Edge Vertex Prime Labeling of Trees

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Abstract : Let $G = (V(G), E(G))$ be a graph with p vertices and q edges. A bijective function $f: V(G) \cup E(G) \to \{1,2,3, \dots, p+q\}$ is said to be an edge vertex prime labeling, for any edge $uv \in E(G)$, if it is satisfies that $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. In this paper, we investigate several families of edge vertex prime labeling for double star $B_{m,n}$, subdivision of $B_{m,n}$ and $K_{1,n}$, comb graph, spider, H-graph of path P_n and coconut tree.

Key Words: prime labeling, edge vertex prime labeling, relatively prime.
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I. INTRODUCTION

Consider finite, simple and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. G is also called a (p, q) graph. We follow the basic notation and terminologies of graph theory as in [1]. *A graph labeling* is an assignment of integers to the vertices or edges or both, subject to certain conditions. An excellent survey on graph labeling is maintained by Gallian [2]. A graph on n vertices admits a *prime labeling* if its vertices can be labeled with the first n natural numbers such way that any two adjacent vertices have relatively prime labels. Such a graph admitting a prime labeling is known as a *prime graph*. The notion of a prime labeling originated with Entriger and was introduced in Tout et. al. [7]. Entriger conjectured that all trees have prime labeling which is not settle today.

A bijective function $f: V(G) \cup E(G) \to \{1,2,3, \dots, |V(G) \cup E(G)|\}$ is said to be an *edge vertex prime labeling* for each edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. A graph that admits an edge vertex prime labeling is called an *edge vertex prime graph*. The notion of an edge vertex prime labeling was introduced by Jagadesh and Basker Babujee [3]. They proved the existence of paths, cycles and star $K_{1,n}$. In [4], they are edge vertex prime labeling of generalised star, generalised cycle star, $p + q$ is odd for $G\hat{O}K_{1,n}$, $G\hat{O}P_n, G\hat{O}C_n$ are proved by Jagadesh et. al. [4]. Parmer [5] proved that wheel W_n, fan f_n, Friendship graph F_n and they also proved in [6], $K_{2,n}$ for all n and $K_{3,n}$ for $2 \le n \le 29$ are edge vertex prime labeling. An edge vertex prime labeling is a variation of prime labeling.

Let $n_1, n_2, \dots, n_r, r \ge 1$, be a positive integers, where $n_1, n_r \ge 1$ and $n_i \ge 0$ for $i \in \{2,3, \dots, r-1\}$. The *Caterpillar* $C(n_1, n_2, \dots, n_r)$ is the tree obtained from the path $P_r = x_1 x_2 \dots x_r$ by joining the x_i to n_i new vertices $x_{i,1}, x_{i,2}, \dots, x_{i,n_i}$ for each i. Clearly, $C(n_1, n_2, \dots, n_r)$ is of diameter $r + 1$.

Star graph is a tree consisting of one apex or center vertex adjacent to all the others. If T is a Caterpillar of diameter 2, then $r = 1$ and hence T is a star. Define a bijection function $f: V\big(C(n_1)\big) \cup E\big(C(n_1)\big) \to \{1,2, \dots, \big|V\big(C(n_1)\big) \cup E\big(C(n_1)\big)\big|\}$. In this case apex vertex or center vertex should be label 1 and label vertices by odd numbers and label edges by even numbers.

Bistar is a graph obtained from K_2 by joining n pendant edges to both the ends of K_2 and is denoted by $B_{n,n}$. It have $2n + 2$ vertices and $2n + 1$ edges.

Double star is the graph obtained from K_2 by joining m pendant edges to one end and n pendant edges to the other end of K_2 and is denoted by $B_{m,n}$. It have $m + n + 2$ vertices and $m + n + 1$ edges. Bistar is a particular case of double star.

The *subdivision* graph $S(G)$ is obtained from G by subdividing each edge of G by a vertex. The graph obtained by joining a pendant edge at each vertex of a path P_n is called a *comb* and is denoted by $P_n.K_1$.

A *spider* is a tree with at most one vertex of degree more than two, is called the center of spider (if no vertex of degree more than two, then any vertex can be the center).

A *leg* of a spider is a path from the center to a vertex of degree one. Thus, a star with k legs is a spider of legs each of length 1.

The H- *graph* of a path P_n is the graph obtained from two copies of P_n with vertices $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ by joining the vertices $x_{\frac{n-1}{2}}$ and $y_{\frac{n-1}{2}}$ if n is odd and the vertices $x_{\frac{n-2}{2}}$ and $y_{\frac{n-2}{2}}$ if n is even.

A *coconut tree* $CT(m,n)$ is the graph obtained from the path P_m by appending n new pendant edges at an end vertex of P_m.

A Y-*tree* , Y_{n+1} is a graph obtained from the path P_n by appending an edge to a vertex of the path P_m adjacent to an end point.

In this paper, we prove that some classes of trees such as double star $B_{m,n}$, subdivision of $B_{m,n}$ and $K_{1,n}$, comb, spider, H- graphs of path P_n and coconut tree are edge vertex prime labeling.

II. MAIN RESULTS

We concentrate on an edge vertex prime labeling of double star.

Theorem 2.1. The double star $B_{m,n}$, $m, n \geq 1$ admits edge vertex prime labeling.

Proof. Let x be the vertex of degree $m+1$ and y be the vertex of degree $n+1$, which is adjacent to x. Let $x_1, x_2, \ldots, x_m$ be the vertices of degree 1 which are adjacent to x and $y_1, y_2, \ldots, y_n$ be the vertices of degree 1 which are adjacent to y. Without loss of generality, assume that $m \leq n$. Then $V(B_{m,n}) = \{x, y, x_i, y_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(B_{m,n}) = \{xy\} \cup \{xx_i : 1 \leq i \leq m\} \cup \{yy_j : 1 \leq j \leq n\}$. Also, $|V(B_{m,n})| = m + n + 2$ and $|E(B_{m,n})| = m + n + 1$. Define a bijective function $f: V(B_{m,n}) \cup E(B_{m,n}) \to \{1, 2, \ldots, 2m + 2n + 3\}$ by $f(x) = 1$, $f(x_i) = 2i + 1$ for $1 \leq i \leq m$, $f(xy) = 2m + 2$.
Consider the following cases.
Case (i). When $p = 2m + 2n + 3$ is a prime congruent to 1 or 3 modulo 4.
$f(y) = 2m + 2n + 3$.
For each $1 \leq j \leq n$, $f(y_j) = 2m + 2j + 1$, $f(yy_j) = 2m + 2j + 2$.
Case (ii). When $p = 2m + 2n + 3$ is not a prime, label the apex vertex y choose the largest one prime number between $2m + 3$ to $2m + 2n + 3$, label the vertices y_j's are remaining the odd numbers and label the edges yy_j's are even numbers.
Now, our claims are (i) $f(x), f(x_i)$ and $f(xx_i)$, (ii) $f(x), f(y)$ and $f(xy)$,
(iii) $f(y), f(y_j)$ and $f(yy_j)$ are pairwise relatively prime.
(i) For each $1 \leq i \leq m$,
$gcd(f(x), f(x_i)) = gcd(1, 2i + 1) = 1$,
$gcd(f(x), f(xx_i)) = gcd(1, 2i) = 1$,
$gcd(f(x_i), f(xx_i)) = gcd(2i + 1, 2i) = 1$, which is two consecutive numbers.
(ii) $gcd(f(x), f(y)) = gcd(1, 2m + 2n + 3) = 1$,
$gcd(f(x), f(xy)) = gcd(1, 2m + 2) = 1$,
$gcd(f(y), f(xy)) = gcd(2m + 2n + 3,\ 2m + 2) = 1$.
Since p is the largest prime in the corresponding set and there are no such product in the set, which has p as a multiplier.
(iii) For each $1 \leq j \leq n$,
$gcd\left(f(y), f(y_j)\right) = gcd(2m + 2n + 3, 2m + 2j + 1) = 1$,
$gcd\left(f(y), f(yy_j)\right) = gcd(2m + 2n + 3, 2m + 2j + 2) = 1$,
$gcd\left(f(y_j), f(yy_j)\right) = gcd(2m + 2j + 1, 2m + 2j + 2) = 1$, which is two consecutive numbers.
Therefore, for any edge $uv \in E(B_{m,n})$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime. Hence double star $B_{m,n}$, $m, n \geq 1$ is an edge vertex prime labeling. ■

Theorem 2.2. The graph obtained by subdivision of the central edge of the double star $B_{m,n}$ is an edge vertex prime labeling.

Proof. On subdivision of the central edge of the double star $B_{m,n}$, we get the graph G which has $m + n + 3$ vertices and $m + n + 2$ edges. Without loss of generality, assume that $m \leq n$. Here the subdivision is at the bridge xy forming the new vertex z. Let x be the vertex of degree $m + 1$ and y be the vertex of degree $n + 1$ and z be the central vertex which is adjacent to x and y. Let $x_1, x_2, \ldots, x_m$ be the vertices of degree 1 which are adjacent to x and $y_1, y_2, \ldots, y_n$ be the vertices of degree 1 which are adjacent to y. Then

$V(G) = \{x, y, z, x_i : 1 \le i \le m, y_j : 1 \le j \le n\}$ and $E(G) = \{xz, yz, xx_i : 1 \le i \le m, yy_j : 1 \le j \le n\}$.
Define a bijective function $f: V(G) \cup E(G) \to \{1,2, \dots ,2m + 2n + 5\}$ by $f(x) = 1$,
$f(x_i) = 2i + 1$ for $1 \le i \le m$,
$f(xz) = 2m + 2, f(z) = 2m + 3, f(yz) = 2m + 4$.
Consider the following cases.
Case (i). When $2m + 2n + 5$ is a prime congruent to 1 or 3 modulo 4.
$f(y) = 2m + 2n + 5$.
For each $1 \le j \le n$, $f(y_j) = 2m + 2j + 3$, $f(yy_j) = 2m + 2j + 4$.
Case (ii). When $2m + 2n + 5$ is not a prime.
Label the apex vertex y, choose the largest one prime number between $2m + 5$ to $2m + 2n + 5$, lable the vertices $y_j's$ are remaining the odd numbers and label the edges $yy_j's$ are even numbers. The property of an edge vertex prime labeling is similar to the double star $B_{m,n}$. Additionally,
$gcd(f(x), f(z)) = gcd(1, 2m + 3) = 1$,
$gcd(f(x), f(xz)) = gcd(1, 2m + 2) = 1$,
$gcd(f(z), f(xz)) = gcd(2m + 3, 2m + 2) = 1$, which is two consecutive numbers.
Clearly, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime.
Hence the subdivision of central edge of the double star $B_{m,n}$ is an edge vertex prime labeling. ■

Theorem 2.3. The subdivision of $K_{1,n}$ is an edge vertex prime labeling.

Proof. Let G be the subdivision of $K_{1,n}$.Then $V(G) = \{u, u_i, v_i : 1 \le i \le n\}$ and
$E(G) = \{uu_i, u_iv_i : 1 \le i \le n\}$. Also, $|V(G)| = 2n + 1$ and $|E(G)| = 2n$.
Define a bijective function $f: V(G) \cup E(G) \to \{1,2, \dots ,4n + 1\}$ by $f(u) = 1$.
For each $1 \le i \le n$, $f(u_i) = 4i - 1, f(v_i) = 4i + 1, f(uu_i) = 4i - 2, f(u_iv_i) = 4i$.
Next, we show the property of an edge vertex prime labeling.
For each $1 \le i \le n$,
$gcd(f(u), f(u_i)) = gcd(1, 4i - 1) = 1$,
$gcd(f(u), f(uu_i)) = gcd(1, 4i - 2) = 1$,
$gcd(f(u_i), f(uu_i)) = gcd(4i - 1, 4i - 2) = 1$,
$gcd(f(u_i), f(v_i)) = gcd(4i - 1, 4i + 1) = 1$,
$gcd(f(u_i), f(u_iv_i)) = gcd(4i - 1, 4i) = 1$,
$gcd(f(v_i), f(u_iv_i)) = gcd(4i + 1, 4i) = 1$.
Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime.
Hence the subdivision of $K_{1,n}$ has an edge vertex prime labeling. ■

Theorem 2.4. Every comb graph $P_n.K_1$ is an edge vertex prime labeling.

Proof. Let $u_1, u_2, \dots, u_n$ be the vertices of a path P_n and $v_1, v_2, \dots, v_n$ be the vertices adjacent to each vertex of the path P_n. Let $G = P_n.K_1$ be a graph. Then $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and
$E(G) = \{u_iu_{i+1} : 1 \le i \le n - 1\} \cup \{u_iv_i : 1 \le i \le n\}$.
Also, $|V(G)| = 2n$ and $|E(G)| = 2n - 1$.
Define a bijective function $f: V(G) \cup E(G) \to \{1,2, \dots ,4n - 1\}$ as follows.
Case (i). If $n \equiv 0(mod\ 3)$
For each $1 \le i \le \frac{n}{3}$,
$f(u_{3i-2}) = 12i - 11, f(u_{3i-1}) = 12i - 7, f(u_{3i}) = 12i - 1$,
$f(v_{3i-2}) = 12i - 9, f(v_{3i-1}) = 12i - 5, f(v_{3i}) = 12i - 3$, $f(u_{3i-2}u_{3i-1}) = 12i - 8, f(u_{3i-1}u_{3i}) = 12i - 4$,
$f(u_{3i-2}v_{3i-2}) = 12i - 10$, $f(u_{3i-1}v_{3i-1}) = 12i - 6, f(u_{3i}v_{3i}) = 12i - 2$.
For each $1 \le i \le \frac{n}{3} - 1$,
$f(u_{3i+1}) = 12i + 1, f(u_{3i}u_{3i+1}) = 12i$.
Case (ii). If $n \equiv 1(mod\ 3)$
For each $1 \le i \le \frac{n+2}{3}$,
$f(u_{3i-2}) = 12i - 11, f(v_{3i-2}) = 12i - 9, f(u_{3i-2}v_{3i-2}) = 12i - 10$.
For each $1 \le i \le \frac{n-1}{3}$,

$f(u_{3i-1}) = 12i-7, f(u_{3i}) = 12i-1, f(v_{3i-1}) = 12i-5, f(v_{3i}) = 12i-3,$
$f(u_{3i-2}u_{3i-1}) = 12i-8, f(u_{3i-1}u_{3i}) = 12i-4, f(u_{3i-1}v_{3i-1}) = 12i-6, f(u_{3i}v_{3i}) = 12i-2,$
$f(u_{3i+1}) = 12i+1, f(u_{3i}u_{3i+1}) = 12i.$

Case (iii). If $n \equiv 2(mod\ 3)$

For each $1 \le i \le \frac{n+1}{3}$,

$f(u_{3i-2}) = 12i-11, f(v_{3i-2}) = 12i-9, f(u_{3i-1}) = 12i-7, f(v_{3i-1}) = 12i-5,$
$f(u_{3i-2}u_{3i-1}) = 12i-8, f(u_{3i-2}v_{3i-2}) = 12i-10, f(u_{3i-1}v_{3i-1}) = 12i-6.$

For each $1 \le i \le \frac{n-2}{3}$,

$f(u_{3i}) = 12i-1, f(v_{3i}) = 12i-3, f(u_{3i-1}u_{3i}) = 12i-4, f(u_{3i}u_{3i+1}) = 12i, f(u_{3i}v_{3i}) = 12i-2.$

Now, our claims are (i) $f(u_{3i-2}), f(u_{3i-1})$ and $f(u_{3i-2}u_{3i-1})$, (ii) $f(u_{3i-1}), f(u_{3i})$ and $f(u_{3i-1}u_{3i})$, (iii) $f(u_{3i}), f(u_{3i+1})$ and $f(u_{3i}u_{3i+1})$, (iv) $f(u_{3i-2}), f(v_{3i-2})$ and $f(u_{3i-2}v_{3i-2})$, (v) $f(u_{3i-1}), f(v_{3i-1})$ and $f(u_{3i-1}v_{3i-1})$, (vi) $f(u_{3i}), f(v_{3i})$ and $f(u_{3i}v_{3i})$ are pairwise relatively prime.

For each $1 \le i \le \frac{n}{3}$,

$gcd(f(u_{3i-2}), f(u_{3i-1})) = gcd(12i-11,\ 12i-7) = 1,$
$gcd\ (f(u_{3i-2}), f(u_{3i-2}u_{3i-1})) = gcd(12i-11,\ 12i-8) = 1,$
$gcd\ (f(u_{3i-1}), f(u_{3i-2}u_{3i-1})) = gcd(12i-7,\ 12i-8) = 1,$
$gcd(f(u_{3i-1}), f(u_{3i})) = gcd(12i-7,\ 12i-1) = 1,$
$gcd\ (f(u_{3i-1}), f(u_{3i-1}u_{3i})) = gcd(12i-7,\ 12i-4) = 1,$
$gcd\ (f(u_{3i}), f(u_{3i-1}u_{3i})) = gcd(12i-1,\ 12i-4) = 1,$
$gcd\ (f(u_{3i-2}), f(v_{3i-2})) = gcd(12\mathrm{i}-11,\ 12\mathrm{i}-9) = 1,$
$gcd\ (f(u_{3i-2}), f(u_{3i-2}v_{3i-2})) = gcd(12\mathrm{i}-11,\ 12\mathrm{i}-10) = 1,$
$gcd\ (f(v_{3i-2}), f(u_{3i-2}v_{3i-2})) = gcd(12\mathrm{i}-9, 12\mathrm{i}-10) = 1,$
$gcd\ (f(u_{3i-1}), f(v_{3i-1})) = gcd(12i-7,\ 12i-5) = 1,$
$gcd\ (f(u_{3i-1}), f(u_{3i-1}v_{3i-1})) = gcd(12i-7,\ 12i-6) = 1,$
$gcd\ (f(v_{3i-1}), f(u_{3i-1}v_{3i-1})) = gcd(12i-5,\ 12i-6) = 1,$
$gcd\ (f(u_{3i}), f(v_{3i})) = gcd(12i-1,\ 12i-3) = 1,$
$gcd\ (f(u_{3i}), f(v_{3i}v_{3i})) = gcd(12i-1,\ 12i-2) = 1,$
$gcd\ (f(v_{3i}), f(u_{3i}v_{3i})) = gcd(12i-3,\ 12i-2) = 1.$

For each $1 \le i \le \frac{n}{3} - 1$,

$gcd(f(u_{3i}), f(u_{3i+1})) = gcd(12i-1,\ 12i+1) = 1,$
$gcd\ (f(u_{3i}), f(u_{3i}u_{3i+1})) = gcd(12i-1,\ 12i) = 1,$
$gcd\ (f(u_{3i+1}), f(u_{3i}u_{3i+1})) = gcd(12i+1,\ 12i) = 1.$

Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime.
Hence the comb graph $P_n.K_1$ is an edge vertex prime labeling. ■

Jagadesh and Baskar Babujee [4] proved that every generalized star $K_{1,n_1,n_2,\ldots,n_m}$ admits an edge vertex prime labeling. In this case, only label the apex vertex should be larger prime label.

A spider is another class of trees that can be shown to an edge vertex prime labeling. It is defined as a tree with only one vertex of degree 3 or more, but a spider can also be viewed as a collection of paths $P_{n_1}, P_{n_2}, \ldots, P_{n_k}$ with one end of each path adjoined to a central vertex.

Theorem 2.5. All spiders are an edge vertex prime labeling.

Proof. For each P_{n_j}, consider the vertices as $v_1, v_2, \ldots, v_{n_j}$ with v_1 being adjacent to the centre and v_{n_j} as the leaf. We begin the labeling by assigning the label 1 to the central vertex. For each path, we apply a similar function to the path labeling from the generalised star. We assign the even numbers label to the edges and assigning odd numbers label to the vertices of the first path. Continuing in this way, another paths. Consider a vertex v with degree at least 2. If v is on a path P_{n_j} and is adjacent to the central vertex, then $f(v_1), f(v_i)$ and $f(v_1v_i)$ are pairwise relatively prime. If v is an a path, but not adjacent to central vertex, its vertices and edges are labeled by consecutive integers. Hence all spiders have an edge vertex prime labeling. ■

Theorem 2.6. The H −graph of path P_n is an edge vertex prime labeling.

Proof. Let H_n be the $H-$graph of path P_n. Then $V(H_n) = \{x_i, y_i : 1 \le i \le n\}$ and $E(H_n) = \{x_i x_{i+1}, y_i y_{i+1} : 1 \le i \le n-1\} \cup \left\{x_{\frac{n-1}{2}} y_{\frac{n-1}{2}}, \text{if } n \text{ is odd}\right\}$ or $\left\{x_{\frac{n-2}{2}} y_{\frac{n-2}{2}}, \text{if } n \text{ is even}\right\}$.
Also, $|V(H_n)| = 2n$ and $|E(H_n)| = 2n$. Define a bijective function $f: V(H_n) \cup E(H_n) \to \{1,2,\ldots,4n\}$ as follows.
For each $1 \le i \le n, f(x_i) = 2i-1, f(y_i) = 2n+2i-1$.
For each $1 \le i \le n-1$, $f(x_i x_{i+1}) = 2i-2$, $f(y_i y_{i+1}) = 2n+2i$.
Consider the following cases.

Case (i). When n is odd. $f\left(x_{\frac{n-1}{2}} y_{\frac{n-1}{2}}\right) = 2n$.

Case(ii). When n is even. $f\left(x_{\frac{n-2}{2}} y_{\frac{n-2}{2}}\right) = 2n$.

Clearly, (i) $f(x_i), f(x_{i+1})$ and $f(x_i x_{i+1})$, (ii) $f(y_i), f(y_{i+1})$ and $f(y_i y_{i+1})$, (iii) if n is odd, then $f\left(x_{\frac{n-1}{2}}\right), f\left(y_{\frac{n-1}{2}}\right)$ and $f\left(x_{\frac{n-1}{2}} y_{\frac{n-1}{2}}\right)$, (iv) if n is even, then $f\left(x_{\frac{n-2}{2}}\right), f\left(y_{\frac{n-2}{2}}\right)$ and $f\left(x_{\frac{n-2}{2}} y_{\frac{n-2}{2}}\right)$ are pairwise relatively prime.
Hence the $H-$graph of path P_n is an edge vertex prime labeling. ■

Theorem 2.7. The coconut tree $CT(m,n)$ admits edge vertex prime labeling.

Proof. Let $G(V,E) = CT(m,n)$. Then $V(G) = \{u_i, v_j : 1 \le i \le m, 1 \le j \le n\}$ and $E(G) = \{u_1 v_j : 1 \le j \le n\} \cup \{u_i u_{i+1} : 1 \le i \le m\}$. Also, $|V(G)| = m+n$ and $|E(G)| = m+n-1$.
Define a bijective function $f: V(G) \cup E(G) \to \{1,2,\ldots,2m+2n-1\}$ by $f(u_1) = 1$.
For each $1 \le j \le n$,
$f(v_j) = 2j+1$, $f(u_1 v_j) = 2j$.
$f(u_i) = 2n+2i-1$ for $2 \le i \le m-1, f(u_i u_{i+1}) = 2n+2i$ for $1 \le i \le m$.
Now, our claims are (i) $f(u_1), f(v_j)$ and $f(u_1 v_j)$, (ii) $f(u_i), f(u_{i+1})$ and $f(u_i u_{i+1})$, are pairwise relatively prime.
Verification of (i). For each $1 \le j \le n$,
$gcd\left(f(u_1), f(v_j)\right) = gcd(1, 2j+1) = 1$,
$gcd\left(f(u_1), f(u_1 v_j)\right) = gcd(1, 2j) = 1$,
$gcd\left(f(v_j), f(u_1 v_j)\right) = gcd(2j+1, 2j) = 1$,
$gcd(f(u_1), f(u_2)) = gcd(1, 2n+3) = 1$,
$gcd(f(u_1), f(u_1 u_2)) = gcd(1, 2n+2) = 1$,
$gcd(f(u_2), f(u_1 u_2)) = gcd(2n+3, 2n+2) = 1$.
Verification of (ii). For each $2 \le i \le m$,
$gcd(f(u_i), f(u_{i+1})) = gcd(2n+2i-1, 2n+2i+1) = 1$,
$gcd(f(u_i), f(u_i u_{i+1})) = gcd(2n+2i-1, 2n+2i) = 1$,
$gcd(f(u_{i+1}), f(u_i u_{i+1})) = gcd(2n+2i+1, 2n+2i) = 1$.
Therefore, for any edge $uv \in E(G)$, the numbers $f(u), f(v)$ and $f(uv)$ are pairwise relatively prime.
Hence $CT(m,n)$ has an edge vertex prime labeling. ■

Corollary 2.8. For $m \ge 3$ the $Y-$tree, Y_{m+1} is an edge vertex prime labeling.

Proof. It is easily observed that $Y_{m+1} \cong CT(m-1, 2)$ for $m \ge 3$. Hence by Theorem 2.7, Y_{m+1} has an edge vertex prime labeling. ■

References

[1] Balakrishnan, R. and Ranganathan, K. (2012): A Text Book of Graph Theory, Second Edition, Springer. New York.

[2] Gallian, J, A. (2015): A Dynamic Survey of Graph Labeling, Electronic Journal of Combinatorics, DS6.

[3] Jagadesh, R. Baskar Babujee, J. (2017): Edge Vertex Prime Labeling for some class of Graphs, National Conference on Recent Trends in Mathematics and its Applications, 24-25 February, SRM University, Vadapalani, Chennai, India.

[4] Jagadesh, R. Baskar Babujee, J. (2017): On Edge Vertex Prime Labeling, International Journal of Pure and Applied Mathematics, Vol.114 No.6, 209-218.

[5] Parmar, Y. (2017): Edge Vertex Prime Labeling for Wheel, Fan and Friendship Graph, International Journal of Mathematics and Statistics Invention,Vol.5 Issue 8, October, 23-29.

[6] Parmar, Y. (2018): Vertex Prime Labeling for $K_{2,n}$ and $K_{3,n}$ Graphs, Mathematical Journal of Interdisciplinary Sciences, Vol.6 No.2, March, 167-180.

[7] Tout, A. Dabboucy, A, N. Howalla, K.(1982): Prime Labeling of Graphs, National Academy Science, Letters, 11,365-368.