Fuzzy split independent dominating sets in Fuzzy graphs

V.T.Chandrasekaran* and R.Sangeetha+
Department of Mathematics, Jawahar Science College
Neyveli-607803, India

Abstract

In this paper, we define fuzzy vertex cut and fuzzy split independent sets. Some of the relations between the fuzzy dominating sets and the fuzzy split independent sets were discussed. Bounds of fuzzy split domination were derived.

Keywords: Fuzzy vertex cut, Fuzzy split independent dominating sets and Fuzzy split independence domination number.

1. **Introduction**

The study of dominating sets in graphs was introduced by Berge. The domination number and independent domination are introduced by Cockayene and Hedetniemi. A.Nagoorgani and V.T.Chandrasekaran discussed about the domination in fuzzy graphs. A. Somasundharam and S. Somasundharam presented the concepts of independent domination in connected fuzzy graphs. P.Gladyis, C.V.Harinarayanan and R. Muthuraj have presented split domination in fuzzy graphs.

In this paper, we have introduced the concept of fuzzy split independent domination in fuzzy graphs.

2. Preliminaries

Let V be a finite non empty set. A fuzzy graph $G(\sigma,\mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ where $\sigma(u) \geq 0$ for all $u \in V$ and $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all $(u,v) \in V \times V$. The fuzzy graph $H(\tau,\eta)$ is called a fuzzy sub graph of $G(\sigma,\mu)$ if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\eta(u,v) \leq \mu(u,v)$ for all $(u,v) \in V \times V$ .A path ρ in a fuzzy graph is a sequence of distinct vertices $u_0, u_1, u_2 ... u_n$ such that $\mu(u_{i-1}, u_i) > 0, 1 \leq i \leq n$; here $n \geq 0$ is called the length of the path ρ .The consecutive pairs (u_{i-1}, u_i) are called the arcs of the path. The strength of a path is defined as $\wedge_{i=1}^{n} \mu(u_{i-1}, u_i)$. In other words, the strength of a path is defined to be the weight of the weakest arc of the path. Two vertices are joined by a path are said to be connected. Clearly u and v is connected iff $\mu^{\infty}(u,v) > 0$ .A strongest path joining any two vertices u and v has strength $\mu^{\infty}(u,v)$. We shall sometimes refer to this as the strength of connectedness between the vertices. The underlying crisp graph of the fuzzy graph $G(\sigma,\mu)$ is denoted by the pair of set $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V : \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in V \times V : \mu(u,v) > 0\}$

Throughout, we assume that $G(\sigma,\mu)$ is a finite fuzzy graph. $ie.\sigma^*$ is finite and by G, we mean the fuzzy graph $G(\sigma,\mu)$.

3. Basic Definitions

Let us recollect the definition of fuzzy subset of a fuzzy set. The function α is said to be a fuzzy subset of σ if $\alpha,\sigma : V \to [0,1]$ and $\alpha(x) \le \sigma(x)\ \forall x \in X$. Further α is said to be a proper fuzzy subset of σ if $\alpha(x) < \sigma(x)$, when $\sigma(x) > 0$. Also the cardinality of a fuzzy set σ is defined as $|\sigma| = \sum \sigma(u)$ for all $u \in V$. An arc (u,v) is said to be strong [2] if $\mu(u,v) \ge \mu^{\infty}(u,v)$. The strong neighborhood of u is $N_s(u) = \{v \in \sigma^* : (u,v)\text{is a strong arc}\}$. We use the notation $u \in \sigma$, we mean that $\sigma(u) > 0$ and otherwise $u \notin \sigma$. A fuzzy vertex of $u \in G(\sigma,\mu)$ is said to be an effective vertex if $\sigma(u) = \max_{v \in N_s(u)} \mu(u,v)$.A fuzzy arc $(u,v) \in G(\sigma,\mu)$ is said to be an effective arc if $\min(\sigma(u),\sigma(v)) = \mu(u,v)$. A fuzzy graph G having every arc as effective arc is called an effective fuzzy graph. A fuzzy graph G having every vertex as effective vertex is called a vertex effective fuzzy graph. A fuzzy graph G is fuzzy bipartite if the vertex set σ can be partitioned into two nonempty sets σ_1 and σ_2 such that σ_1 and σ_2 are fuzzy independent sets. These σ_1 and σ_2 are called fuzzy bipartition of σ Thus every strong arc of G has one end in σ_1 and other end in σ_2. A fuzzy bipartite graph G with fuzzy bipartition σ_1 and σ_2 is said to be a complete fuzzy bipartite if for each vertex of σ_1, every vertex of σ_2 is a strong neighbor. Let u and v be two fuzzy vertices of σ in a fuzzy graph G. Then we say that either u dominates v or vice versa if (i) the membership values of u and v are $\ge$ $\mu(u,v)$ in the reference set. (ii) (u,v) is a strong arc. A fuzzy subset α_d of σ is called a fuzzy dominating set of G if for every $v \in \sigma - \alpha_d^{*}$, there exists $u \in \alpha_d$ such that u dominates v . A fuzzy dominating set α_d is called a minimal fuzzy dominating set if no proper subset of α_d is a fuzzy dominating set. The cardinality of any minimum fuzzy dominating subset of σ is called its fuzzy domination number and is denoted by $\gamma_f(G)$. Two fuzzy vertices of a fuzzy graph are said to be fuzzy independent if there is no strong arc between them. A fuzzy subset α_i of σ is said to be fuzzy independent set of G if any two fuzzy vertices of α_i are fuzzy independent. A fuzzy vertex independent set α_i is called maximal fuzzy independent set if no superset of α_i is a fuzzy vertex independent set. The cardinality of any maximal fuzzy independent set of G is called its fuzzy vertex independence number and is denoted by β . A fuzzy subset α_c of σ such that every strong of G has at least one end in α_c is called fuzzy vertex covering of G.The cardinality of any minimum fuzzy vertex covering of G is called its fuzzy vertex covering number and it is denoted by ε.

4. Fuzzy vertex cut and Fuzzy split domination in fuzzy graphs

Definition 4.1

A fuzzy subset ν of σ is said to be a fuzzy vertex cut of G if $G(\sigma - \nu)$ is a disconnected graph.

Definition 4.2

A fuzzy dominating set α_d is called fuzzy independent dominating set if α_d is fuzzy independent. It is denoted by α_{id}.

Definition 4.3

The minimum cardinality of a maximum fuzzy independent dominating set of G is called the fuzzy independence domination number of G and is denoted by $\gamma_{fid}(G)$.

Definition 4.4

A fuzzy independent dominating set α_{id} of a fuzzy graph G is called a fuzzy split independent dominating set if α_{id} is a fuzzy vertex cut. It is denoted by α_{sid} .

Definition 4.5

The minimum scalar cardinality of a maximum fuzzy split independent dominating set of G is called the fuzzy split independence domination number of G and is denoted by $\gamma_{fsid}(G)$.

Note

Fuzzy split independent domination cannot be defined in the complete fuzzy graphs.

5. Some Results

Theorem 5.1

Let α_{sid} be a fuzzy split independent dominating set of the connected fuzzy graph G. Suppose $u,v \in V-\alpha_{sid}$. Then each $u-v$ path contains a fuzzy vertex of α_{sid} iff they are not in the same component of the fuzzy graph $G(\sigma-\alpha_{sid})$.

Proof

Let α_{sid} be a fuzzy split dominating set and $u,v \in V-\alpha_{sid}$.

Consider any $u-v$ path in G which contains some fuzzy vertex of α_{sid} .

Since $G(\sigma-\alpha_{sid})$ is disconnected, u and v are not belonging to the same component of $G\left(\sigma-\alpha_{sid}\right)$.

Conversely, Let u and v do not belong to the same component of $G(\sigma-\alpha_{sid})$ in G.

Since G is a connected fuzzy graph; there exists a $u-v$ path in G, which must contain a fuzzy vertex of α_{sid}. Hence the proof.

Theorem 5.2

For any non-complete connected fuzzy graph G,

$$\gamma_f \le \gamma_{fsid} \le \beta$$

Proof

By definition (4.5), $\gamma_{fsid} \le \beta \quad \rightarrow (1)$

Since γ_f is a fuzzy domination number,

$$\gamma_f \leq \gamma_{fsid} \rightarrow (2)$$

From (1) and (2), we get result.

Theorem 5.3

For any non-complete connected effective fuzzy graph G $\gamma_f + \gamma_{fsid} \leq |\sigma|$

Proof

Since $\gamma_f \leq \beta$ and $\gamma_{fsid} \leq \varepsilon$,

$$\gamma_f + \gamma_{fsid} \leq \beta + \varepsilon$$

Also $\varepsilon + \beta \leq |\sigma|$ [3]

Thus $\gamma_f + \gamma_{fsid} \leq |\sigma|$.

Theorem 5.4

For a connected fuzzy bipartite graph G, $\gamma_{fsid} \leq \max\{|\sigma_1|, |\sigma_2|\}$.

Proof

Let σ_1 and σ_2 be the two fuzzy bipartitions of a fuzzy bipartite graph G .

Since σ_1 and σ_2 are fuzzy independent sets, then by the defn/-(4.5), the result follows.

Theorem 5.5

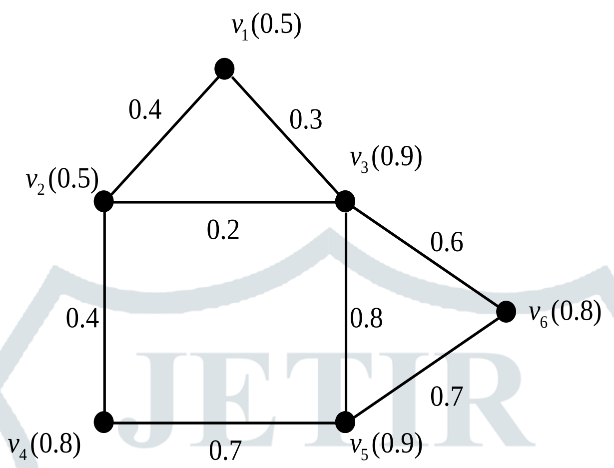
For any connected fuzzy graph G, $\gamma_{fsid} \leq |\sigma| - \Delta(G)$.

Proof

Let u be a fuzzy vertex of G such that $N_s(u)$ be its strong neighbors. Clearly u dominates $N_s(u)$.Then $\sigma - N_s(u)$ is a fuzzy dominating set.

Therefore $\gamma_{fsid}(G) \leq |\sigma - N_s(u)|$.Since u is any arbitrary fuzzy vertex we've $\gamma_{fsid} \leq |\sigma| - \Delta(G)$.

Example



$\gamma_{fsid} = 1.5$

Reference

[1]A.Nagoorgani and V.T.Chandrasekaran, Independent sets in fuzzy graphs, Adv. In fuzzy sets and systems-I. (2006), 83-85.

[2]V.T.Chandrasekaran and R.Sangeetha, Fuzzy dominating sets, IJMAA.,6(3)(2018), 301-305.

[3]V.T.Chandrasekaran and R.Sangeetha, Fuzzy independent sets and Fuzzy covering, accepted in Bulletin of Pure and Applied Sciences.

[4]P.Gladyis, C.V.Harinarayanan and R.Muthuraj, Strong Split Independent Domination in Fuzzy Graphs, IJMAA, Volume 4, Issue 1 –A(2016), 159-163.