# STEADY STATE DISPERSION OF AIR POLLUTANT FROM A POINT SOURCE IN A TWO PATCH ATMOSPHERE

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***Abstract :*** Over the years mathematical modellers and atmospheric scientists have laid considerable emphasis on studying and modelling various atmospheric processes so as to estimate pollutant concentration in the atmosphere and adopt several methods for reduction of it. For the purpose in this paper a mathematical model is developed in two patches with variable wind speed, diffusivity and removal term. It is supposed that uniformly distributed sinks are present in second patch. It is also supposed that the ground surface and inversion layer are impermeable. From the above discussion it may be concluded that the presence of sink phase in the second patch in the form of rain droplets, fog droplets or even extremely introduced species (liquid or gas) can be very effective in removing the pollutants from the atmosphere. The analysis also suggests a mechanism by which toxic gases leaked out in the atmosphere due to accidental discharge etc. can be removed by introducing suitable liquid or gaseous phase in the environment using mechanical means. It can also be utilized for studying risk analysis particularly in cases of accidental leakages of pollutants / toxicants in the atmosphere.

***IndexTerms* – Mathematical modelling, pollution, two patch atmosphere, point source.**

## I. INTRODUCTION

There exist natural removal mechanisms (fog, droplets, vegetative canopies etc.) in environment, which play a vital role in keeping the level of pollution under control. In order to develop meaningful models, it is necessary to characterize mathematically these complex chemical and physical processes. Several investigations have been made by mathematicians (Pasquill [1]; Alam and Sienfeld [2]; Meeder and Nieuwstadt [3]; Malcolm et.al., [4]) to study the dispersion of air pollutants in the atmosphere taking removal mechanisms into consideration.

As in real situations, the ecological conditions on the ground and meteorological conditions in the atmosphere are different at different places leading to patchiness in the environment. The effect of variable wind velocity and diffusion coefficients on the dispersion of air pollutants can also be studied by splitting the environment in patches. Shukla and Chauhan [5], have studied the effect of grrenbelt on reduction of pollutant concentration by making use of two layered environment. Also in order to study the removal of pollutants by artificial means, the existence of a patchy atmosphere may be assumed (Shukla et. al., [6]). But these works were done by considering constant wind velocity, diffusivity and removal parameters, while it is often seen that the above parameters changes from place to place i.e., having dependent nature.

In this paper, a study is done by steady state dispersion of air pollutant emitted from a point source, taking into account the effect of removal mechanism in a two patch environment, assuming the existence of sinks uniformly distributed throughout the second patch causing removal of air pollutants from the atmosphere. In the atmosphere, these sinks may represent rain droplets, fog droplets or even externally introduced species (liquid or gas) which are effective in the removal of pollutants by several process such as chemical reactions etc. This problem have been studied by solving coupled three dimensional steady state convective diffusion equations involving the pollutants with removal term taking as a dependent function of vertical height incorporating the zero flux condition at the ground and at the top of inversion layer with suitable matching condition at the interface of two patches. The effect of removal mechanism in the second patch on the concentration of pollutants is discussed in above case.

## II. MATHEMATICAL MODEL

Consider the steady state dispersion of air pollutant from a point source of strength Q located at height $h_s$ from the ground z = 0 in a two patch environment under atmospheric inversion as shown in figure 1. It is assumed here that point source is located in the first patch, with removal negligible in comparison to second patch, whereas there exist a phase of uniformly distributed sinks depending on vertical height which are responsible for removing pollutants from the atmosphere. In the first and second patches, the two boundaries i.e., ground and inversion layer are assumed to be totally reflective. Under these assumptions, the partial differential equations governing the concentration of pollutants in both the patches, with appropriate initial and boundary conditions can be written as follows,

**Patch – 1** $(0 \le x \le x_1$ , $0 \le z \le H)$

The differential equation governing the concentration $C_1(x, y, z)$ of the pollutant in the first patch is given by,

$$u_1(z)\frac{\partial C_1}{\partial x} = K_{y_1}(z)\frac{\partial^2 C}{\partial y^2} + \frac{\partial}{\partial z}\left(K_{z_1}(z)\frac{\partial C_1}{\partial z}\right), \qquad (1)$$

where, x, y, z are cartesian coordinates, x–axis is taken in the downwind direction and z–axis vertically upwards, $u_1(z)$ is mean wind velocity and $K_{y_1}$ and $K_{z_1}(z)$ are diffusivities in y and z direction respectively in patch 1, taken as function of z.

Here, the wind is assumed to be blowing in the x direction, the turbulent fluxes are approximated by gradient transport and the turbulent diffusion is neglected as compared to advection since the turbulent transport is smaller than plume dimensions. The following initial and boundary conditions for equation (1) are taken into consideration,

I. A point source of strength Q is assumed to be located at $(0, 0, h_s)$ as,

$$C_1(x,y,z) = \frac{Q}{u_1(z)}\delta(y)\delta(z-h_s), \quad \text{at } x = 0, \tag{2}$$

where, 'δ' is Dirac–delta function.

II. At infinite lateral distance, the concentration of pollutant is assumed to approach zero as,

$$C_1(x, y, z) = 0, \qquad y \to \pm\infty. \tag{3}$$

III. The pollutants are assumed to be reflected back in the atmosphere from the ground surface z = 0, i.e.,

$$K_{z_1}\frac{\partial C_1}{\partial z} = 0, \qquad z = 0. \tag{4}$$

IV. The pollutants are reflected by inversion layer situated at height H, i.e.,

$$K_{z_1}\frac{\partial C_1}{\partial z} = 0, \qquad z = H. \tag{5}$$

**Patch – 2** $(x \geq x_1, 0 \leq z \leq H)$

The differential equation governing the concentration $C_2(x, y, z)$ of the pollutant in the second patch is given by,

$$u_2(z)\frac{\partial C_2}{\partial x} = K_{y_2}(z)\frac{\partial^2 C_2}{\partial y^2} + \frac{\partial}{\partial z}\left[K_{z_2}(z)\frac{\partial C_2}{\partial z}\right] - R(z)C_2, \tag{6}$$

where, $u_2(z)$ is mean wind velocity and $K_{y_2}(z)$ and $K_{z_2}(z)$ are diffusivities in y and z direction respectively, taken as function of z, R(z) is removal rate coefficient.

The initial and boundary conditions for equation (6) are assumed as,

$$C_2(x, y, z) = C_1(x, y, z), \qquad x = x_1, \tag{7}$$

$$C_2(x, y, z) = 0, \qquad y \to \pm\infty, \tag{8}$$

$$K_{z_2}\frac{\partial C_2}{\partial z} = 0, \qquad z = 0, \tag{9}$$

$$K_{z_2}\frac{\partial C_2}{\partial z} = 0, \qquad z = H, \tag{10}$$

where, the condition (7) represents the matching condition at the boundary of two patches $x = x_1$.

Here, in this paper, the wind speed $u_i(z)$, eddy diffusivities $K_{y_i}(z)$ and $K_{z_i}(z)$ (i = 1, 2) and removal rate coefficient R(z) are assumed to vary with height and taken as following law profiles;

$$\left.\begin{array}{l} u_i(z) = \dfrac{u_H z^{\alpha}}{H^{\alpha}}, \quad K_{y_i} = \dfrac{K_H z^{\gamma}}{H^{\gamma}}, \quad K_{z_i} = \dfrac{K_H z^{\beta}}{H^{\beta}} \\ \text{and} \\ \\ \end{array}\right\}$$

$$R(z) = R_0 + R_1 z^{\delta} \quad , \tag{11}$$

where, $u_H$ and $K_H$ are wind speed and eddy diffusivities respectively at the reference height H. For simplicity, here it is assumed that, $\alpha = \beta = y = \delta = 1/2$.

It is convenient to cast the problem in dimensionless form. For the purpose, following dimensionless quantities are assumed,

$$\bar{x} = \frac{K_H x}{u_H H^2} \quad , \quad \bar{y} = \frac{y}{H} \quad , \quad \bar{z} = \frac{z}{H} \quad , \quad \bar{h}_s = \frac{h_s}{H} \quad ,$$

$$\bar{C}_i = \frac{u_H H^2}{Q_0} C_i \quad , \quad \bar{Q} = \frac{Q}{Q_0} \quad , \quad \bar{\delta} = \delta H \, ,$$

where, $K_H$ and $u_H$ are reference diffusion coefficient and wind velocity respectively.

The equations can be rewritten in dimensionless form, after removing the bars for convenience, as follows;

**Patch 1**

$$\frac{\partial C_1}{\partial x} = \frac{\partial^2 C_1}{\partial y^2} + \frac{1}{2z}\frac{\partial C_1}{\partial z} + \frac{\partial^2 C_1}{\partial z^2} \, . \tag{12}$$

B.C.

$$C_1 = \frac{Q}{\sqrt{z}}\delta(z - h_s)\delta(y) \quad , \quad x = 0, \tag{13}$$

$$C_1 = 0 \quad , \quad y \to \pm\infty, \tag{14}$$

$$\frac{\partial C_1}{\partial z} = 0 \quad , \quad z = 0, \tag{15}$$

$$\frac{\partial C_1}{\partial z} = 0 \quad , \quad z = 1. \tag{16}$$

**Patch 2**

$$\frac{\partial C_2}{\partial x} = \frac{\partial^2 C_2}{\partial y^2} + \frac{1}{2z}\frac{\partial C_2}{\partial z} + \frac{\partial^2 C_2}{\partial z^2} - \frac{R_0}{\sqrt{z}} C_2 - R_1 C_2 \, . \tag{17}$$

B.C.

$$C_2 = C_1 \quad , \quad x = x_1, \tag{18}$$

$$C_2 = 0 \quad , \quad y \to \pm\infty, \tag{19}$$

$$\frac{\partial C_2}{\partial z} = 0 \quad , \quad z = 0, \tag{20}$$

$$\frac{\partial C_2}{\partial z} = 0 \quad , \quad z = 1. \tag{21}$$

## III. METHOD OF SOLUTION

On applying Fourier transform with respect to y, the above equations transformed to,

**Patch 1**

$$\frac{\partial \hat{C}_1}{\partial x} = -p^2\hat{C}_1 + \frac{\partial^2 \hat{C}_1}{\partial z^2} + \frac{1}{2z}\frac{\partial \hat{C}_1}{\partial z}. \tag{1}$$

B.C. $$\hat{C}_1 = \frac{Q}{\sqrt{z}\sqrt{2\pi}}\delta(z-h_s) \quad , \quad x = 0, \tag{2}$$

$$\hat{C}_1 = 0 \quad , \quad y \to \pm\infty, \tag{3}$$

$$\frac{\partial \hat{C}_1}{\partial z} = 0 \quad , \quad z = 0, \tag{4}$$

$$\frac{\partial \hat{C}_1}{\partial z} = 0 \quad , \quad z = 1, \tag{5}$$

and in patch 2,

**Patch 2**

$$\frac{\partial \hat{C}_2}{\partial x} = -p^2\hat{C}_2 + \frac{1}{2z}\frac{\partial \hat{C}_2}{\partial z} + \frac{\partial^2 \hat{C}_2}{\partial z^2} - \frac{R_0}{\sqrt{z}}C_2 - R_1C_2. \tag{6}$$

B.C. $$\hat{C}_2 = \hat{C}_1 \quad , \quad x = x_1, \tag{7}$$

$$\hat{C}_2 = 0 \quad , \quad y \to \pm\infty, \tag{8}$$

$$\frac{\partial \hat{C}_2}{\partial z} = 0 \quad , \quad z = 0, \tag{9}$$

$$\frac{\partial \hat{C}_2}{\partial z} = 0 \quad , \quad z = 1. \tag{10}$$

where,

$$\hat{C}_i = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} C_i e^{ipy}\,dy, \text{ (i=1,2).}$$

Now, for solving equation (1), in patch 1, applying variable separable method, let,

$$\hat{C}_1 = M(x)\,N(z), \tag{11}$$

we get,

$$\hat{C}_1 = e^{-(p^2+\alpha^2)x}\left[A_1\left\{1-\frac{\alpha^2}{2.\frac{3}{2}}z^2+\dots\right\}+B_1\left\{\sqrt{z}-\frac{\alpha^2 z^{5/2}}{\frac{5}{2}.2}+\dots\right\}\right] \tag{12}$$

where, $A_1$ and $B_1$ are arbitrary constants.

To determine constants $A_1$ and $B_1$ applying boundary condition (5.3.4) to equation (5.3.20), it is obtained that,

$$C_1 = \sum_{n=1}^{\infty} \frac{Q}{4\sqrt{\pi}x} \frac{P_n(h_s)}{\int_0^1 \sqrt{z} P_n^2(z)dz} e^{-\alpha_n^2 x - y^2/4x} P(z). \tag{13}$$

Now for patch 2, applying variable separable method to equation (2), the two separate differential equations may be written as;

$$\frac{dS}{dx} = -(p^2 + R_1 + \beta^2)S, \tag{14}$$

and

$$\frac{d^2T}{dz^2} + \frac{1}{2z}\frac{dT}{dz} - \frac{R_0}{\sqrt{z}}T + \beta^2 T = 0 , \tag{15}$$

where, β is the separation constant.

On solving it we get that,

$$\hat{C}_2 = e^{-(p^2+R_1+\beta^2)x}\left[U'\left\{1 + \frac{2R_0\left(2\sqrt{z}\right)^3}{4.2.3} + ....\right\}\right.$$

$$\left. + V'\left(2\sqrt{z}\right)\left\{1 + \frac{2R_0\left(2\sqrt{z}\right)^3}{4.3.4} + ...\right\}\right] \tag{16}$$

where, $U'$ and $V'$ are arbitrary constants.

After finding constants, the required concentration of pollutant in second patch is obtained as,

$$C_2 = \frac{Qe^{-y^2/4x}}{4\sqrt{\pi}x} P(z) \sum_{m=1}^{\infty} \exp[-(R_1 + \beta_m^2)(x - x_1)] \frac{S(z)}{S_m(z)}.$$

$$\sum_{n=1}^{\infty} \exp(-\alpha_n^2 x_1) \frac{P_n(h_s)}{\int_0^1 \sqrt{z} P_n^2(z)dz}. \tag{17}$$

## IV. RESULT AND DISCUSSION

To analyze the variation in concentration of pollutant in the region of interest, certain plots are drawn for different values of parameters. The values of parameters used in this case in dimensionless form are as follows, (Alam and Sienfeld, [2]);

$u_H = 1.0$, $h_s = 0.20$.

Figure (2), illustrates the change in concentration of pollutants with respect to the crosswind distance ($0 \le y \le 0.4$) for different downwind distances ($0.02 \le x \le 0.01$) at constant vertical height ($z = 0.6$) in patch 1. It is seen that at $x = 0.02$, concentration of pollutant decreases steeply for $y \le 0.25$, while for $y > 0.25$, the concentration profile is almost uniform. As downwind distance increases, the concentration of pollutant approaches a uniform distribution with extended spreading.

In figure (3), concentration profile are plotted against crosswind distance for different downwind location at vertical height z = 0.6 in path 2. Here, a steep decrease is occurred in concentration level at x = 0.06 which remains almost uniform for y > 0.8. Since in this patch, sinks are assumed to be uniformly distributed in the atmosphere, to account for natural or artificial removal processes. Therefore it can be seen easily that, the value of concentration of pollutants is much lower as compared to figure (2) for different crosswind locations. It can also be seen that concentration level decreases at faraway downwind distances due to removal process.

Figure (4), depicts the variation of concentration of pollutants along vertical distance for different downwind distances at constant crosswind location (y=0.6). It can be seen that, as the vertical distance increases, concentration of pollutant also increases and has its maximum value at z = 0.22, which is near to the source and after that there is reduction in pollutant concentration. It is also observed that, as downwind distance increases, the peak value decreases and at higher value of x there is almost a straight line without peak, i.e., introduction of sinks reduces the concentration level to a considerable amount.

In figure (5), the variation of pollutants concentration with respect to crosswind distance for different vertical height ($0 \leq z \leq 0.6$), at constant downwind distance (x = 0.06) is studied. It is to be noted here that patch width is taken as $x_1 = 0.5$ i.e., after this downwind location there exist uniformly distributed sinks, which play a vital role in removing pollutants. It is seen that at z = 0 there is steep decrease in concentration level for $y \leq 0.6$, while for $y > 0.6$ the concentration profile is almost uniform. Also it can be depicted that, there is a significant decrease in concentration level as vertical distance increases.

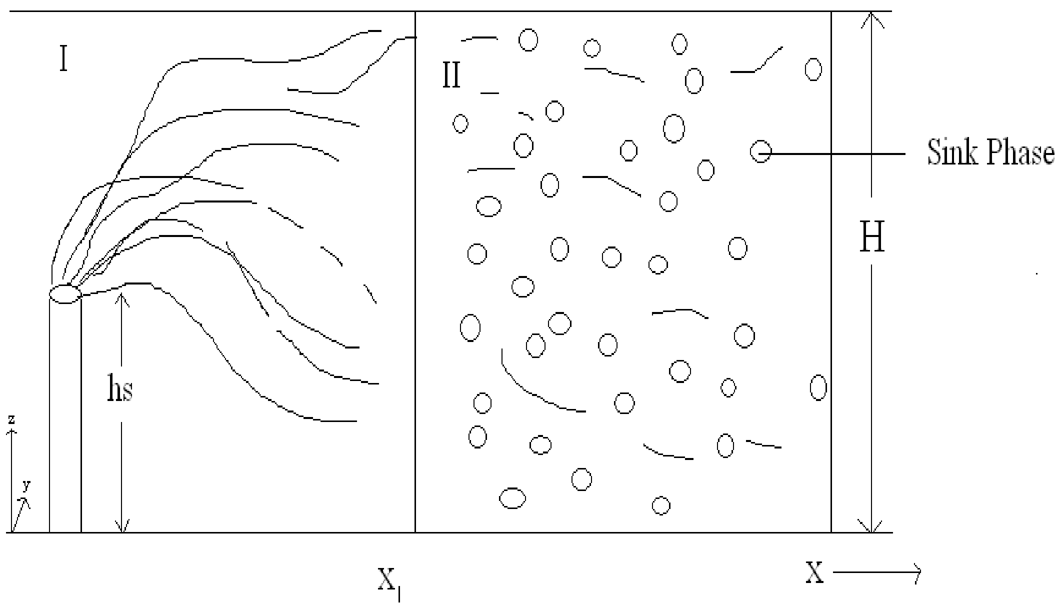


**Figure (1):** Dispersion of air pollutant in a two patch environment.

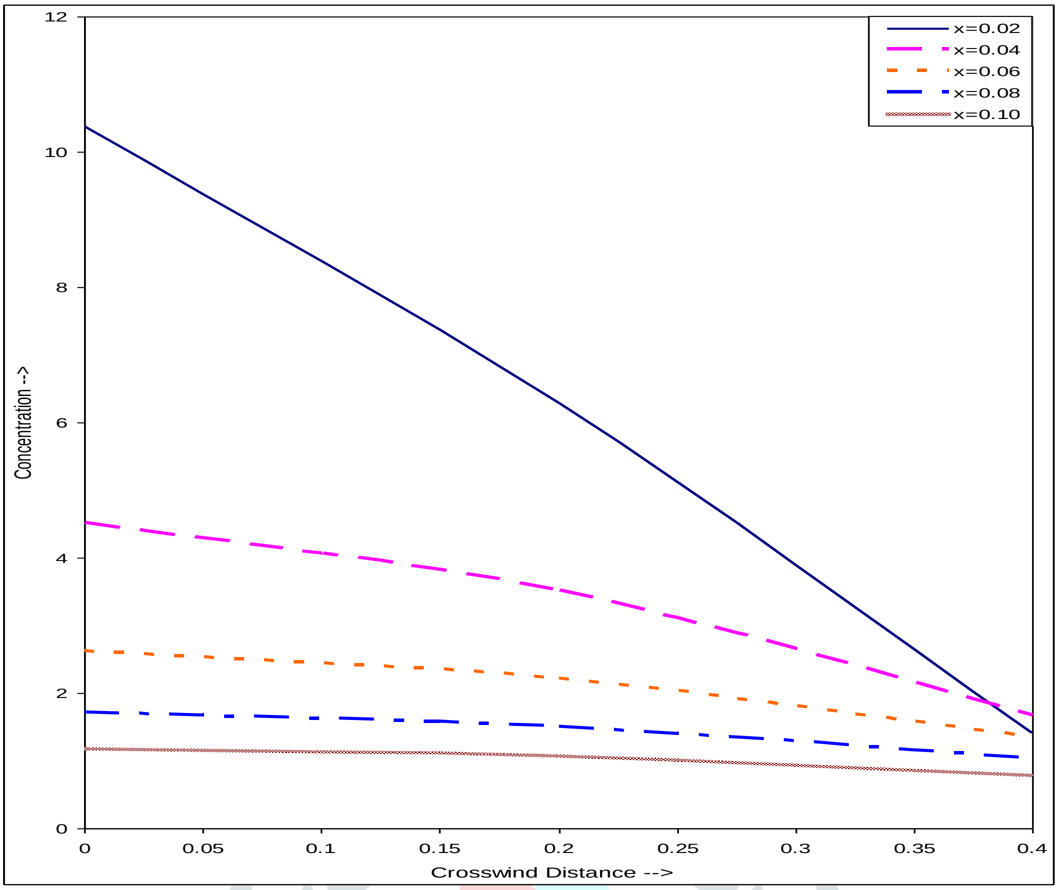


**Figure (2):** Variation of concentration in region one at different crosswind distances with constant z=0.6 for different downwind distances.

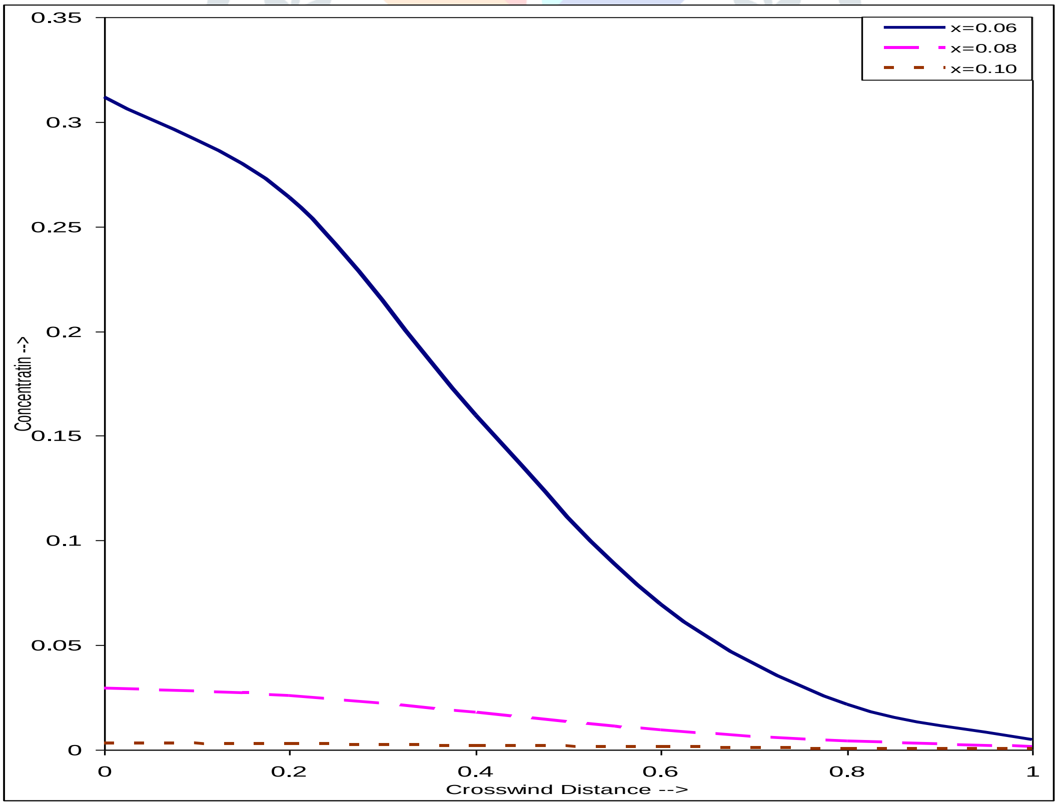


**Figure (3):** Variation of concentration in region two at different crosswind distances with constant z=0.6 for different downwind distances.

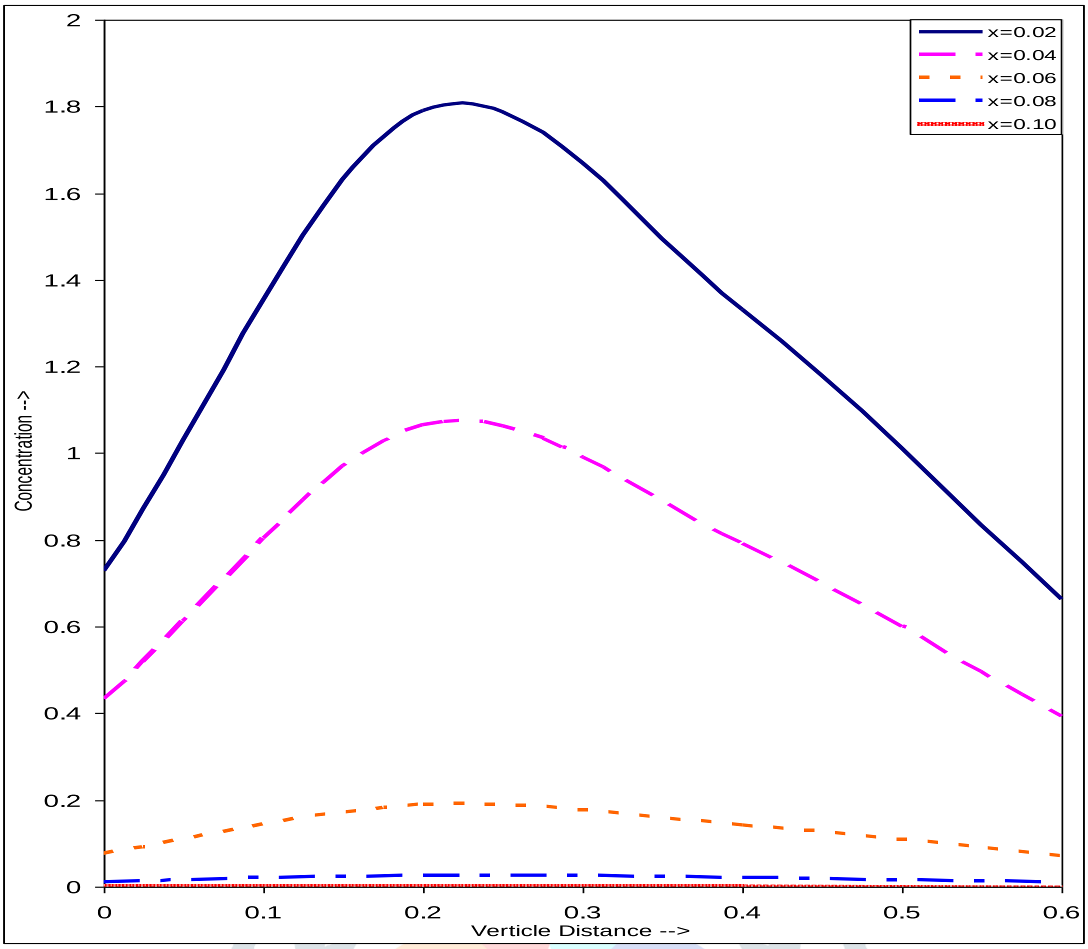


**Figure (4):** Variation of concentration in region two at different vertical distances with constant y=0.6 for different downwind distances.

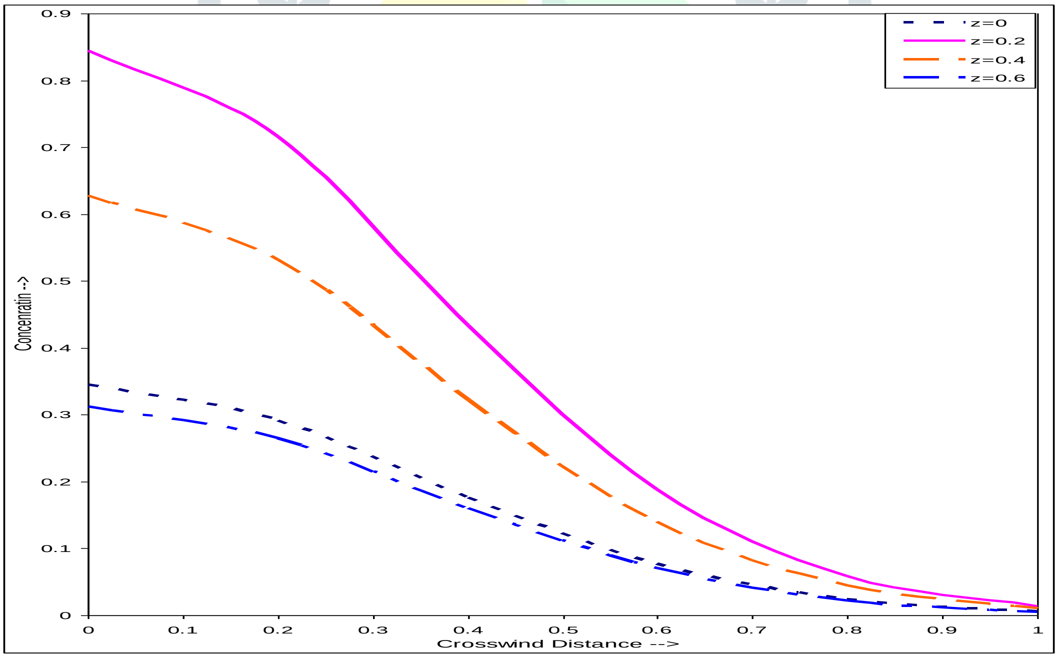


**Figure (5):** Variation of concentration in region two at different crosswind distances with constant x=0.06 for different vertical distances.

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