ A(z) + \xi}}{z - \psi z\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\overline{B_4}(h)}\right]\left[\frac{1-\overline{B_1}(g)}{g}\right]\left[\frac{1-\overline{B_2}(h)}{h}\right]$$

$$\pi^{*(b)}(z) = \psi z\left[\frac{\lambda^+ F(A(z)-1) + \frac{z\xi\lambda^+ A(z)F}{\lambda^+ - \lambda^+ A(z) + \xi}}{z - \psi z\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\overline{B_4}(h)}\right]\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\left[\frac{1-\overline{B_3}(h)}{h}\right]$$

$$\pi^{*(c)}(z) = \psi z\left[\frac{\lambda^+ F(A(z)-1) + \frac{z\xi\lambda^+ A(z)F}{\lambda^+ - \lambda^+ A(z) + \xi}}{z - \psi z\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\overline{B_4}(h)}\right]\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\left[\frac{1-\overline{B_4}(h)}{h}\right]$$

Eq. (27)

7. PROBABILITY GENERATING FUNCTION OF THE QUEUE SIZE

Let $P(z)$ denote the P.G.F of queue size irrespective of the state of the system.Then adding Eq. (27) and $\theta(z)$ from Eq. (12) , we get

$$P(z) = \theta(z) + Y^*(z) + \pi^{*(a)}(z) + \pi^{*(b)}(z) + \pi^{*(c)}(z)$$

$$= \frac{\left[\lambda^+ F(A(z)-1) + \frac{z\xi\lambda^+ A(z)F}{\lambda^+ - \lambda^+ A(z) + \xi}\right]\left[\frac{1-\overline{B_1}(g)}{g}\right]\left\{1 + \psi z\left[\left[\frac{1-\overline{B_2}(h)}{h}\right] + \overline{B_2}(h)\left[\frac{1-\overline{B_3}(h)}{h}\right] + \overline{B_2}(h)\overline{B_3}(h)\left[\frac{1-\overline{B_4}(h)}{h}\right]\right]\right\}}{z - \psi z\left[\frac{1-\overline{B_1}(g)}{g}\right]\overline{B_2}(h)\overline{B_3}(h)\overline{B_4}(h)}$$

Eq. (28)

8. STEADY STATE QUEUE SIZE DISTRIBUTION

In order to obtain F, we use the normalization condition $P(1) + F = 1$.

For $z = 1$, $P^*(z)$ is indeterminate of $\frac{0}{0}$ form.

Therefore, using L'Hopitals rule on Eq. (28), we get

$$P^*(1) = \frac{\lambda^+ + 2\xi\overline{B_1}'(\psi)}{1 + \psi\overline{B_1}'(\psi)}$$

9. PERFORMANCE MEASURES OF THE MODEL DEFINED

Let L_q denote the mean queue size under the steady state. Then

$$L_q = \frac{d}{dz}P^*(z)_{z=1} = \frac{D'(1)N''(1) - D''(1)N'(1)}{2(D'(1))^2} \quad \text{Eq. (29)}$$

where primes and two fold primes in the above condition indicate first and second subsidiary at $z = 1$ individually. Completing the subordinates at $= 1$, we have

$$N'(1) = \lambda^+ + 2\xi\overline{B_1}'(\psi)$$

$$N''(1) = \lambda^+\overline{B_1}'(\psi)[\lambda^+ + 2\xi\overline{B_1}'(\psi)] + (\lambda^+ + 2\xi\overline{B_1}'(\psi))(-\overline{B_1}'(\psi)) - \overline{B_1}''(\psi){\lambda^+}^2 - 2\lambda^+\overline{B_1}'(\psi)[\psi\{E(B_2) + E(B_3) + E(B_4)\}]$$

$$D'(1) = 1 + \psi\overline{B_1}'(\psi)$$

$$D''(1) = -\psi\left[-2\overline{B_1}'(\psi) + \lambda^+\left[\overline{B_1}''(\psi) - 2\overline{B_1}'(\psi)\{E(B_2) + E(B_3) + E(B_4)\}\right]\right]$$

10. NUMERICAL JUSTIFICATION OF THE MODEL

10.1. Consider the following set of values:

$\lambda^+ = 3$, $\psi = 2.5$, $\varphi = 3.5$, $\delta_a = 4$, $\delta_b = 4.5$, $\delta_c = 5$, $\xi = 3.6$

Table 1. Effect of change of Set up time ($\xi = 3.6, 4.5, 5.4, 6.2, 7.3$)

ρ	Q	L_q	L	W_q	W
0.2545	0.7455	4.9851	5.7306	1.6617	1.9102
0.2296	0.7704	5.5635	6.3339	1.8545	2.1113
0.2901	0.7909	6.142	6.9329	2.0473	2.311
0.1937	0.8063	6.6562	7.4625	2.2187	2.4875
0.1759	0.8241	7.3633	8.1874	2.4544	2.7291

Figure 1. Effect of change of Set up time

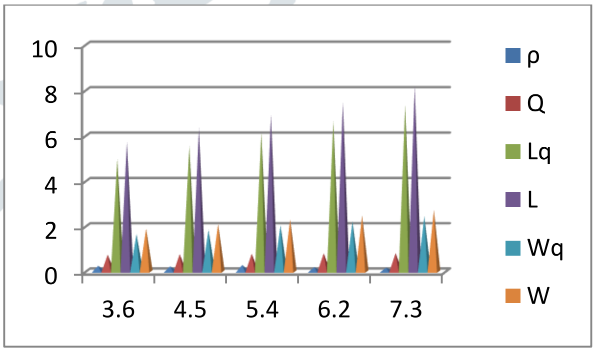


Table 2. Effect of change of Breakdown $\psi = 1, 1.5, 2, 2.5, 3$

Q	ρ	L_q	L	W_q	W
0.1713	0.8287	3.7720	4.6007	1.2573	1.5336
0.2032	0.7968	4.6427	5.4395	1.5476	1.8132
0.2306	0.7694	4.9224	5.6918	1.6408	1.8973
0.2545	0.7455	4.9850	5.7305	1.6617	1.9102
0.2395	0.7605	4.9610	5.7215	1.6537	1.9072

Figure 2. Effect of change of Breakdown

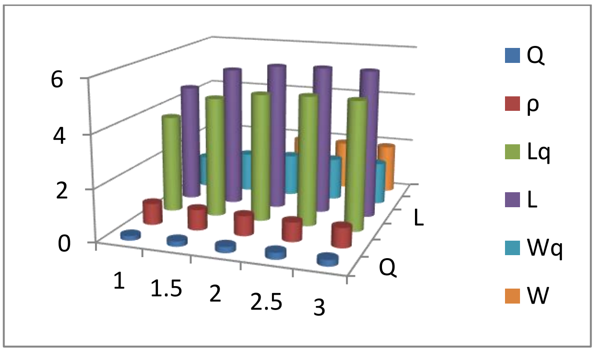
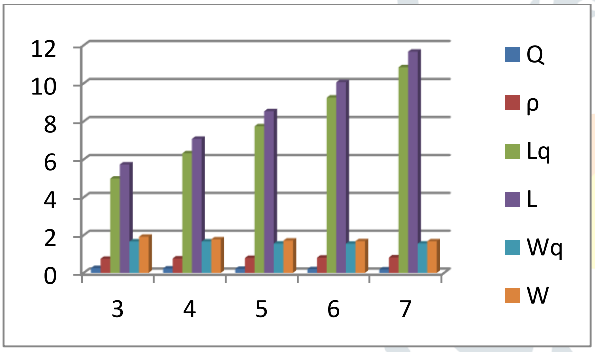


Table 3.Effect of change of Arrival ($\lambda^{+} = 3,4,5,6,7$)

Q	ρ	L_q	L	W_q	W
0.2545	0.7455	4.9851	5.7306	1.6617	1.9102
0.2307	0.7633	6.3185	7.0818	1.6617	1.7705
0.2109	0.7891	7.7398	8.5289	1.5480	1.7058
0.1942	0.8058	9.2488	10.0546	1.5415	1.6758
0.1800	0.8200	10.846	11.6656	1.5494	1.6665

Figure 3. Effect of change of Arrival



11. *Numerical analysis*

From the table qualities, as the set up time increments, there is no slack in the administration, the administration interference is negligible and furthermore the work gets finished sooner which prompts the expansion out of gear time of the server. What's more, despite the fact that the administration intrusion rate increments, because of the phases of patch up procedure, separate is corrected and the framework runs easily. Next because of the speedier transmission of the framework, the landing of clients expands which prompts the length of the line and furthermore the other execution to gets expanded.

12. *Conclusion*

The model is all around vindicated by various trades in graphical survey. The peculiarity of this work is the introduction of set up time and fix up work which gives complete satisfaction to all the arriving customers. This model has idle valuable solid life application in media transmission structure, remedial interpretation and gathering organizations. As a pending work, additional components of Priority organization, shying away, reneging and shut down time can be joined and a comparable examination of execution strategies with this model can be figured. In light of interchange techniques and diverse interruptions in versatile correspondence, new lining models can be delivered and the execution study can be picked up for the grounds of the correspondence compose.

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