GENERALIZED HYERS - ULAM STABILITY OF AC FUNCTIONAL EQUATION IN FUZZY NORMED SPACE

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ABSTRACT. In this paper, the author has investigate the generalized Hyers-Ulam stability of additive cubic functional equation of the form

f (2x + x) f (2x x) = 4f (x + x) 4f (x x) 6f (x)

in fuzzy normed spaces using Hyers method.

1. INTRODUCTION AND PRELIMINARIES

The stability problem of functional equations originated from a question of S.M. Ulam [40] concerning the stability of group homomorphisms. D.H. Hyers [15] gave a first affirmative partial answer to the question of Ulam for Banach spaces.

Hyers' theorem was generalized by T. Aoki [2] for additive mappings and by Th.M. Rassias [35] for linear mappings by considering an unbounded Cauchy difference. The paper of Th.M. Rassias [35] has provided a lot of influence in the development of what we call generalized Hyers-Ulam stability of functional equations. A generalization of the Th.M. Rassias theorem was obtained by P. Gavruta [14] by replacing the unbounded Cauchy difference by a general control function in the spirit of Rassias approach. In 1982, J.M. Rassias [32] followed the innovative approach of the Th.M. Rassias theorem [35] in which he replaced the factor $jjxjj^p$ + $jjxjj^p$ by $jjxjj^p jjxjj^q$ for p; q 2 R with p + q = 1:
In 2008, a special case of Gavruta's theorem for the unbounded Cauchy dif-ference was obtained by Ravi etal., [37] by considering the summation of both the sum and the product of two p norms in the sprit of Rassias approach. The stability problems of several functional equations have been extensively investi-gated by a number of authors and there are many interesting results concerning

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this problem (see [1, 12, 16, 20]).

A.K. Katsaras [22] defined a fuzzy norm on a vector space to construct a fuzzy vector topological structure on the space. Some mathematicians have de-fined fuzzy norms on a vector space from various points of view [13, 24, 41]. In particular, T. Bag and S.K. Samanta [8], following S.C. Cheng and J.N. Morde-son [10], gave an idea of fuzzy norm in such a manner that the corresponding fuzzy metric is of Kramosil and Michalek type [23]. They established a decom-position theorem of a fuzzy norm into a family of crisp norms and investigated some properties of fuzzy normed spaces [9].

We use the definition of fuzzy normed spaces given in [8] and [27, 28, 29, 30].

Definition 1.1. Let X be a real linear space. A function N : X R ! [0; 1](the so-called fuzzy subset) is said to be a fuzzy norm on X if for all x; y 2 X and all s; t 2 R,

(F 1) N(x; c) = 0 for c 0;

(F 2) x = 0 if and only if N(x; c) = 1 for all c > 0;

(F 3) N(cx; t) = N x; $\frac{t}{jcj}$ if c 6= 0;

(F 4) N(x + y; s + t) min N x; s ; N y; t f ( ) ( )g;

(F 5) N(x;) is a non-decreasing function on R and $\lim_{t!1}$N(x; t) = 1;

(F 6) for x 6= 0; N(x;) is (upper semi) continuous on R.

The pair (X; N) is called a fuzzy normed linear space. One may regard N(X; t) as the truth-value of the statement the norm of x is less than or equal to the real number t'.

Example 1.2. Let (X; jj jj) be a normed linear space. Then

$$N (x; t) = 8 \frac{t}{t + kxk} ;t > 0; x 2 X; \quad <0; \; t \; 0;x \; X \quad : \; 2$$

is a fuzzy norm on X.

Definition 1.3. Let (X; N) be a fuzzy normed linear space. Let x_n be a sequence in X. Then x_n is said to be convergent if there exists x 2 X such that $\lim_{n!1}$ N(x_n x; t) = 1 for all t > 0. In that case, x is called the limit of the sequence x_n and we denote it by N $\lim_{n!1}$ x_n = x:

Definition 1.4. A sequence x_n in X is called Cauchy if for each > 0 and each t > 0 there exists n_0 such that for all n n_0 and all p > 0, we have N(x_{n+p} x_n; t) > 1 :

Definition 1.5. Every convergent sequence in a fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed space is called a fuzzy Banach space.

The stability of various functional equations in fuzzy normed spaces was in-vestigated in [3, 4, 6, 17, 26, 27, 28, 29, 30, 38].

In this paper, the author has investigate the generalized Hyers-Ulam stability of additive cubic functional equation of the form

$$f(2x + x) \quad f(2x \quad x) = 4f(x + x) \quad 4f(x \quad x) \quad 6f(x) \tag{1.1}$$

in fuzzy normed spaces using Hyers method.

2. FUZZY STABILITY RESULTS: DIRECT METHOD

Throughout this section, assume that X; (Z; N^0) and (Y; N^0) are linear space, fuzzy normed space and fuzzy Banach space, respectively. Now use the following notation for a given mapping f : X ! Y

D f(x; x) = f(2x + x) f(2x x) 4f(x + x) + 4f(x x) + 6f(x)

for all x; y 2 X:

Now, we investigate the generalized Ulam-Hyers stability of AQCQ functional equation (1.1).

Theorem 2.1. Let 2 f 1; 1g be fixed and let : X^2 ! Z be a mapping such that for d

$$N^0 ((2 x; 2 x) ; r) \quad N^0 (d \quad (x; x) ; r) \tag{2.1}$$

for all x 2 X and all r > 0, d > 0, and

$$\lim_{k!1} N^0 \quad 2^{k}x; 2^{k}y \quad ; 2^{k}r = 1 \tag{2.2}$$

for all x; y 2 X and all r > 0. Suppose that a function f : X ! Y satisfies the inequality

$$N (D f(x; y); r) \quad N^0 (\quad (x; x); r) \tag{2.3}$$

for all r > 0 and all x; y 2 X. Then the limit

$$A(x) = N \lim_{k!1} \frac{f(2^{k}x)}{2^{k}} \tag{2.4}$$

exists for all x 2 X and the mapping A : X ! Y is a unique additive mapping such that

$$N (f(2x) \quad 8f(x) \quad A(x); r) \quad \min N^0 \quad (x; x); \frac{8}{(2 \quad d)r} ; N^0 \quad (x; 2x); \frac{4}{(2 \quad d)r} \tag{2.5}$$

for all x 2 X and all r > 0.

Proof. First assume = 1. Replacing (x; y) by (x; x) in (2.3), we get

$$N\,(f(3x)\ \ 4f(2x) + 5f(x); r)\ \ N^{0}\,(\ (x; x); r) \qquad (2.6)$$

for all x 2 X and all r > 0. Replacing (x; y) by (x; 2x) in (2.3), we obtain

$$N\,(f(4x)\ \ 4f(3x) + 6f(2x)\ \ 4f(x); r)\ \ N^{0}\,(\ (x; 2x); r) \qquad (2.7)$$

for all x 2 X and all r > 0. Now, from (2.6) and (2.7), we have

$$N\ \ (f(4x)\ \ 10f(2x) + 16f(x); r)$$
$$\min\ N\ \ 4\,(f(3x)\ \ 4f(2x) + 5f(x))\,;\,\frac{r}{}\ ;\ N\ \ f(4x)\ \ 4f(3x) + 6f(2x)\ \ 4f(x);\ \frac{r}{}\ o$$
$$\min{}^{n}N_0\ \ (x; x);\ \frac{r}{8};\ N_0\ \ (x; 2x);\ \frac{r}{2}\ o \qquad (2.8)$$

for all x 2 X and all r > 0. Let a : X ! Y be a mapping defined by a(x) = f(2x) 8f(x). Then we conclude that

$$N\,(a(2x)\ \ 2a(x); r)\ \ \min{}^{n}N^{0}\ \ (x; x);\ \frac{r}{8};\ N_0\ \ (x; 2x);\ \frac{r}{2}\ o \qquad (2.9)$$

for all x 2 X and all r > 0. Thus, we have

$$N\ \ \frac{a(2x)}{2}\ \ a(x);\ \frac{r}{2}\ \ \min{}^{n}N_0\ \ (x; x);\ \frac{r}{8};\ N_0\ \ (x; 2x);\ \frac{r}{4}\ o \qquad (2.10)$$

for all x 2 X and all r > 0. Replace x by $2^k y$ in (2.10), we get

$$N\ \ \frac{a(2^{k+1}x)}{}\ \ \frac{f(2^{k}x)}{}\ \ \frac{r}{}{}_{n}\ \ \min\ N_0\ \ \frac{r}{}\ ;\ N^{0}\ \ \frac{r}{}\ o \qquad (2.11)$$

for all x 2 X and all r > 0. Using (2.1), (F 3) in (2.11), we arrive

$$N\ \ \frac{a(2^{k+1}x)}{2^{k+1}}\ \ \frac{a(2^{k}x)}{2^{k}}\ \ \frac{r}{}\ \ \min{}_{n}N_0\ \ \frac{r}{}\ ;\ N^{0}\ \ \frac{r}{}\ o \qquad (2.12)$$

for all x 2 X and all r > 0. Replacing r by $d^k r$ in (2.12), we get

$$N\ \ \frac{a(2^{k+1}x)}{2^{k+1}}\ \ \frac{a(2^{k}x)\ d^{k}r}{2^{k}}{}_{n}\ \ \min\ N_0\ \ (x; x)\ 8\ \ \frac{r}{}\ ;\ N^{0}\ \ (x; 2x);\ \frac{r}{4}\ o \qquad (2.13)$$

for all x 2 X and all r > 0. It is easy to see that

$$\frac{a(2^{k}x)}{}\ X\ \ k\ 1\ \ \frac{a(2^{i+1}x)}{}\ \ \frac{a(2^{i}x)}{} \qquad (2.14)$$

for all x 2 X. From equations (2.13) and (2.14), we have

$$N\left(\frac{a(2^k x)}{2^k} - a(x); \sum_{i=0}^{k-1} \frac{d^i r}{2^{i+1}}\right) \geq \min \bigcup_{i=0}^{k-1} \left\{ N\left(\frac{a(2^{i+1}x)}{2^{i+1}} - \frac{a(2^i x)}{2^i}; \frac{d^i r}{2^{i+1}}\right) \right\}$$

$$\geq \min\left\{ N_0\left(\varphi(x;x); \frac{r}{8}\right); N_0\left(\varphi(x;2x); \frac{r}{4}\right) \right\} \quad (2.15)$$

for all x 2 X and all r > 0. Replacing x by $2^m x$ in (2.15) and using (2.1), (F 3), we obtain

$$N\left(\frac{a(2^{k+m}x)}{2^{(k+m)}} - \frac{a(2^m x)}{2^m}; \sum_{i=0}^{k-1} \frac{d^i r}{2^{i+m}2}\right) \geq \min\left\{ N_0\left(\varphi(x;x); \frac{r}{8d^m}\right); N_0\left(\varphi(x;2x); \frac{r}{4d^m}\right) \right\} \quad (2.16)$$

for all x 2 X and all r > 0 and all m; k 0. Replacing r by $d^m r$ in (2.16), we get

$$N\left(\frac{a(2^{k+m}x)}{2^{(k+m)}} - \frac{a(2^m x)}{2^m}; \sum_{i=m}^{m+k-1} \frac{d^{i+m} r}{2^{i+m}2}\right) \geq \min\left\{ N_0\left(\varphi(x;x); \frac{r}{8}\right); N_0\left(\varphi(x;2x); \frac{r}{4}\right) \right\} \quad (2.17)$$

for all x 2 X and all r > 0 and all m; k 0. Using (F 3) in (2.17), we obtain

$$N\left(\frac{a(2^{k+m}x)}{2^{(k+m)}} - \frac{a(2^m x)}{2^m}; r\right) \geq \min\left\{ N_0\left(\varphi(x;x); \frac{r}{8\sum_{i=m}^{m+k-1} \frac{d^i}{2\cdot 2}}\right); N_0\left(\varphi(x;2x); \frac{r}{4\sum_{i=m}^{m+k-1} \frac{d^i}{2\cdot 2}}\right) \right\} \quad (2.18)$$

for all x 2 X and all r > 0 and all m; k 0. Since 0 < d < 2 and $\sum_{i=0}^{k} \left(\frac{d}{2}\right)^i < 1$, the cauchy criterion for convergence and (F 5) implies that $\left\{\frac{a(2^k x)}{2^k}\right\}$ is a Cauchy sequence in (Y; N). Since (Y; N) is a fuzzy Banach space, this sequence converges to some point A(x) 2 Y. So one can define the mapping A : X ! Y by $A(x) = N - \lim_{k\to\infty} \frac{a(2^k x)}{2^k}$ for all x 2 X: Letting m = 0 in (2.18), we get

$$N\left(\frac{a(2^k x)}{2^k} - a(x); r\right) \geq \min\left\{ N_0\left(\varphi(x;x); \frac{r}{8\sum_{i=0}^{k-1} \frac{d^i}{2\cdot 2}}\right); N_0\left(\varphi(x;2x); \frac{r}{4\sum_{i=0}^{k-1} \frac{d^i}{2\cdot 2}}\right) \right\} \quad (2.19)$$

for all x 2 X and all r > 0. Letting k ! 1 in (2.19) and using (F 6), we arrive

$$N(a(x) - A(x); r) \geq \min\left\{ N_0\left(\varphi(x;x); \frac{(2-d)r}{8}\right); N_0\left(\varphi(x;2x); \frac{(2-d)r}{4}\right) \right\}$$

for all $x \in X$ and all $r > 0$. To prove A satisfies the (1.1), replacing $(x; x)$ by $(2^k x; 2^k x)$ in (2.3), respectively, we obtain

$$N_{\frac{1}{2^k} Df(2^k x; 2^k x); r} N^0 (2^k x; 2^k x); 2^k r \quad (2.20)$$

for all $r > 0$ and all $x; y \in X$. Now,

$$N(A(2x+y) \quad A(2x \quad y) \quad 4A(x+y) + 4A(x \quad y) + 6A(y); r)$$

$$\min N_{A(2x+y) \frac{1}{2^k} f(2x+y); \frac{r}{6}}; N_{A(2x \ y) + \frac{1}{2^k} f(2x \ y); \frac{r}{6}};$$

$$N_{4A(x+y)+4\frac{1}{2^k} f(x+y); \frac{r}{6}}; N_{+4A(x \ y) \ 4\frac{1}{2^k} f(x \ y); \frac{r}{6}};$$

$$N_{6A(y) \ 6 \frac{1}{2^k} f(y); \frac{r}{6}};$$

$$N_{\frac{1}{2^k} f(2x+y) \ \frac{1}{2^k} f(2x \ y) \ \frac{1}{2^k} 4f(x+y) + \frac{1}{2^k} 4f(x \ y) + \frac{1}{2^k} 6f(y); \frac{r}{6}}$$

(2.21)

for all $x; y \in X$ and all $r > 0$. Using (2.20) and (F 5) in (2.21), we arrive

$$N(A(2x+y) \quad A(2x \quad y) \quad 4A(x+y) + 4A(x \quad y) + 6A(y); r)$$

$$\min \quad 1; 1; 1; 1; 1; N^0 (2^k y; 2^k x); \frac{(2 \quad d)2^k r}{8}; N^0 (2{:}2^k y; 2^k x); \frac{(2 \quad d)2^k r}{4}$$

$$\min \quad N^0 (2^k y; 2^k x); \frac{(2 \quad d)2^k r}{8}; N^0 (2{:}2^k y; 2^k x); \frac{(2 \quad d)r}{42^k}$$

(2.22)

for all $x; y \in X$ and all $r > 0$. Letting $k \to \infty$ in (2.22) and using (2.2), we see that

$N(A(2x+y)\ A(2x\ y)\ 4A(x+y) + 4A(x\ y) + 6A(y); r) = 1$ (2.23) for all $x; y \in X$ and all $r > 0$. Using (F 2) in the above inequality gives

$$A(2x+y) \quad A(2x \quad y) = 4A(x+y) \quad 4A(x \quad y) \quad 6A(y)$$

for all $x; y \in X$. Hence A satisfies the cubic functional equation (1.1). In order to prove $A(x)$ is unique, let $A^0(x)$ be another additive functional equation satisfying (1.1) and (2.5). Hence,

$$N(A(x) \quad A^0(x); r) \quad \min_n \frac{A(2^k x)}{} \frac{f(2^k x)}{} r; N_o \frac{f(2^k x)}{} \frac{A(2^k x)}{} r$$

$$\min \quad N^0 (2^k y; 2^k x); \frac{2^k(2 \quad d)r}{8}; N^0 (2^k 2y; 2^k x); \frac{2^k(2 \quad d)r}{4}$$

$$\min \quad N^0 (x; x); \frac{2^k(2 \quad d)r}{8d^k}; N^0 (x; 2x); \frac{2^k(2 \quad d)r}{4d^k}$$

for all x 2 X and all r > 0. Since

$$\lim_{k!1} \frac{2^k(2\ \ d)r}{8d^K} = 1 \text{ and } \lim_{k!1} \frac{2^k(2\ \ d)r}{4d^K} = 1 ;$$

we obtain

$$\lim_{k!1} N^0 (x; x); \frac{2^k(2\ \ d)r}{8d^k} = 1 \text{ and } \lim_{k!1} N^0 (x; 2x); \frac{2^k(2\ \ d)r}{4d^k} = 1$$

for all x 2 X and all r > 0. Thus

N(A(x) A^0(x); r) = 1

for all x 2 X and all r > 0, hence A(x) = A^0(x). Therefore A(x) is unique.

For = 1, we can prove the result by a similar method. This completes the proof of the theorem.

From Theorem 2.1, we obtain the following corollaries concerning the Hyers-Ulam-Rassias and JMRassias stabilities for the functional equation (1.1).

Corollary 2.2. Suppose that a function f : X ! Y satisfies the inequality

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>
< N^0 (fjjxjj^s + jjxjj^{s}g ; r) ; N^0 (; r) ;
N (D f(x; y); r) N^0 (fjjxjj^{s}jjxjj^{s}g ; r) ;
:
s 6= 1;
s 6= 1/2 ; (2.24)
s 6= 1/2 ;

for all x; y 2 X and all r > 0, where ; s are constants with > 0. Then there exists a unique additive mapping A : X ! Y such that

N (f(2x) 8f(x) A(x); r)

8min N^0; j2jr/8 ; N^0 ; j2jr/4 ; 1 + 2 s x s ; r
>
> 2 jj jj 4 2 2 2 jj jj 2 2 2
>
<
> 2^{2s} jj jj 4 2^{2s} 2 2s jj jj s 2. 2^{2s} 2 ; 3 2s ; j r j 1 + 2 j + 1 j x 2s ; r
>
> 2 jj jj 4 2 2 2 2 jj jj 2 2 2
>
> j j j j

> (2.25)

for all x 2 X and all r > 0.

Theorem 2.3. Let 2 f 1; 1g be fixed and let : X^2 ! Z be a mapping such that for
d

$$N^0 ((2 y; 2 y) ; r) \quad N^0 (d \quad (y; y) ; r) \qquad (2.26)$$

for all x 2 X and all r > 0, d > 0, and

$$\lim_{k!1} N^{U} \left(\frac{x;_k y}{2^{2k}} ; \frac{r}{2^k} \right) = 1 \quad (2.27)$$

for all x; y 2 X and all r > 0. Suppose that a function f : X ! Y satisfies the inequality

$$N (Df(x; y); r) \quad N^0 ((x; x); r) \quad (2.28)$$

for all r > 0 and all x; y 2 X. Then the limit

$$C(x) = N \lim_{k!1} \frac{a(2^k x)}{2^{3k}} \quad (2.29)$$

exists for all x 2 X and the mapping C : X ! Y is a unique cubic mapping such that

$$N (f(2x) \quad 2f(x) \quad C(x); r) \quad \min \left\{ N^0(x; x); \frac{(2^3 \quad d)r}{8} ; N^0 \quad (x; 2x); \frac{(2^3 \quad d)r}{4} \right\} \quad (2.30)$$

for all x 2 X and all r > 0.

Proof. It is easy to see from (2.8) that

$$N \quad (f(4x) \quad 2f(2x) \quad 8f(2x); r)$$

$$\min \left\{ N \quad 4 (f(3x) \quad 4f(2x) + 5f(x)) ; \frac{r}{2} ; N \quad f(4x) \quad 4f(3x) + 6f(2x) \quad 4f(x); \frac{r}{2} \right\}$$

$$\min \left\{ N^0 \quad (x; x); \frac{r}{8} ; N^0 \quad (x; 2x); \frac{r}{2} \right\} \quad (2.31)$$

for all x 2 X and all r > 0. Let h : X ! Y be a mapping defined by h(x) = f(2x) 2f(x). Then we conclude that

$$N (h(2x) \quad 8h(x); r) \quad \min \left\{ N^0 \quad (x; x); \frac{r}{8} ; N^0 \quad (x; 2x); \frac{r}{2} \right\} \quad (2.32)$$

for all x 2 X and all r > 0. The rest of the proof is similar to that of Theorem 2.1.

The following corollary is an immediate consequence of Theorem 2.3 concern-ing the Ulam-Hyers stability of the functional equation(1.1).

Corollary 2.4. Suppose that a function f : X ! Y satisfies the inequality

$$N (Df(x; y); r) \quad \begin{cases} N^0 (; r) ; & s \, 6= 3; \\ N^0 (fjjxjj^s + jjxjj^s g ; r) ; & s \, 6= \frac{3}{2} ; \\ N^0 (fjjxjj^s jjxjj^s g ; r) ; & s \, 6= \frac{3}{2} ; \\ N^0 (fjjxjj^s jjxjj^s + (jjxjj^{2s} + jjxjj^{2s}) g ; r) ; & \end{cases} \quad (2.33)$$

for all x; y 2 X and all r > 0, where ; s are constants with > 0. Then there exists a unique Cubic mapping C : X ! Y such that

N (f(2x) 2f(x) C(x); r)

8min N0 ; 7 ; N0 ; 27r s
r

>j j >min N⁰ x ; sr ; N⁰ j j 1 + 2 x s ; 2r
> s s 3 s s 3
>
> 2 jj jj 2 2 2 jj jj 2 2
>
>j j j j
>2s r 2s r
>min N⁰ x ; ; N⁰ x ;
< 22s jj jj 22s 23 2s jj jj s 2 22s 23
> 3 2s j r j 1 + 2 j 1 j 2s 2r
>min N⁰ x ; ; N⁰ + x ;
> 2s 2s 3 s 2s 2s 3
>
> 2 jj jj 2 2 2 2 jj jj 2 2
>
>j j j j >

>(2.34)

for all x 2 X and all r > 0.

Theorem 2.5. Let 2 f 1; 1g be fixed and let : X² ! Z be a mapping such that for some d with the condition given (2.1) and (2.26) and 0 < d/2 < 1, 0 < d/2³ < 1.

Suppose that a function f : X ! Y satisfies the inequality

N (Df(x; y); r) N⁰ ((x; x); r) (2.35)

for all r > 0 and all x; y 2 X. Then there exists a additive mapping A : X ! Y and unique cubic mapping C : X ! Y satisfying the functional equation (1.1) and

N (f(x) A(x) C(x); r)

min N0 (x; x); (2 d)r/8 ; N⁰ (x; 2x); (2 d)r/4 ; N⁰ (x; x); (2³ d)r/8 ;
(x; 2x);
4

for all x 2 X and all r >0.

Proof. By Theorems 2.1 and 2.3, there exists a unique additive function A₁ : X ! Y and a unique cubic function C₁ : X ! Y such that

N (f(2x) 8f(x) A 1 (x); r) min N⁰ (x; x); (2 d)r 8 ; N⁰ (x; 2x); (2 d)r 4

(2.37)

for all x 2 X and all r > 0 and

N (f(2x) 2f(x) C $_1$ (x); r) min N^0 (x; x); $\frac{(2^3 \quad d)r}{8}$; N^0 (x; 2x); $\frac{(2^3 \quad d)r}{4}$

(2.38)

for all x 2 X and all r > 0. Now from (2.37) and (2.38), one can see that

N f(x) + 6A1(x) 6C1(x); 2r

min N $\frac{1}{f(2x)}$ 6 6f(x) $\frac{1}{8}$ $6^{A_1(x);}$ $\frac{6}{r}$; N $\frac{1}{f(2x)}$ 66f(x) $6^{C_1(x);}$ $\frac{6}{r}$ 21

min fN (f(2x) 8f(x) A_1(x); r) ; N (f(2x) 2f(x) C_1(x); r)g

min N^0 (x; x); $\frac{(2 \quad d)r}{8}$; N^0 (x; 2x); $\frac{(2 \quad d)r}{4}$; N^0 (x; x); $\frac{(2^3 \quad d)r}{8}$;

4

for all x 12 X and all r > 0. Thus we obtain (2.36) by defining A(x) = 61 A_1(x) and

C(x) = $\frac{1}{6}$ C_1(x) for all x 2 X and all r > 0.

The following corollary is an immediate consequence of Theorem 2.5 concern-ing the Ulam-Hyers stability of the functional equation(1.1).

Corollary 2.6. Suppose that a function f : X ! Y satisfies the inequality

8
>
N (Df(x; y); r) < N^0 (; r) ; N^0 (x_s + x ^{s} ; r) ; 2s s = 1; 3 (2.39)
> N^0 ((jjxjj jjxjj + jjxjj + jjxjj) ; r) ; s 6= 2 ; 2 ; 61 3
:

for all x; y 2 X and all r > 0, where ; s are constants with > 0. Then there exists a unique additive mapping A : X ! Y and a unique Cubic mapping C : X ! Y such that

N (f(x) A(x) C(x); r)

8 > min N^0 ; j2jr ; N^0 ; j2jr ; N^0 ; r ; N^0 ; 2r o
> 84 jj jj j j7j j7j
> 1+2s N^0 x ; s 3 ; N^0 s x ; s 3
> 2 2 2 2 2 2
> jj jj j j jj jj j j
> 2s r 2s r o

2s 2s s 2s

> 2 4 2 2 2 2 2 2

>

> jj jj j j jj jj j j

>

< n 2s r 2s r

> > min N0 2s 3x jj2s jj rj ; 2s ; N j 0 1+2 s jjs + jj 2s 1 j x 2sj o ; r 2s ;

> N0 22s x ; 24 2 22s 2 23 ; N 0 2s 2 x ; 2 2 22s 2 23 2 2 2

>

> jj jj j j jj jj j j >

> >n N0 3 2s x 2s ; r 2s 3; N0 1+2 s + s 1 2s x 2s ; 2r 2s 3

>2 2 2 2 22 2

>

>jj jj j j jj jj j j

>

> o

>

> :

for all $x \in X$ and all $r > 0$.

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