A STUDY ON THE ANALYSIS OF A BULK ARRIVAL FEEDBACK SERVICE QUEUEING MODEL WITH BALKING AND RENEGING IN GRID COMPUTING

[1] Dr.S.Maragathasundari,[2]N.Murugeswari,[3]S.Radha
Department 0f Mathematics
Kalasalingam Academy of Research and Education,Krishnankoil, India

*Abstract:*The present paper goes for focus a Queueing model of single server giving the administration in which the customers arrive in packs. Administration time takes after general dispersion. After the satisfaction of the administration the server goes for a mandatory get-away taken after by an optional expanded get-away. In this model when we talk about get-away, it suggests truly the help work will be done in the midst of the period of vacation. The minor support work will be finished in compulsory excursion. On the off chance that requiring genuine upkeep work, it will be passed on in a optional extended vacation is taken after. Also at whatever point the arriving customer isn't satisfied by the administration he has gotten, he has the option of getting a re benefit for his fullest satisfaction. Likewise we have included Balking and reneging which mirrors customers' on edge lead where an arriving bunch picks not to join the line for a couple of reasons. For the proposed demonstrate, the likelihood producing capacity for the line estimate dispersion at a discretionary age is inferred utilizing supplementary variable method. Distinctive execution measures are determined. The model is upheld by a consistent application to such an extent that the model audits the execution measures of the application. Numerical depiction is given to legitimize the proposed show.

***Index Terms* - Bulk arrival, maintenance time (vacation time), Balking, Reneging, optional extended vacation and optional re service.**
***Mathematics subject classification* – 60k25, 60k20, 90B22**

I. INTRODUCTION

Queueing systems with vacations have been developed for wide range applications in production. They have many applications in machining systems working in industrial environment such as manufacturing and production systems. Server vacation models are useful for the system in which a server wants to utilize the idle time for different purposes. A few creators have examined queueing models in sorts of services in differing charges that incorporate unpredictable service interruption and repair. Diverse vacation approaches have been presented by understood scientists in the before studies. Cox.D.R [2] made an examination on Non-Markovian Stochastic Processes by the incorporation of Supplementary Variables. Madan and Anabosi [12] concentrated two sorts of services with single vacation and Bernoulli plan excursion. Maragathasundari [13] concentrated a mass arriving queueing model with three phases of services took after by service interruption and delay time. Choudhury [1] explored a bulb arrival queue with an excursion time under single vacation strategy. Madan and Abu-Dayyeh [8] concentrated a solitary server line with stage sort server departure and optional stage sort server vacation. Maraghi et.al [23] made an examination on the bunch landing queueing framework with second discretionary administration and irregular breakdown. Maragathasundari and Srinivasan [14] made an examination on a Non Markovian line with three phases of service and numerous vacations. Kavitha and Maragathasundari [6] researched the idea of limited tolerability and optional sorts of repair in a Non Markovian queue. A Non Markovian queue with discretionary services has been investigated by Srinivasan and Maragathasundari [19].A Non Markovian line with multi phases of service and reneging have been considered by Maragathasundariet.al[15]. Karthikeyan and Maragathasundari [5] made an examination on a clump arrival of two phases of service with standby server amid general vacation time and general repair time. Kumar.R and Sharma.S.K [4] reneging and Balking in a non markovian line. Madan. K.C. what's more, Chodhury. G [10] made an examination on M[x]/G/1 line with Bernoulli get-away timetable under limited admissibility. Madan. K.C [9], examined the line with two phase heterogeneous administration and binomial calendar server vacations. Maraghi. Et.al [17], concentrated a clump Arrival lining framework with Random Breakdowns and Bernoulli Schedule server vacation taking after general distribution. Ranjitham.A. what's more, Maragathasundari .S [18], studied the two phases of service in mass landing queueing model. In their review, if an arriving group of clients discover the server occupied or in excursion,, then the whole bunch joins the circle so as to look for the administration once more. Sowmiyah and Maragathasundari [20] researched a mass queueing models with amplified excursion and stages in repair. Kendall. D.G [7] concentrated the stochastic Processes happening in the hypothesis of lines and their examination by the strategy for implanted Markov chains. Based on all the above research work, another queueing model has been surrounded for the above versatile correspondence framework. We surmise that the Queueing model here proposed speaks to a stage toward the advancement of a general and diagnostic tractable demonstrating instrument for the plan of portable correspondence organizes under more reasonable contemplations.

II. Practical Justification Of The Model

Performance estimation of Grid computing using queuing theory

In the recent years, high performance computing (HPC)applications which require high processing power, vast memory storage and large volumes of data(in the order of terabytes and petabytes in size) pose a challenge to the existing stand-alone computers/workstation . Grid computing presents a resource sharing facility in which a complex task is handled collectively by a group of machines connected to the grid. The complex task is split into a group of independent subtasks, which are queued for execution. Job scheduler distributes these subtasks to the available computing systems and manages the workflow These systems share various resources like computing hardware, data storage capacity, communications, software and licenses, special equipments and capacities. Using grid computing, an application can be remotely run on any idle computer in the grid that has the necessary computing resources, whenever the host computer is busy. A complex task can be split into 'n' number of subtasks and run on n systems concurrently and hence the task can be completed 'n' times faster than the speed of a single system. Thus grid computing can be used to achieve a maximum resource utilization efficiency. The grid can comprise of homogeneous systems or heterogeneous systems with different operating systems. Grid is constructed across Local area network (LAN) or Wide area network (WAN) at regional, national, or global scales. The hardware resources used in a grid are workstations, servers, clusters, and supercomputers, Personal computers, laptops and PDAs.

In the queuing model for a grid computing system, we consider the job scheduler (control node) as the single server. The scheduler is responsible for sending a job to a given machine to be executed. This module manages the scheduling, distribution, and prioritization of jobs and processes running on the grid. Load balancing is a process of improving the performance of computational grid system in such a way that all the computing nodes involved in the grid are uniformly utilized as much as possible so that the throughput is improved, and the execution times are minimized. So the job scheduler can migrate jobs to less busy parts of the grid to balance resource loads and absorb unexpected peaks of organization activity. The scheduler could schedule jobs to reduce the communications traffic or to minimize the distance of the communications. Finally, the results of all jobs are collected and assembled to produce the ultimate answer for the application. The independent subtasks from all nodes in the network form the single execution queue. The scheduler is capable of batch processing i.e it handles a batch of subtasks at the same time in the input of the system. The arrival rate, λ follows poisson distribution and service rate μ follows general distribution. The following parameters are analysed to estimate the performance of the system

i) expected number of customers in the system
ii) expected number of customers in the queue
iii) expected amount of time customer spends in system
iv) expected waiting time in queue
v) server utilization rate
vi)mean service rate

During the time t, when the server goes for a vacation, the arriving customers balk with a probability p. When the server goes to vacation, the service becomes unavailable to other jobs in the queue and customers decide to renege during vacation times. We assume reneging follows exponential distribution

Vacation- Secondary Task

Since the high performance computing (HPC) applications are run on different hardware resources connected to the grid, security in these systems is monitored as a secondary task by the server during vacation. The security issues include preventing manipulation, misuse, unauthorized access, denial of service, hijacking, stalling of processes, stealing of information stored on the grid and stealing of computing power provided by the grid.

The server launches a security scan to detect malware, unauthorized users, virus attack, SQL injection attack to protect the data integrity and confidentiality of the users.

Grid communication uses public key cryptography to authenticate clients to protect the server and ensure protection against attackers. Whenever a new task enters the grid system, the host user credentials and network certificates are verified. This prevents the unauthorisduser's from hacking the grid system and stealing confidential data.

These secondary tasks (security scan) are performed by the server during vacation when there are no jobs in the queue.

Extended Vacation - Off Peak Time

During off peak time the server takes a extended vacation in which the idle systems are put in power saving mode. This increases the energy efficiency in the grid system by reducing energy consumption and excess heat. The server employs DVFS (Dynamic voltage frequency scaling) technique to change the voltage and frequency supply at the each node depending on the application. The nodes include memory, data path, clock generation and distribution network, memory, and hard drive. Depending on the voltage and frequency, the set of power states available for each device are sleep, stand-by or busy mode

Balking

For real time applications carried out in the grid, like online banking and fund transfer applications, waits for a time t and then the session expires (i.e) the job is in the system but doesn't join the execution queue. If the waiting time in the system $W_s < t$, then the job joins the queue. If $W_s > t$, the operation is aborted and the session window expires. The waiting time t is monitored by the inactivity timer and operation is aborted for security reasons to prevent hackers from accessing sensitive data.

Other examples for balking are online games with two players. The player A is given specified time t_1 to make a move .Irrespective of the player's action or inactivity, the session expires after the specified time and player B is given a specified time t_2.

For a system size of s and n number of customers in the queue upon arrival, let us represent a probability p_{s_n} with which the job joins the queue or a balks with a probability $1 - p_{s_n}$.

Reneging

A job joins the queue and after waiting for a specified time leaves the queue without being served. Examples for such jobs are short jobs like ZIP files, lossy audio files, email chat message (application with kilobyte data) which leave the queue. At the same time, data-intensive jobs(like application with terabyte data) stay in the queue irrespective of the waiting time as the cost to move the job out of the queue is larger balking, reneging -cost of delay

III. Notations

$P_n(x,t)$: Probability that at time t, the server is active providing service and there are $n(n \geq 0)$customers in the queue excluding the one being served and the elapsed service time for this customer is x.

$R_n(x,t)$: Probability that at time$t$, the server is active providing service and there are customers $n(n \geq 0)$ in the queue excluding the one being served the feedback service and the elapsed service time for this customer is x.

$V_n(x,t)$: Probability that at time t, the server is on compulsory vacation with elapsed vacation time x and there are $n(n \geq 0)$customers waiting in the queue for service.

$E_n(x,t)$: Probability that at time $t$the server is on optional extended vacation with elapsed vacation time x and there are $n(n \geq 0)$customers waiting in the queue for service.

$Q(t)$: Probability that at time$t$there are no customers in the system and the server is idle but available in the system.

IV.Assumptions And Mathematical Description Of The Model

a)Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a 'first come'-first served basis'. Let $\lambda a_i, (i = 1,2,3 \dots)$ be the first order probability that a batch of $i$customers arrives at the system during a short interval of time $(t, t + dt)$, where $0 \leq a_i \leq 1$ and $\sum_{i=1}^{\infty} a_i = 1$ and $\lambda > 0$is the mean arrival rate of batches.

b)There is a single server and the service time follows general (arbitrary)distribution with distribution function $B_1(s)$and density function $b_1(s)$.Let $\mu_1(x)dx$be the conditional probability density of service completion during the interval $(x, x + dx)$,given that the elapsed time is x,so that

$$\mu_1(x) = \frac{b_1(x)}{1-B_1(x)} \quad (1)$$

and therefore

$$b_1(s) = \mu_1(s)e^{-\int_0^s \mu_1(x)dx} \quad (2)$$

c)If the customer is dissatisfied with the service, he has the option of getting a re service with probability$r$. This re service time follows general(arbitrary) distribution with distribution function $B_2(s)$and density function $b_2(s)$.Let $\mu_2(x)dx$be the conditional probability density of service completion during the interval $(x, x + dx)$,given that the elapsed time is x, so that

$$\mu_2(x) = \frac{b_2(x)}{1-B_2(x)} \quad (3)$$

and therefore

$$b_2(s) = \mu_2(s)e^{-\int_0^s \mu_2(x)dx} \quad (4)$$

d) As soon as a service is completed, then the server goes for a compulsory vacation.

e)The server's vacation time follows a general(arbitrary) distribution with distribution function $D(v)$and density function $d(v)$.Let $\varphi_1(x)dx$be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$, so that

$$\varphi_1(x) = \frac{d(x)}{1-D(x)} \quad (5)$$

and, therefore

$$d(v) = \varphi_1(v)e^{-\int_0^v \varphi_1(x)dx} \quad (6)$$

f)After the completion of the vacation time, the server has the option of taking a extended vacation which follows a general(arbitrary) distribution with distribution function $E(v)$and density function $e(v)$. Let $\varphi_2(x)dx$be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$, so that

$$\varphi_2(x) = \frac{e(x)}{1-E(x)} \quad (7)$$

and, therefore

$$e(v) = \varphi_2(v)e^{-\int_0^v \varphi_2(x)dx} \quad (8)$$

g) In addition, we assume that customers arriving for service may become impatient by observing the queue size or the server being busy, may balk (refuse to join the queue).Here we assume that $c_1$is the probability that arriving batch joins the system at the time when the server is busy and $c_2$is the probability that an arriving batch joins the system during the period when server is on vacation.

h) In queueing situations, the process of arrival is stochastic and it is significant to know the probability distribution describing the times between successive customer arrivals. Also the reaction of the customer while entering the system should be known. In some cases, a customer may enter the queue upon arrival, but after some time he may lose his patience and decide to depart from the system. Such a kind of situation is defined as Reneging. Reneging is assumed to follow exponential distribution with parameter ψ. Thus, $f(t) = \psi e^{-\psi t}dt$, where $\psi > 0$. Thus ψdt is the probability that a customer can renege during a short interval of time $(t, t + dt)$.

V.STEADY STATE EQUATIONS GOVERNING THE SYSTEM

Cox (1955) has analyzed Non-markovian models by transforming them into Markovian ones, through the introduction of one or more supplementary variables. A stable recursive scheme for the estimation of the limiting probabilities can be developed, based on a versatile regenerative approach.

$$\frac{d}{dx}P_n(x) + (\lambda + \mu_1(x))P_n(x) = \lambda(1-b_1)P_n(x) + \lambda\sum_{i=1}^{n} a_i P_{n-i}(x) \quad (9)$$

$$\frac{d}{dx}R_n(x) + (\lambda + \mu_2(x))R_n(x) = \lambda(1-b_1)R_n(x) + \lambda b_1\sum_{i=1}^{n} a_i R_{n-i}(x) \quad (10)$$

$$\frac{d}{dx}V_n(x) + (\lambda + \varphi_1(x) + \psi)V_n(x) = \lambda(1-b_2)V_n(x) + \lambda b_2\sum_{i=1}^{n} a_i V_{n-i}(x) + \psi V_{n+1}(x) \quad (11)$$

$$\frac{d}{dx}V_0(x) + (\lambda + \varphi_1(x) + \psi)V_0(x) = \lambda(1-b_2)V_0(x) + \psi V_0(x) \quad (12)$$

$$\frac{d}{dx}E_n(x) + (\lambda + \varphi_2(x) + \psi)E_n(x) = \lambda(1-b_2)E_n(x) + \lambda b_2\sum_{i=1}^{n} a_i E_{n-i}(x) + \psi E_{n+1}(x) \quad (13)$$

$$\frac{d}{dx}E_0(x) + (\lambda + \varphi_2(x) + \psi)E_0(x) = \lambda(1-b_2)E_0(x) + \psi E_0(x) \quad (14)$$

$$\lambda Q = (1-p)\int_0^\infty V_n(x)\varphi_1(x) + \lambda(1-b_1)Q + \int_0^\infty E_n(x)\varphi_2(x)dx$$

The initial and boundary conditions of the system is as follows:

$$P_n(0) = (1-p)\int_0^\infty V_n(x)\varphi_1(x)dx + \lambda(b_1 a_n)Q + \int_0^\infty E_n(x)\varphi_2(x)dx \quad (15)$$

$$R_n(0) = r\int_0^\infty P_n(x)\mu_1(x)dx \quad (16)$$

$$V_n(0) = (1-r)\int_0^\infty P_{n+1}(x)\mu_1(x)dx + \int_0^\infty R_{n+1}(x)\,\varphi_2(x)dx \quad (17)$$

$$E_n(0) = (p)\int_0^\infty V_n(x)\varphi_1(x)dx \quad (18)$$

VI.PROBABILITY GENERATING FUNCTION

We define the probability generation function as follows:

$$P_q^{(i)}(x,z) = \sum_{n=0}^{\infty} z^n P_q^{(i)}(x), \qquad P_q^{(i)}(z,t) = \sum_{n=0}^{\infty} z^n P_q^{(i)}, i = 1 \text{ to } N$$

$$V_q^{(i)}(x,z) = \sum_{n=0}^{\infty} z^n V_n^{(i)}(x,t) \quad (19)$$

For $i = 1,2$

$$V_q^{(i)}(z,t) = \sum_{n=0}^{\infty} z^n V_n^{(i)}$$

For $i = 1,2 R_q^{(i)}(x) = \sum_{n=0}^{\infty} z^n R_n^{(i)}(x)\, ;\, R_q^{(i)}(z) = \sum_{n=0}^{\infty} z^n R_n^{(i)}$

Multiplying Eq. 1&Eq.2 by z^n and summing over n from 1to ∞, yields

$$\frac{d}{dx}P(x,z) + \{b_1(\lambda - \lambda A(z)) + \mu_1(x)\}P(x,z) = 0 \quad (20)$$

Applying the same process in Eq.3,Eq. 4 and Eq.5, Eq.6 we get,

$$\frac{d}{dx}R(x,z) + \{b_1(\lambda - \lambda A(z)) + \mu_2(x)\}R(x,z) = 0 \quad (21)$$

$$\frac{d}{dx}V(x,z) + \left\{b_2(\lambda - \lambda A(z)) + \varphi_1(x) + \psi - \frac{\psi}{z}\right\}V(x,z) = 0 \quad (22)$$

Similarly from Eq.7, Eq.8 and Eq.9, Eq.10 we get,

$$\frac{d}{dx}E(x,z) + \left\{b_2(\lambda - \lambda A(z)) + \varphi_2(x) + \psi - \frac{\psi}{z}\right\}E(x,z) = 0 \quad (23)$$

Now integrating Eq.20-Eq.23, we get

$$P(x,z) = P(0,z)e^{-b_1(\lambda - \lambda A(z))x} - \int_0^x \mu_1(t)dt \quad (24)$$

$$R(x,z) = R(0,z)e^{-b_1(\lambda-\lambda A(z))x} - \int_0^x \mu_2(t)dt \tag{25}$$

$$V(x,z) = V(0,z)e^{-\left(b_2(\lambda-\lambda A(z))+\psi-\frac{\psi}{z}\right)x} - \int_0^x \varphi_1(t)dt \tag{26}$$

$$E(x,z) = E(0,z)e^{-\left(b_2(\lambda-\lambda A(z))+\psi-\frac{\psi}{z}\right)x} - \int_0^x \varphi_2(t)dt \tag{27}$$

Next by multiplying the boundary conditions by suitable powers of z^n and taking summation over all possible values of$n$, we get

$$zP(0,z) = (1-p)z\int_0^\infty V(x,z)\varphi_1(x)dx + z\lambda b_1(A(z)-1)Q + z\int_0^\infty E(x,z)\varphi_2(x)dx \tag{28}$$

$$R(0,z) = r\int_0^\infty P(x,z)\mu_1(x)dx \tag{29}$$

$$zV(0,z) = (1-r)\int_0^\infty P(x,z)\mu_1(x)dx + \int_0^\infty R(x,z)\varphi_2(x)dx \tag{30}$$

$$E(0,z) = (p)\int_0^\infty V(x,z)\varphi_1(x)dx \tag{31}$$

Again multiplying Eq. 24-Eq.27 by $\mu_1(x)$,$\mu_2(x)$,$\varphi_1(x)$, $\varphi_2(x)$ respectively and integrating by parts with respect to x between 0 and ∞, we get

$$\int_0^\infty P(x,z)\mu_1(x)dx = P(0,z)B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right) \tag{32}$$

$$\int_0^\infty R(x,z)\mu_2(x)dx = R(0,z)B_2^*\left(b_1\left(\lambda-\lambda A(z)\right)\right) \tag{33}$$

$$\int_0^\infty V(x,z)\varphi_1(x)dx = V(0,z)V^*\left(b_2(\lambda-\lambda A(z))+\psi-\frac{\psi}{z}\right) \tag{34}$$

$$\int_0^\infty E(x,z)\varphi_2(x)dx = E(0,z)E^*\left(b_2(\lambda-\lambda A(z))+\psi-\frac{\psi}{z}\right) \tag{35}$$

Now using Eq.32-Eq.35 in Eq. 28-Eq.31 we get,

$$zP(0,z) = (1-p)zV(0,z)V^*\left(b_2\left(\lambda-\lambda A(z)\right)+\psi-\frac{\psi}{z}\right) + zE(0,z)E^*\left(b_2\left(\lambda-\lambda A(z)\right)+\psi-\frac{\psi}{z}\right) + z\lambda\left(b_1(A(z)-1)\right)Q \tag{36}$$

$$R(0,z) = rR(0,z)B_2^*\left(b_1(\lambda-\lambda A(z))\right) \tag{37}$$

$$zV(0,z) = (1-r)P(0,z)B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right) + rP(0,z)B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)B_2^*\left(b_1\left(\lambda-\lambda A(z)\right)\right) \tag{38}$$

$$zE(0,z) = p\{(1-r)P(0,z)B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right) + rP(0,z)B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)B_2^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)\}V^*\left(b_2(\lambda-\lambda A(z))+\psi-\frac{\psi}{z}\right) \tag{39}$$

Using the Eq. 37-Eq.39 in Eq. 36, we get

$$P(0,z) = \frac{zb_1(\lambda-\lambda A(z))Q}{z-(1-p)(1-r)B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)-rB_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)V^*\left(b_2\left(\lambda-\lambda A(z)\right)+\psi-\frac{\psi}{z}\right) -pB_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)V^*\left(b_2(\lambda-\lambda A(z))+\psi-\frac{\psi}{z}\right)E^*\left(b_2\left(\lambda-\lambda A(z)\right)+\psi-\frac{\psi}{z}\right)} \tag{40}$$

Further integrate Eq.16-Eq.19 with respect to x, we get

$$P(z) = \frac{zQ\{B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)-1\right)\}}{D(z)} \tag{41}$$

$$R(z) = \frac{rzQqB_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)\{B_2^*\left(b_1\left(\lambda-\lambda A(z)\right)-1\right)\}}{D(z)} \tag{42}$$

$$V(z) = \frac{-b_1\left(\lambda-\lambda A(z)\right)Q\left[(1-r)B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)+rB_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)\{B_2^*\left(b_1\left(\lambda-\lambda A(z)\right)-1\right)\}\right]\left[1-V^*\left(b_2(\lambda-\lambda A(z))+\psi-\frac{\psi}{z}\right)\right]}{D(z)\left(b_2\left(\lambda-\lambda A(z)\right)+\psi-\frac{\psi}{z}\right)} \tag{43}$$

$$E(z) = \frac{\left[(1-r)B_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)+rB_1^*\left(b_1\left(\lambda-\lambda A(z)\right)\right)\{B_2^*\left(b_1\left(\lambda-\lambda A(z)\right)-1\right)\}\right]\left[V^*\left(b_2(\lambda-\lambda A(z))+\psi-\frac{\psi}{z}\right)\right]\left[1-E^*\left(b_2\left(\lambda-\lambda A(z)\right)+\psi-\frac{\psi}{z}\right)\right]\left(-pQb_1\left(\lambda-\lambda A(z)\right)\right)}{D(z)\left(b_2\left(\lambda-\lambda A(z)\right)+\psi-\frac{\psi}{z}\right)} \tag{44}$$

Let $S_q(z)$ be the probability generating function irrespective of the service being provided by the server such that we have,

$$S_q(z) = P(z)+R(z)+V(z)+E(z) = \frac{N(z)}{D(z)} \tag{45}$$

VII.STEADY STATE QUEUE SIZE DISTRIBUTION

To find, we use the normalizing condition $P(1)+R(1)+V(1)+E(1)+Q=1$ and proceed as follows:

Since the RHS of the above equation 41-44 is indeterminate of the $0/0$ form at $z=1$, applying L'Hopial's rule we obtain

$$P(1) = \lim_{z\to 1} P(z) = \frac{b_1\lambda QE(I)E(B_1)}{d(z)} \tag{46}$$

This is the steady state probability that at any random epoch, the server is providing the essential service.

$$R(1) = \lim_{z\to 1} R(z) = \frac{rb_1\lambda QE(I)E(B_2)}{d(z)} \tag{47}$$

This is the steady state probability that at any random epoch, the server is busy providing the optional re service.

$$V(1) = \lim_{z\to 1} V(z) = \frac{-b_1\lambda Q E(I)E(V)}{d(z)} \quad (48)$$

This is the steady state probability that at any random epoch, the server is on compulsory vacation.

$$E(1) = \lim_{z\to 1} E(z) = \frac{-pb_1\lambda Q E(I)E(V)E(s)}{d(z)} \quad (49)$$

This is the steady state probability that at any random epoch, the server is on optional extended vacation.

Where

$$D(z) = (1-p)(1-r)B_1^*\left(b_1(\lambda - \lambda A(z))\right) + r\left[V^*\left(b_2(\lambda-\lambda A(z)) + \psi - \frac{\psi}{z}\right)\right]B_1^*\left(b_1(\lambda-\lambda A(z))\right) + p\left[B_1^*\left(b_1(\lambda - \lambda A(z))\right)\right]\left[V^*\left(b_2(\lambda-\lambda A(z)) + \psi - \frac{\psi}{z}\right)\right]\left[E^*\left(b_2(\lambda-\lambda A(z)) + \psi - \frac{\psi}{z}\right)\right] - z$$

$$d(z) = (1-p)(1-r)b_1\lambda E(I)E(B_1) + r[b_2[\lambda E(I) - \psi]E(v) + b_1[\lambda E(I)]E(B_1)] + p\big[[b_2[\lambda E(I)] - \psi]E(V) + b_1[\lambda E(I)]E(B_1) + [b_2[\lambda E(I)] - \psi]E(S)\big]$$

Substituting Eq.46-Eq.49 in the normalization condition, and on simplification, we get the idle time of the server Q as

$$Q = \frac{(1-p)(1-r)b_1\lambda E(I)E(B_1) + r[b_2[\lambda E(I)-\psi]E(v) + b_1[\lambda E(I)]E(B_1)] + p\big[[b_2[\lambda E(I)]-\psi][E(V)+E(s)] + b_1[\lambda E(I)]E(B_1) + [b_2[\lambda E(I)]-\psi]E(S)\big]}{(1-p)(1-r)b_1\lambda E(I)E(B_1) + r[b_2[\lambda E(I)-\psi]E(v) + b_1[\lambda E(I)]E(B_1)] + p\big[[b_2[\lambda E(I)]-\psi][E(V)+E(S)] + b_1[\lambda E(I)]E(B_1)\big] + [E(B_1) + rE(B_2) - E(V) - pE(V)E(s)]\lambda E(I)E(B_1)} \quad (50)$$

VIII.PERFORMANCE MEASURES OF THE MODEL DEFINED

Let L_q denote the mean queue size under the steady state.

Then

$$L_q = \frac{d}{dz}M_q(z)_{z=1} = \frac{D'(1)N''(1) - D''(1)N'(1)}{2\left(D'(1)\right)^2} \quad (51)$$

where primes and two fold primes in the above condition indicate first and second subsidiary at $z = 1$ individually.

Completing the subordinates at $= 1$, we have

$$D'(1) = (1-p)(1-r)b_1\lambda E(I)E(B_1) + r[b_2[\lambda E(I) - \psi]E(v) + b_1[\lambda E(I)]E(B_1)] + p\big[[b_2[\lambda E(I)] - \psi]E(V) + b_1[\lambda E(I)]E(B_1) + [b_2[\lambda E(I)] - \psi]E(S)\big] - 1 \quad (52)$$

$$D''(1) = (1-p)(1-r)b_1^2\{\lambda E(I)\}^2E(B_1^2) + r[b_1^2[[\lambda E(I)]^2E(B_1^2) - \psi] + b_2b_1[\lambda E(I)]E(B_1)E(V)\{\lambda E(I) - \psi\}] + b_1[\lambda E(I)]E(B_1) + [-[b_2[\lambda E(I)] + \psi]E(V) + 2\lambda E(V) + E(V^2)[b_2[\lambda E(I)] + \psi]^2] + p\{b_1[\lambda E(I)]E(B_1)[-[b_2[\lambda E(I)] + \psi]E(V) + E(S)][E(V^2) + E(s)[b_2[\lambda E(I)] + \psi]^2E(B_1)\{\lambda E(I)\}[b_2[\lambda E(I)] + \psi]E(V) + E(s^2) + E(V)][b_2[\lambda E(I)] + \psi]^2 + E(B_1)\{\lambda E(I)\}[b_2[\lambda E(I)] + \psi]E(s) \quad (53)$$

$$N'(1) = [E(B_1) + rE(B_2) - E(V) - pE(V)E(s)]\lambda E(I)E(B_1) \quad (54)$$

$$N''(1) = b_1[[\lambda E(I)]^2E(B_1^2) - \psi] + 2[\lambda E(I)E(B_1)] + r[1 + 2b_1[\lambda E(I)][E(B_1) + E(B_2)] + b_1\{E(B_1^2) + E(B_2^2)\} + 2b_1E(B_1)E(B_2)]b_1[\lambda E(I)][E(V) + E(s)] \quad (55)$$

Substituting $N'(1)$,$D'(1)$,$N''(1)$,$D''(1)$ in (50) L_q can be found.

Other performance measures L,W, W_q can be derived using Little's formula $L = L_q + \rho$, $W_q = \frac{L_q}{\lambda}$, $W = \frac{L}{\lambda}$

IX.Numerical Justification Of The Model

Assume that service time follows exponential distribution in particular and based on this condition, the numerical justification is elaborated below:

Consider $p = 0.5$,$\psi = 4$, $b_1 = 0.5$,$\lambda = 3$,$b_2 = 0.6$,$\mu_1 = 3$,$\mu_2 = 4$,$\varphi_1 = 4$,$\varphi_2 = 5$.

Let $(B_1) = \frac{1}{\mu_1}$,$E(B_2) = \frac{1}{\mu_2}$, $(V) = \frac{1}{\varphi_1}$,$E(s) = \frac{1}{\varphi_2}$

Table– 1: Effect of Re service r on the server idle time and utilization factor

r	Q	ρ
0.4	0.7316	0.2684
0.6	0.6784	0.3216
0.8	0.6344	0.3656

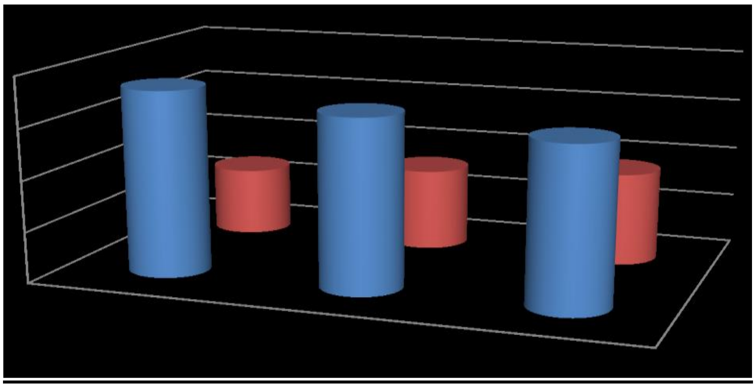

Fig 1:Graphical study

Discussion on Analysis-1: As the probability of getting a feedback service builds, time spent for benefit increments and consequently the sit without moving time of the server gets diminished. This is not surprisingly. In addition, because of dis fulfillment in the administration, the clients settle on a re benefit. In the event that this draws out, the quantity of clients going into the system for service will get diminished and thus the framework bombs totally.

Table – 2: Effect of Re service r on various performance measures

r	L_q	L	W_q	W
0.4	2.7236	2.9920	0.9078	0.9973
0.6	2.9172	3.2388	0.9724	1.0796
0.8	3.0616	3.4272	1.0205	1.1424

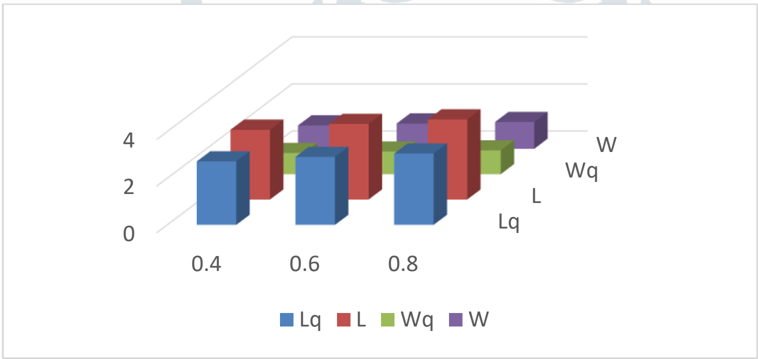

Fig 2.Graphical study

Discussion on Analysis-2: Because of an expansion in probability of getting re sevice, an exceptional change happens in the different execution measures of the model. As the quantity of clients getting a criticism benefit expands, it prompts an increase in the quantity of clients in the queue and in the system. At the same time these outcomes to an expansion in the holding up time of the clients in the line and in addition in the system. Hence the entire framework turns out to be moderate and the clients will experience recoiling if this procedure proceeds.

Table - 3 : Effect of Reneging ψon various performance measures of the system

ψ	Q	ρ	L_q	L	W_q	W
4	0.7316	0.2684	2.7236	2.9920	0.9078	0.9973
5	0.7655	0.2345	1.6305	1.8650	0.5435	0.6217
6	0.7912	0.2088	1.4027	1.6115	0.4676	0.5372

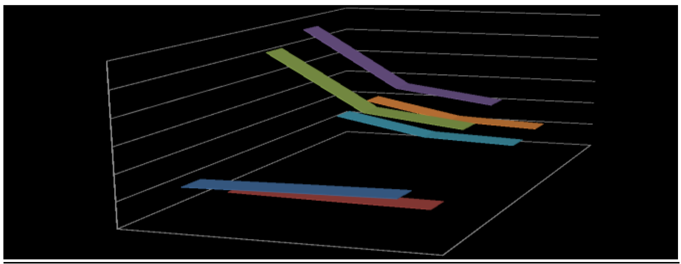

Fig 3. Graphical Study

Discussion on Analysis- 3: If the customer gets dissatisfied with the quality of the service or if the service gets delayed due to any of the reasons or due to increase in waiting time, all these conditions lead to the customers reneging in the system. Hence the number of customers becomes less and it results in boost in idle time of the server and on par with the utilization factor gets decreased. Also the other performance gets decreased. If this situation continues, the whole system gets collapsed and brings a great loss to the system.

XI.METHODS TO MITIGATE THE PROBLEM OF BALKING, RENEGING AND FEEDBACK DELAY

Based on the numerical justification, we interpret that feedback service, reneging and balking in a queue reduces the performance and efficiency of a grid system as the critical jobs get lost from the system. The following techniques are employed in a grid computing system to mitigate the problem of reneging and balking in a system

1.Network Oriented Approach- A method to diminish the problem of balking and reneging

Blacklisting of unauthorised IP address : Due to heavy data traffic in the network, some authorised critical jobs balk or renege from the queue .A hacker can purposely create a heavy data traffic by uploading erroneous data in the grid network. To prevent this situation, the control node (or job scheduler) keeps track of the IP address of all client systems. In the previous history, if a system takes a long time to answer a question asked in a Turing test, the system is labelled as aunauthorised system and the IP address of the system is black listed. In this way the request of the unauthorised systems get automatically rejected. Thus the authorised critical systems gets the service from the grid on time, avoiding reneging and balking.

2.HardwareOriented Approach- A method to moderate the problem of balking and reneging

Pipelining of hardware resources : Pipelining is a technique in which the hardware is divided into modules and the instruction execution is overlapped. A instruction is divided into fetch, decode and execute cycles. When the first instruction is being decoded, the second instruction gets fetched. So if a particular module gets filled, another module can keep on doing its functionality to speed up the process and reduce the delay time. The last job in the queue enters in the fetch stage while the others jobs are being decoded or executed. As the last job is being served, it reduces the possibility of balking and reneging.

3.SoftwareOriented Approach- A method to purge the feedback delay problem

Insertion of check points : The waiting time of a task in a queue increases if there is a fault incidence in the previous task. The task is fed back into the queue to be served again. The grid system has to re-run the entire task after the fault is detected, which in turn increases the waiting time of jobs in the queue. To mitigate this problem, the check-points are inserted during the task-scheduling process to identify fault incidence. When a fault is identified, the task is restarted only at the previous check point which takes less computation and less time. This eliminates the need to restart the full task, thus reducing the delay time.

Replication: Replication is another effective technique in fault tolerance. In this method, there are various replicates of any job on different resources. When any fault occurs in the system, then the recovery process is carried out on the replica in a separate system. The execution of the main server
continues with the next job in the queue

XII. CONCLUSION

The view of optional re service has been presented in this model since it set up the genuine circumstances nearly. Additionally for the maintaining to be completed, all things considered, amid the vacation time, an alternate get-away arrangement is presented in this model. As the reneging expanding; the length of the line gets lessened. Subsequently, the system moves toward becoming worse. Moreover, the expansion in separate makes the sit still time of the server increasingly and henceforth the length of the line and holding up time of the clients in the line and additionally in the system gets increased. All the execution measures of the model are all around actualized. Unique cases on intrigue are clarified. The model is well sensible by methods for computational review.

REFERENCES

[1]Choudhury, G., (2002), "A batch arrival queue with a vacation time under single vacation policy", *Computers and Operations Research,* Vol.29, No.14, pp.1941-1955.

[2]Cox.D.R(1955), "The analysis of Non-Markovian Stochastic Processes by the inclusion of Supplementary Variables", *Proceedings of the Cambridge Philosophical Society*, Vol.51, pp.433-441.

[3]Kella.O, Zwart.B&Boxma.O (2005), "Some time-dependent properties of symmetric M/G/1 queues", *Journal of Applied Probability*, Vol.42, No.1, pp.223-234.

[4]Kumar.R and Sharma.S.K(2012), "A Markovian Feedback Queue with Retention of Reneged Customers and Balking", *AMO-Advanced Modeling and Optimization*, Vol.14, No.3, pp.681-688.

[5]Karthikeyan.K and Maragathasundari.S (2015), Batch arrival of two stages with standby server during general vacation time & general repair time", *International Journal of Mathematical archive*, Vol.6, No.4,
pp.43-48.

[6]Kavitha.K and Maragathasundari.S (2012),"Analysis of Non Markovian queue with restricted admissibility and optional types of repair", *International journal of scientific research and managementstudies*, Vol.2, No.5, pp.244-252.

[7]Kendall. D.G (1953), "Stochastic Processes occurring in the theory of queues and their analysis by the method of embedded Markov chain", *Annals of Mathematical Statistics*, Vol.24.

[8]Madan.K.C and Abu-Dayyeh. A.Z (2004), "On a single server queue with optional phase type server vacations based on exhaustive deterministic service and a single vacation policy", *Applied Mathematics andComputation*, Vol.149, No.3, pp. 723-734.

[9]Madan. K.C (2000), "An M/G/1 queue with second optional service", *Queueing Systems* Vol.34, pp.37- 46.

[10]Madan.K.C. and Chodhury.G (2004), "An M[x]/G/1 queue with Bernoulli vacation schedule under restricted admissibility policy",*Sankhaya*,Vol.66, pp.172-193.

[11]Madan.K.C (2000), "On a single server queue with two stage heterogeneous service and binomial schedule server vacations", *The Egyptian Statistical Journal*, Vol.44, pp.39-55.

[12]Madan.K.C and Anabosi.R.F (2003), "A single server queue with two types of service, Bernoulli schedule server vacations and a single vacation policy", *Pakistan Journal of Statistics*, Vol.19, pp.331-442.

[13]Maragathasundari.S (2015), "A bulk arrival queuing model of three stages of service with different vacation policies, service interruption and delay time", *American International Journal of Research inScience, Technology, Engineering& Mathematics*, Vol.11, No.1, pp.52-56.

[14]Maragathasundari.S and Srinivasan.S (2012), "Analysis of M/G/1 feedback queue with three stage and multiple server vacation", *Applied mathematical sciences*, Vol.6, No.125, pp.6221-6240.

[15]Maragathasundari.S and Srinivasan.S (2015), "A Non-Markovian Multistage Batch arrival queue with breakdown and reneging", *Mathematical problems in engineering*, Volume 2014/16 pages/ Article ID 519579/ http: // dx. Doi. Org / 10.1155/2014/ 519579.

[16]Maraghi. F. A, Madan.K.C and Darby-Dowman.K (2010), "Batch Arrival vacation queue with second optional service and Random Breakdowns", *International Journal of Statistical Theory and Practice*, Vol.4, No.1, pp.137-153.

[17]Maraghi.F.A, Madan.K.C and Darby-Dowman. K (2009), "Batch Arrival queuing system with Random Breakdowns and Bernoulli Schedule server vacations having General vacation Time Distribution", *International Journal of Information and Management Sciences*, Vol.20, pp.55-70.

[18] Ranjitham.A. andMaragathasundari.S (2014), "Batch arrival queueing system with two stages of service", *International journal of Math.Analysis*, Vol.8 (6), pp.247-258

[19]Srinivasan.S and Maragathasundari.S (2017), "Optional services in a Non Markovian Queue", *International Journal of Operational Research*, Vol.28, No.1, pp.1-17.

[20]Sowmiah.S and Maragathasundari.S (2015), "M/G/1 queueing system with extended vacation, service interruption, Delay time and stages in repair", *Journal of Computer and Mathematical Sciences*,
Vol.6 (7), 363-370, July 2015