MULTIPLICATIVE F-INDICES OF NANOTUBES

[1]T. Nandhini, [2]Dr.N.Srinivasan,

[1]Research scholar, Department of Mathematics, SPIHER, Avadi, Chennai -54.

[2] Professor, Department of Mathematics, SPIHER, Avadi, Chennai -54.

Abstract: In this paper, we compute the multiplicative second Hyper F-index, general multiplicative F-indices, multiplicative sum connectivity F-index, multiplicative product connectivity F-index, multiplicative atom bond connectivity F-index and multiplicative geometric-arithmetic F-index of $TUC_4C_8(m,n)$ nanotubes.

Keywords: **Molecular graph, Degree-based topological indices, Nanotubes.**

1. Introduction

Chemical graph theory is a branch of mathematical chemistry which has an important effect on the development of the chemical sciences. A Topological Index for a chemical graph is a numerical representation of the molecular structure derived from the molecular graph [3].

In this paper, we consider the graph which is finite, undirected, loopless and without multiple edges. Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. Recently, V.R.Kulli [2] introduced the multiplicative second F-index, the Multiplicative first and second Hyper F-indices, General Multiplicative first and second F-indices, Multiplicative product connectivity F-index, Multiplicative sum connectivity F-index, Multiplicative ABC F-index and Multiplicative GA F-index of a graph are as follows,

The multiplicative second F-index of a graph G is defined as

$$F_2II\,(G) = \prod_{uv\in E(G)} (d_G(u)^2 d_G(v)^2)$$

The multiplicative first and second Hyper F-indices of a graph *G* are defined as

$$HF_1II(G) = \prod_{uv\in E(G)} (d_G(u)^2 + d_G(v)^2)^2$$

$$HF_2II(G) = \prod_{uv\in E(G)} (d_G(u)^2 d_G(v)^2)^2$$

The general multiplicative first and second F-indices of a graph *G* are defined as

$$F_1^a II(G) = \prod_{uv\in E(G)} (d_G(u)^2 + d_G(v)^2)^a \tag{1}$$

$$F_2^a II(G) = \prod_{uv\in E(G)} (d_G(u)^2 d_G(v)^2)^a \tag{2}$$

The multiplicative sum connectivity F-index and the multiplicative product connectivity F-index of a graph *G* is defined as

$$SFII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2}}$$

$$PFII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)^2 d_G(v)^2}}$$

The multiplicative atom bond connectivity F-index of a graph *G* is defined as

$$ABCFII(G) = \prod_{uv\in E(G)} \sqrt{\frac{d_G(u)^2 + d_G(v)^2 - 2}{d_G(u)^2 d_G(v)^2}} \tag{3}$$

The multiplicative geometric-arithmetic F-index of a graph *G* is defined as

$$GAFII(G) = \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)^2 d_G(v)^2}}{d_G(u)^2 + d_G(v)^2} \tag{4}$$

In this paper, we investigate the above presented topological indices of $TUC_4C_8(m,n)$nanotubes.

3. Main Results

Nanotubes are the important category of nanostructured materials which can be prepared from carbon. These materials are usually represented as molecular graph where the vertices of graph correspond to the atoms and the edges correspond to the chemical bonds. We first present following definition [4] that will be useful to divide the vertex set $V(G)$ and edge set $E(G)$ of a molecular graph G based on its degree of vertices.

Definition 3.1

Let G and d_v $(1 \leq d_v \leq n-1)$ be a simple connected molecular graph and the vertex degrees of vertices/atom v in G. We divide the vertex set $V(G)$ and edge set $E(G)$ of G into several partitions based on d_v $\left(for\ all\ v \in (G)\right)$ as follows,

$$V_k = \{v \in V(G) | d_v = k\} \qquad for\ all\ k, \delta \leq k \leq \Delta,$$

$$E_i = \{e = uv \in E(G) | d_v + d_u = i\} \qquad for\ all\ i, 2\delta \leq i \leq 2\Delta,$$

$$E_j^* = \{e = uv \in E(G) | d_v\, d_u = j\} \qquad for\ all\ j,\ \delta^2 \leq j \leq \Delta^2,$$

Where δ and Δ are the minimum and maximum degree of d_v for all $v \in (G)$.

3.1 Result for $TUC_4C_8(m,n)$ nanotubes

In $TUC_4C_8(m,n)$ nanotubes, the degree of an arbitrary vertex/atom of a molecular graph is equal to 1, 2 or 3. Since the hydrogen atoms in molecular graphs (i.e., vertices of degree 1) are often omitted, thus we will ignore the vertex set V_1.
Therefore, we have two partitions of vertex set $V(G)$ are as follows,

$$V_2 = \{v \in V(G) | d_v = 2\} \qquad |V_2| = 4m$$

$$V_3 = \{v \in V(G) | d_v = 3\} \qquad |V_3| = 8mn$$

Next, the three partitions of edge set $E(G)$ are as follows,

$$E_4 = E_4^* = \{u, v \in V(G) | d_u = d_v = 2\} \qquad |E_4| = |E_4^*| = \frac{1}{2}\,|V_2| = 2m$$

$$E_5 = E_6^* = \{u, v \in V(G) | d_u = 2\ \&\ d_v = 3\} \qquad |E_5| = |E_6^*| = |V_2| = 4m$$

$$E_6 = E_9^* = \{u, v \in V(G) | d_u = d_v = 3\} \qquad |E_6| = |E_9^*| = 12mn - 2m$$

Let G be a $TUC_4C_8(m,n)$ nanotubes, where M.V.Diudea denoted the numbers of octagons C_8 in the first row of G by m and the numbers of octagons C_8 in the first column of G by n of the corresponding 3-Dimensional and 2-Dimensional lattices of $TUC_4C_8(m,n)$ which is shown in Figure 3.1.

The Number of vertices/atoms and edges/bonds in $TUC_4C_8(m,n)$ (for all $m, n \in \mathbb{N}$) is equal to $|V| = 8mn + 4m$ and $|E| = 12mn + 4m$.

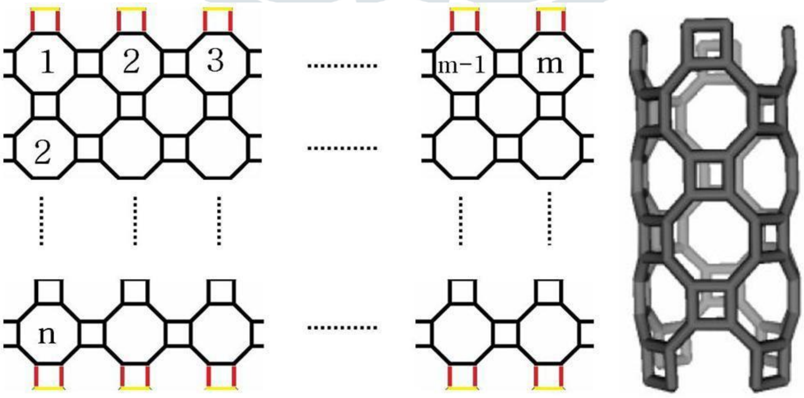

Figure 3.1 - The 2-Dimensional Lattice of G = $TUC_4C_8(m,n)$

In the following theorem, we compute the general multiplicative first F-index of $TUC_4C_8(m,n)$ nanotubes,

Theorem 3.1. The general multiplicative first F-index of $TUC_4C_8(m,n)$ is

$$F_1^a II\ (TUC_4C_8[m,n]) = (8)^{2am} \times (13)^{4am} \times (18)^{a(12mn-2m)} \tag{5}$$

Proof. Let G = $TUC_4C_8(m,n)$. From equation (1) and by cardinalities of edge partition of G, we have

$$\begin{aligned} F_1^a II(G) &= \prod_{uv\in E(G)} (d_G(u)^2 + d_G(v)^2)^a \\ &= \prod_{uv\in E_4} (2^2+2^2)^a \times \prod_{uv\in E_5} (2^2+3^2)^a \times \prod_{uv\in E_6} (3^2+3^2)^a \\ &= [(2^2+2^2)^a]^{2m} \times [(2^2+3^2)^a]^{4m} \times [(3^2+3^2)^a]^{12mn-2m} \\ &= (8)^{2am} \times (13)^{4am} \times (18)^{a(12mn-2m)} \end{aligned}$$

We obtain the following results by using Theorem 3.1.

Corollary 3.2. The multiplicative first hyper F-index of $TUC_4C_8(m,n)$ is

$$HF_1II\,(TUC_4C_8[m,n]) = (8)^{4m} \times (13)^{8m} \times (18)^{(24mn-4m)}$$

Proof. Put a=2 in equation (5), we get the desired result.

Corollary 3.3. The multiplicative sum connectivity F-index of $TUC_4C_8(m,n)$ is

$$SFII\,(TUC_4C_8[m,n]) = \left(\frac{1}{\sqrt{8}}\right)^{2m} \times \left(\frac{1}{\sqrt{13}}\right)^{4m} \times \left(\frac{1}{\sqrt{18}}\right)^{12mn-2m}$$

Proof. Put a = $\frac{-1}{2}$ in equation (5), we get the desired result.

We now compute the general multiplicative second F-index of $TUC_4C_8(m,n)$

Theorem 3.4. The general multiplicative second F-index of $TUC_4C_8(m,n)$ is

$$F_2^a II\,(TUC_4C_8[m,n]) = (16)^{2am} \times (36)^{4am} \times (81)^{a(12mn-2m)} \qquad (6)$$

Proof. Let G = $TUC_4C_8(m,n)$. From equation (2) and by cardinalities of edge partition of G, we have

$$\begin{aligned} F_2^a II(G) &= \prod_{uv\in E(G)} (d_G(u)^2 \times d_G(v)^2)^a \\ &= \prod_{uv\in E_4^*} (2^2\times 2^2)^a \times \prod_{uv\in E_6^*} (2^2\times 3^2)^a \times \prod_{uv\in E_9^*} (3^2\times 3^2)^a \\ &= [(2^2\times 2^2)^a]^{2m} \times [(2^2\times 3^2)^a]^{4m} \times [(3^2\times 3^2)^a]^{12mn-2m} \\ &= (16)^{2am} \times (36)^{4am} \times (81)^{a(12mn-2m)} \end{aligned}$$

We obtain the following results by using Theorem 3.4.

Corollary 3.5. The multiplicative second F-index of $TUC_4C_8(m,n)$ is

$$F_2II\,(TUC_4C_8[m,n]) = (16)^{2m} \times (36)^{4m} \times (81)^{(12mn-2m)}$$

Proof: Put a=1 in equation (6), we get the desired result.

Corollary 3.6. The multiplicative second hyper F-index of $TUC_4C_8(m,n)$ is

$$HF_2II\,(TUC_4C_8[m,n]) = (16)^{4m} \times (36)^{8m} \times (81)^{(24mn-4m)}$$

Proof: Put a=2 in equation (6), we get the desired result.

Corollary 3.7. The multiplicative product connectivity F-index of $TUC_4C_8(m,n)$ is

$$PFII\,(TUC_4C_8[m,n]) = \left(\frac{1}{4}\right)^{2m} \times \left(\frac{1}{6}\right)^{4m} \times \left(\frac{1}{9}\right)^{12mn-2m}$$

Proof. Put a = $\frac{-1}{2}$ in equation (6), we get the desired result.

In the following theorems, we determine the Multiplicative Atom bond connectivity (ABC) F-index and the Multiplicative Geometric-Arithmetic (GA) F-index of $TUC_4C_8(m,n)$ nanotubes.

Theorem 3.8. The multiplicative Atom Bond Connectivity F-index of $TUC_4C_8(m,n)$ is

$$ABCFII\,(TUC_4C_8[m,n]) = \left(\frac{3}{8}\right)^{m} \times \left(\frac{11}{36}\right)^{2m} \times \left(\frac{4}{9}\right)^{12mn-2m}$$

Proof. Let G = $TUC_4C_8(m,n)$. From equation (3) and by cardinalities of edge partition of G, we have

$$ABCFII(G) = \prod_{uv\in E(G)} \sqrt{\frac{d_G(u)^2 + d_G(v)^2 - 2}{d_G(u)^2\, d_G(v)^2}}$$

$$= \prod_{uv\in E_4^*}\left(\sqrt{\frac{2^2+2^2-2}{2^2\times 2^2}}\right) \times \prod_{uv\in E_6^*}\left(\sqrt{\frac{2^2+3^2-2}{2^2\times 3^2}}\right) \times \prod_{uv\in E_9^*}\left(\sqrt{\frac{3^2+3^2-2}{3^2\times 3^2}}\right)$$

$$= \left(\frac{3}{8}\right)^m \times \left(\frac{11}{36}\right)^{2m} \times \left(\frac{4}{9}\right)^{12mn-2m}$$

Theorem 3.9. The multiplicative Geometric-Arithmetic F -index of $TUC_4C_8(m,n)$ is

$$GAFII\ (TUC_4C_8[m,n]) = \left(\frac{12}{13}\right)^{4m}$$

Proof. Let G = $TUC_4C_8(m,n)$. From equation (4) and by cardinalities of edge partition of G, we have

$$GAFII(G) = \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)^2\, d_G(v)^2}}{d_G(u)^2 + d_G(v)^2}$$

$$= \prod_{uv\in E_4}\left(\frac{2\sqrt{2^2\times 2^2}}{2^2+2^2}\right) \times \prod_{uv\in E_5}\left(\frac{2\sqrt{2^2\times 3^2}}{2^2+3^2}\right) \times \prod_{uv\in E_6}\left(\frac{2\sqrt{3^2\times 3^2}}{3^2+3^2}\right)$$

$$= \left(\frac{12}{13}\right)^{4m}$$

Conclusion

In this paper, some new topological indices for a family of Carbon Nanotubes namely $TUC_4C_8(m,n)$ Nanotubes were investigated. These multiplicative groups of F-indices are useful for surveying structure of some connected molecular graphs and nanostructures, which is based on degrees of their vertices/edges.

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