CHROMATIC NUMBER TO THE TRANSFORMATION GRAPH G+-+

[1]Stephen John B and [2] Ashlin Suji M. D
[1]Associate Professor & [2]M.Phil Scholar (Reg. No.: 20183013545202), P.G & Research Department of Mathematics, Annai Velankanni College, Tholayavattam, Kanyakumari District- 629157.
Affiliated By Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.

Abstract

Chromatic number and Graph transformation are the recently developing area in Graph theory. It has many real life applications such as communication network and image recognition. In this paper we have studied the transformation of some graph such as Fan graph, Cycle graph, Path graph, Centipede graph, star graph, Pan graph. Also, we have established the Chromatic number to these transformations.

Keywords:

Fan graph, Cycle graph, Path graph, Centipede graph, star graph, Pan graph, Transformation, Vertex Coloring, Chromatic number.

Introduction: 1.0

In this paper, we are concerned with finite, undirected, loop less graph without multiple edges. Let G = (V(G),E(G)) be a graph. For two vertices u and v of G, if there is an edge e joining them, we say u and v are adjacent. Here, both u and v are end vertices of e and u and e or v and e are said to be incident.

A vertex coloring of G is an assignment of colors to its vertices so that no two adjacent vertices receive the same color G. The chromatic number χ (G) is the minimum number of colors needed to color G. In year 2001, Wu and Meng introduced some new graphical transformations which generalized the concept of total graph.

Here, We shall investigate the transformation graph G^{-+-} of some graphs.

Definition: 1.1

A **graph** G is an ordered pair (V(G),E(G)) consisting of a non- empty set V(G) of vertices and a set E(G) , disjoint from V(G) of edges together with an incidence function Ψ_G that associates with each edge of G is an ordered pair of vertices of G.

Definition: 1.2

walk is an alternating sequence of vertices and edges starting and ending with vertices.

A walk in which all the vertices are distinct is called a **Path**. A path containing n- vertices is denoted by P_n.

A closed path is called **Cycle.** A cycle containing n – vertices is denoted by C_n

Definition: 1.3

A **Star graph** is a graph in which n-1 vertices have degree 1 and a single vertex have degree n-1. The n-1 vertex are connected to a single central vertex. A star graph with total n-vertex is

termed as S_n.

Definition: 1.4

A **Fan graph** F_n is a planar undirected graph with 2n+1 vertices and 3n edges . The Fan graph F_n can be constructed by joining n copies of the cycle C_3 with a common vertex .

Definition: 1.5

The **n- Centipede** is a tree with 2n vertices and 2n-1 edges obtained by joining the bottoms of n copies of the path graph P_2 laid in a row with edges .

Definition:1.6

Let $G = \big(V(G), E(G)\big)$ be a graph and x,y,z be three variables taking values $+$ or $-$. The **transformation graph** G^{xyz} is the graph having $V(G) \cup E(G)$ as the vertex set and for $\propto, \beta \in V(G) \cup E(G)$ $\propto$ *and* β are adjacent in G^{xyz} if and only if one of the following holds:

(i) $\propto, \beta \in V(G). \propto$ *and* β are adjacent in G if $x = +$; $\propto$ *and* β are not adjacent in G if $x = -$.

(ii) $\propto, \beta \in E(G). \propto$ *and* β are adjacent in G if $y = +$; $\propto$ *and* β are not adjacent in G if $y = -$.

(iii) $\propto \in V(G)$, $\beta \in E(G). \propto$ *and* β are incidentt in G if $z = +$; $\propto$ *and* β are not incident in G if $z = -$.

Theorem: 2.1

Let G = P_n (n ≥ 4) be any path graph with n-vertices, then χ (G^{+-+}) = $\left\lfloor\frac{n}{2}\right\rfloor + 2$

Proof:

Let G = P_n be a path graph with 'n' vertices

To prove χ (G^{+-+}) = $\left\lfloor\frac{n}{2}\right\rfloor + 2$

The vertex set of G is V(G) = $\{v_i \,/ 1 \le i \le n\}$ and the edge set of G is E(G) = $\{e_i \,/ 1 \le i \le n-1\}$

The adjacency of G is,

Each v_i is adjacent to $v_{i-1,}$ *and* v_{i+1} for all i ,

That is, $N(v_i) = \{v_{i-1,} v_{i+1} \,/ 2 \le i \le n-1\}$

$N(v_1) = v_2$ and $N(v_n) = v_{n-1}$

Each $e_i \in E(P_n)$ is adjacent to e_{i-1} and e_{i+1} for all i,

That is, $N(e_i) = \{e_{i-1}, e_{i+1}\}$

$N(e_1) = e_2$ and $N(e_n) = e_{n-1}$

Each $e_i \in E(G)$ is incident with v_{i-1} and v_{i+1} where $1 \le i \le n-1$

The vertex set of G^{+--+} is $V(G^{+-+}) = V(P_n) \cup E(P_n)$.

By the definition of the transformation (P_n^{+-+})

The adjacency in $V(G^{-+-})$ is as follows;

Those pair of vertices (v_i, v_j) are adjacent in G, are neighboring vertices in G^{+-+} .

Those pair of edges (e_i, e_j) which are connected in G , are not neighboring vertices in G^{+-+} . In similar, the pair (v_i, e_i) are incident in G , are neighboring vertices in G^{+-+}

Now, the vertices of $V(G^{+-+})$ can be classified as follows:

$$C_1 = \left\{v_{2i-1},\ 1 \le i \le \left\lceil\frac{n}{2}\right\rceil\right\} \qquad \text{........ (1)}$$

$$C_2 = \left\{v_{2i},\ 1 \le i \le \left\lfloor\frac{n}{2}\right\rfloor\right\} \qquad \text{........ (2)}$$

$$C_i = \{e_{2i-5}, e_{2i-4}\ /\ 3 \le i \le \left\lfloor\frac{n}{2}\right\rfloor + 1\ \} \qquad \text{........ (3)}$$

$$C_k = \begin{Bmatrix} e_{n-2},\ e_{n-1} & if\ n\ is\ odd \\ e_{n-1} & if\ n\ is\ even \end{Bmatrix} \text{ where } k = \left\lfloor\frac{n}{2}\right\rfloor + 2 \qquad \text{... .... (4)}$$

The elements of equation (1) , are independent in G^{+-+} . Hence, a particular color C_1 can be given to all the vertices of equation (1). The elements of equation (2) , are independent in G^{+-+} . Hence, a different color C_2 can be given to all the vertices of equation (2). The elements of equation (3) , are independent in G^{+-+} . Therefore the new colors C_i , to apply all the vertices in equation (3) . In similar, The elements of equation (4) , is independent in G^{+-+} Therefore, we use a another color C_k , to give the vertices of equation (4) .

Therefore, the total numbers of color class is the minimum coloring number of the graph G^{+-+}

That is, $\chi\ (G^{+-+}) = 1+1+\left\lfloor\frac{n}{2}\right\rfloor + 1 - 2\ + 1 = \left\lfloor\frac{n}{2}\right\rfloor + 2$

Hence, the proof.

Theorem: 2.2

Let $G = C_n$ $(n = 1, 2, 3,..,)$ be the cycle graph with n-vertices, If $(n \ge 4)$ then

$\chi\ (G^{+-+}) = \left\lfloor\frac{n}{2}\right\rfloor + 2$

Proof:

Let C_n be the cycle graph with 'n' vertices of G

To prove χ $(G^{+-+}) = \left\lfloor\frac{n}{2}\right\rfloor + 2$

The vertex set of G is V(G) = $\{v_i \,/ 1 \leq i \leq n\}$ and the edge set of G is E(G) = $\{e_i \,/ 1 \leq i \leq n\}$

The adjacency of G is,

Each vertex v_i is adjacent to $v_{i-1,}$ and v_{i+1} for all i ,

That is, $N(v_i) = \{v_{i-1,}v_{i+1} \,/\, 2 \leq i \leq n-1\}$

Each e_i is adjecent to $e_{i-1,}$ and e_{i+1} for all i ,

That is, N(e_i) = { e_{i-1}, e_{i+1}}

Each e_i is incident with v_i and v_{i+1} where $1 \leq$ i$\leq$ n-1

N(e_n) = $\{v_1, v_n\}$

The vertex set of G^{+-+} is V(G^{+-+} = V(C_n) ∪E(C_n).

By the definition of the transformation (G^{-+-})

The adjacency in V(G^{+-+} is as follows:

Those pair of vertices (v_i, v_j) are not adjacent in C_n , are neighboring vertices in G^{+-+}.

Those pair of edges $(e_i\,, e_j)$ which are adjacent in C_n , are neighboring vertices in G^{+-+}. In similar, the pair (e_i, v_i) which are not incident in C_n , are neighboring vertices in G^{+-+}.

Now, the vertices of V(G) can be classified as follows:

Case: (i)

If n is odd

$C_1 = \left\{ v_{2i-1} \,/\, 1 \leq \text{ i} \leq \left\lfloor\frac{n}{2}\right\rfloor\right\}$ ………….(1)

$C_2 = \left\{ v_{2j}\,, e_n \;/ 1 \leq \text{ j} \leq \left\lfloor\frac{n}{2}\right\rfloor\right\}$ ………….(2)

$C_3 = \{ v_n\,, e_1\,, e_2 \}$ … … … ….(3)

$C_k = \{e_{2k-5}, e_{2k-4} \;/\, 4 \leq k \leq \left\lfloor\frac{n}{2}\right\rfloor + 2 \}$ ………….(4)

The elements of equation(1),are independent in G^{+-+}.

Hence, a particular color C_1 can be given to the vertices of equation (1).The elements of equation(2), are independent in G^{+-+}. Therefore the new color C_2 , to apply the vertices in equation (2) .The elements of equation (3), are independent in G^{+-+}. Hence, the different color C_3 , to give the vertices in equation (3) . In similar, The elements of equation (4) , are independent in G^{+-+}. Therefore, we use the colors C_k , to apply all the vertices of equation

Case: (ii)

If n is even

$C_1 = \left\{ v_{2i-1} , 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \right\}$ ………….(1)

$C_2 = \left\{ v_{2j} , 1 \leq j \leq \left\lfloor \frac{n}{2} \right\rfloor \right\}$ ………….(2)

$C_3 = \{ e_1 , e_2 \}$ … … … ….(3)

$C_k = \left\{ e_{2k-5}, e_{2k-6} \ / \ 4 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor + 2 \right\}$ ………….(4)

The elements of equation(1),are independent in G^{+-+}.

Hence, a particular color C_1 can be given to the vertices of equation (1).The elements of equation(2),are independent in G^{+-+}. Therefore the new color C_2 , to apply the vertices in equation (2) .The elements of equation (3), are independent in G^{+-+}. Hence, the different color C_3 , to give the vertices in equation (3) . In similar, The elements of equation (4) , are independent in G^{+-+}. Therefore, we use the colors C_k , to apply all the vertices of equation (4) .

Therefore, the total numbers of color class is the minimum coloring number of the graph G^{+-+}.

That is , $\chi(G^{+-+}) = 1 + 1 + 1 + \left\lfloor \frac{n}{2} \right\rfloor + 2 - 3 = \left\lfloor \frac{n}{2} \right\rfloor + 2$

Hence, the proof.

Theorem: 2.3

Let $G = F_{2n+1}$ ($n = 1, 2, \ldots$) be the fan graph , then $\chi(G^{+-+}) = 3$

Proof:

Let $G = F_{2n+1}$ be a fan graph

To prove $\chi(G^{+-+}) = 3$

Choose a vertex v_0 be the center vertex and its degree is 2n

The vertex set of G is V(G) = $\{v_i \ / \ 0 \le i \le 2n\}$ and the edge set of G is

E(G) = $\{e_i \ / 1 \le i \le 2n-1\}$.

The adjacency of G is,

Each v_i is adjacent to v_{i-1}, and v_{i+1} for all i ,

$$N(v_i) = \begin{cases} v_0 , v_{i+1} , & if \ i \ is \ odd \\ v_0 , v_{i-1} , & if \ i \ is \ even \end{cases}$$

Each $e_i \in E(G)$ is adjacent to each other edges except e_{3i-1},

and e_{3i-1} is adjacent to e_{3i-2} and e_{3i} .

Each $e_i \in E(G)$ is incident with ,

N(e_{3i-1}) =$\{v_{2i-1}, v_{2i}\}$

N(e_{3i-2}) =$\{v_0, v_{2i-1}\}$

N(e_{3i}) =$\{v_0, v_{2i}\}$

The vertex set of G^{+-+} is V(G^{+-+}) = $V(G) \cup E(G)$

By the definition of the transformation (G^{+-+})

The adjacency in V(G^{+-+}) is as follows:

Those pair of vertices (v_i , v_j) are adjacent in G, are neighboring vertices in G^{+-+} .Those pair of edges (e_i, e_j) which are connected in G, are not neighboring vertices in G^{+-+} . In similar, the pair (v_i, e_i) are incident in G , are neighboring vertices in G^{+-+}.

Now, the vertices of V(G^{+-+}) can be classified as follows:

$C_1 = \{v_0 , e_{3i-1} \ / \ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\}$ ……..(1)

$C_2 = \{v_{2i-1} , e_{3i} \ / \ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\}$ ……..(2)

$C_3 = \{v_{2i} , e_{3i-2} \ / \ 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor\}$ ……..(3)

The elements of equation (1), are independent in G^{+-+} Hence, a particular color C_1, can be given to all the vertices of equation (1) The elements of equation (2), are independent in G^{+-+} Hence, a different color C_2 , to apply all the vertices of equation (2) . The elements of equation (3), are independent in G^{+-+} Hence, a new color C_3 , can be given to all the vertices of equation (3) .

Therefore, the total numbers of color class is the minimum coloring number of the graph $G^{+ - +}$

That is, $\chi(G^{+ - +}) = 1 + 1 + 1 = 3$

Hence, the proof.

Theorem: 2.4

Let $G = S_n$ ($n = 1, 2, \ldots$) be the Star graph with n-vertices, If $n \geq 3$ then $\chi(G^{+ - +}) = 3$

Proof:

Let S_n be the Star graph with 'n' vertices in G

To prove $\chi(G^{+ - +}) = 3$

Choose a vertex v_0 be the centre vertex and its degree is n-1 .

The vertex set of G is $V(G) = \{v_i \,/\, 0 \leq i \leq n\}$ and the edge set of G is $E(G) = \{e_i \,/ 1 \leq i \leq n\}$

The adjacency of G is as follows:

The vertex v_i is adjacent to , $N(v_i) = \{ v_i\}$ for all n = 1, 2, …

The edge e_i is adjacent to each other edges in E(G)

The edge e_i is incident with v_0 *and* v_i.

The vertex set of $G^{+ - +}$ is $V(G^{+ - +}) = V(G) \cup E(G)$.

By the definition of the transformation $(G^{+ - +})$

The adjacency in $V(G^{+ - +})$ is as follows:

Those pair of vertices (v_i, v_j) are adjacent in G, are neighboring vertices in $G^{+ - +}$. Those pair of edges (e_i, e_j) which are adjacent in G, are not neighboring vertices in $G^{+ - +}$. In similar, the pair (v_i, e_i) which are incident in G, are neighboring vertices in $G^{+ - +}$.

Now, the vertices of V(G) can be classified as follows:

$C_1 = \{v_0\}$ ………(1)

$C_2 = \{v_i \,/\, 1 \leq i \leq n-1\}$ ………(2)

$C_3 = \{e_i \,/\, 1 \leq i \leq n-1\}$ ………..(3)

The elements of equation (1), are independent in $G^{+ - +}$. Hence a particular color C_1, can be given to the vertex of equation (1). The elements of equation (2), are independent in G^{+--}.

Hence a different color C_2, to apply all the vertices of equation (2) . The elements of equation (3), are independent in G^{+--}. Hence a new color C_3, can be given to the vertices of equation (3).

Therefore, the total numbers of the color class is the minimum coloring number of the transformation graph G^{+-+}

That is , $\chi\,(G^{+-+}) = 1+1+1 = 3$

Hence, the proof.

Theorem: 2.5

Let $G = \zeta_{2n}(n = 1, 2, 3, \ldots)$ be the centipede graph, $If\ (n \geq 2)$ then

$$\chi(G^{+-+}) = \left\lceil\frac{2n+1}{2}\right\rceil + 1$$

Proof:

Let ζ_{2n} be the centipede graph of G ,

To prove $\chi(G^{+-+}) = \left\lceil\frac{2n+1}{2}\right\rceil + 1$

The vertex set of ζ_{2n} $is\ V(G) = \{v_i/1 \leq i \leq 2n\}$ and the edge set of ζ_{2n} is

$E(G) = \{e_i/1 \leq i \leq 2n-1\}$

The adjacency of V(G) is, $v_{i-1,}\ and\ v_{i+1}$ for all i ,

$$N(v_i) = \begin{Bmatrix} v_{i-1}, & v_{i+1}, & v_{i+2} & if\ 'i' is\ odd \\ v_{i-1} & & & if\ \ 'i' is\ even \end{Bmatrix}$$

Each $e_i \in E\ (G)\ \ is,\ e_{i-1,}\ and\ e_{i+1}$ for all i ,

$$N(e_i) = \begin{Bmatrix} e_{i+1} & if\ 'i'\ is\ odd \\ e_{i-2}, e_{i-1},\ e_{i+1}, e_{i+2} & if\ 'i'\ is\ even \end{Bmatrix}$$

Each e_i is incident with,

$$N(e_i) = \begin{Bmatrix} v_i, & v_{i+1} & if\ 'i' is\ odd \\ v_{i-1}\ , & v_{i+1} & if\ \ 'i' is\ even \end{Bmatrix}$$

The vertex set of G^{+-+} is V$(G^{+-+}) = V(\zeta_{2n}) \cup E(\zeta_{2n})$.

By the definition of the transformation (G^{+-+})

The adjacency in $V(G^{+-+})$ as follows:

Those pair of vertices (v_i, v_j) are adjacent in G , are neighboring vertices in G^{+-+}.

Those pair of edges (e_i, e_j) which are adjacent in G, are not neighboring vertices in G^{+-+}.

In similar, the pair (v_i, e_i) which are incident in G, are neighboring vertices in G^{+-+} .

Now, the vertices of V(G^{+-+}) can be classified as follows

$$C_1 = \begin{Bmatrix} v_{4i-3}, v_{4j} & /1 \le i \le \left\lceil \frac{2n}{4} \right\rceil \\ & /1 \le j \le \left\lfloor \frac{2n}{4} \right\rfloor \end{Bmatrix} \quad \ldots\ldots\ldots\ldots(1)$$

$$C_2 = \begin{Bmatrix} v_{4i-2}, v_{4j-1} & /1 \le i \le \left\lceil \frac{2n}{4} \right\rceil \\ & /1 \le j \le \left\lfloor \frac{2n}{4} \right\rfloor \end{Bmatrix} \quad \ldots\ldots\ldots\ldots(2)$$

$$C_i = \{e_{2i-5}, e_{2i-4} \ / 3 \le i \le \left\lceil \frac{2n+1}{2} \right\rceil\} \quad \ldots\ldots\ldots\ldots.(3)$$

$$C_k = \{e_{2n-1}\} \qquad \text{where } k = n + 2 \quad \ldots\ldots\ldots\ldots(4)$$

The elements of equation (1) , are independent in G^{+-+} .Hence a particular color C_1, can apply all the vertices of equation (1). The elements of equation (2) , are independent in G^{+-+}, Hence a different color C_2(except C_1) to give the vertices of equation (2). The elements of equation (3), are independent in G^{+-+}, Therefore, the new colors C_i , to apply all the vertices in equation (3). The elements of equation (4) , are independent in G^{+-+}, Hence a specific color C_k, to apply the vertex of equation (4) .

Therefore, the total numbers of color class is the minimum coloring number of the graph G^{+-+}.

That is, $\chi(G^{+-+}) = 1 + 1 + \left\lceil \frac{2n+1}{2} \right\rceil + 1 - 2 = \left\lceil \frac{2n+1}{2} \right\rceil + 1$

Hence, the proof.

Theorem: 2.6

Let G = Ꝑ$_n$ (n ≥ 5) be the Pan graph with n-vertices, then χ (G^{+-+}) = $\left\lceil \frac{n}{2} \right\rceil + 1$.

Proof:

Let Ꝑ$_n$ be the Pan graph with ‘n’ vertices in G

To prove χ ($G^{+ - +}$) $=\left\lceil\frac{n}{2}\right\rceil + 1$

The vertex set of G is V(G) = $\{v_i \ /1\leq i \leq n\}$ and the edge set of G is E(G) $=\{e_i \ /1\leq i \leq n\}$

The adjacency of G is as follows:

Adjacency between any two pair of vertices(v_i, v_j) ∈V(G),

$$N(v_i) = \{v_{i-1,} v_{i+1}\}$$

$N(v_2)$ = $\{v_{1,} v_{3,} v_n\}$ and $N(v_n) = \{v_{2,} v_{n-1}\}$

Adjacency between any pair of edges (e_i, e_j) ∈ E(G) is

N(e_i) = { e_{i-1}, e_{i+1}}

N(e_1) = $\{e_2, e_n\}$ and N(e_2) = $\{e_1, e_{3,} e_n\}$

N(e_n) = $\{e_{1,} e_2, e_{n-1}\}$

Adjacency between any two pairs (e_{i}, v_j) ∈V(G) is,

N(e_n) = $\{v_2, v_n\}$

The vertex set of $G^{- + -}$ is V($G^{+ - +}$) = V(G) ∪ E(G).

The transformation $p_n^{+ - +}$ of p_n consists of 2n vertices.

By the definition of the transformation ($G^{+ - +}$)

The adjacency in V($G^{+ - +}$) is as follows:

Those pair of vertices (v_i, v_j) are not adjacent in G, are neighboring vertices in $G^{+ - +}$. Those pair of edges (e_i , e_j) which are adjacent in G, are neighboring vertices in $G^{+ - +}$. In similar, the pair (v_i , e_i) which are not incident in G, are neighboring vertices in $G^{+ - +}$.

Now, the vertices of V(G) can be classified as follows:

Case: (i)

If n is odd

$C_1 = \{v_{2\,i-1}, 1 \leq j \leq n\}$ …………………(1)

$C_2 = \{v_{2\,j}, 1 \le j \le n-1\}$ ………………….(2)

$C_3 = \{e_1, e_2, e_n\}$ ………………….(3)

$C_k = \{e_{2k-5}, e_{2k-4} \;/\; 4 \le k \le \lceil \frac{n}{2} \rceil + 1\}$ ………………….(4)

From case (i), The elements of equation (1) , are independent in G^{+-+} . Hence, a particular color C_1 can be given to the vertices of equation (1). The elements of equation (2) , are independent in G^{+-+}. Hence, a different color C_2 can be given to the vertices of equation (2).

The elements of equation (3), are independent in G^{+-+} . Therefore the single color C_3 , to apply the vertices in equation (3) . In similar, The elements of equation (4), is independent in G^{+-+} . Therefore, we use a different colors C_k (except C_1, C_2, C_3), to apply all the vertices of equation (4) .

Case: (ii)

If n is even

$C_1 = \{v_{2\,i-1}, e_n / 1 \le i \le n-1\}$ ………………….(5)

$C_2 = \{v_{2\,j}, e_{n-1} / 1 \le j \le n-2\}$ ………………….(6)

$C_3 = \{e_1, e_2, v_n\}$ ………………….(7)

$C_k = \{e_{2k-5}, e_{2k-4} \;/\; 4 \le k \le \lceil \frac{n}{2} \rceil + 1\}$ ………………….(8)

From case (ii), The elements of equation (5) , are independent in G^{+-+} . Hence, a particular color C_1 can be given to the vertices of equation (5). The elements of equation (6) , are independent in G^{+-+} . Hence, a different color C_2 can be given to the vertices of equation (6).

The elements of equation (7), are independent in G^{+-+}. Therefore the single color C_3 , to apply the vertices in equation (7) . In similar, The elements of equation (8), is independent in G^{+-+} . Therefore, we use a different colors Ck (except C_1, C_2, C_3), to apply all the vertices of equation (8) .

Therefore, the total numbers of color class is the minimum coloring number of the graph G^{+-+}

$$\chi(G^{+-+}) = 1+1+1+\lceil \tfrac{n}{2} \rceil + 1 - 3 = \lceil \tfrac{n}{2} \rceil + 1$$

Hence the proof.

References:

1) J. A. Bondy and U. S. R. Murthy "Graph Theory with Applications", American Elsevier, New York, Macilan London,1976.
2) Dr. D. Vijayalakshmi, N. Nithya Devi "A study on Achromatic number of the transformation graph G++-", Mathematical sciences International Research Journal : vol 5 Issue 2 (2016)
3) M.K. Angel Jebitha & J. Paulraj Joseph "Domination in transformation graph G-+-", pages 279-303
4) Preeti Gupta " A study on Vertex – Edge Coloring Techniques with Applications" , IJCEM vol 1, Issue 2, May 2014
5) Vijay. V. Teli and Present V.Patil , "On the Transformation Graph G^{+--}", Haliyal, Belagavi, 2016.
6) Wu. B, ZhangL, zhang Z, "The transformation graph G^{xyz} when xyz = -+-" Discrete Mathematics ,2005.