Fuzzy programming technique with new exponential membership function for the solution of multiobjective transportation problem with mixed constraints

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Abstract: Multiobjective transportation problem with mixed constraints is one of the special class of vector minimum problem in which the objective functions are conflicting in nature, non commensurable and constraints are inequality and equality type. In this paper, we have proposed a method to obtain a solution of multiobjective transportation problem with mixed constraints in which the availability and/or demand constraints are in-equations instead of usual equations using fuzzy programming technique with new exponential membership function. This method gives efficient solutions and the best compromise solution for the multiobjective transportation problem with mixed constraints. LINGO software is used to find the best compromise solution. The proposed algorithm is illustrated by a numerical example.

Key words: multiobjective transportation problem with mixed constraints, fuzzy linear programming, membership function, best compromise solution.

Mathematical Subject Classifications[2010]: 90B06, 90C05, 90B50, 90C11, 90C70

1 Introduction

The purpose of classical single objective transportation problem model with equality constraints is to minimise the total shipping cost which satisfy total supply and demands. This model discuss about the fixed amount of supply and demand. But in real life, most of the problems have inequality and equality type constraints with multiple objectives for example in production inventory, job scheduling, allocation problems and investment analysis.

Appa [4] has discussed about the single objective transportation problem and its variants. He considered 81 problems by taking all combinations of the form of coefficients of objective function, supply constraints, demand constraints and relation of total supply and total demand. He studied 54 problems by eliminating repetitive cases and discussed its solution by converting the inequality constraints into equality constraints. Kligman and Russel [9] transformed the single objective transportation problem with mixed constraint into an equivalent transshipment problem and then the transshipment problem is replaced by standard transportation problem. Brigden [7] has given the necessary and sufficient conditions for the existence of a feasible solution of single objective transportation problem with mixed constraints. He transformed the transportation problem with mixed constraints into classical single objective transportation problem by adding two more constraints and obtained the optimal solution. But Iserman [8] got the solution by adding only one column constraint and one row constraint in transportation problem with mixed constraints of size $m \times n$.

Adlakha and Kowalski [2] provided an heuristic algorithm to find more for less solution of classical and fixed charge transportation problem. Next Adlakha et al. [3] provided a heuristic algorithm to find a solution for transportation problem with mixed constraints where more for less parodox exists. Pandian and Natarajan [12,13] developed a new approach which uses shadow prices (Modi Index) for solving transportation problem with mixed constraints and also for finding an optimal more for less solution. Mondal et al [10] obtained optimal solution of a single objective transportation problem with mixed constraints by using an algorithm based on VAM method. Acharya et al [1] has developed an efficient algorithm for finding more-for-less paradoxical solution under fuzzy environments. Pandian and Anuradha [11] developed path method for finding an optimal more for less solution to a transportation problem.

2 Mathematical model for the multiobjective transportation problem with mixed constraints

In real world, all transportation problem with mixed constraints are not always single objective type. We have more than one objective function in transportation problem with mixed constraints.

Consider m origins O_i (i = 1,2,...., m) and n destinations D_j (j = 1,2,..., n). At each origin O_i, let a_i be the quantity of a homogeneous product which we want to transport to j^{th} destinations D_j to satisfy the demand for b_j units of the product there. Z_k is the k^{th} objective function and c^k_{ij} is the k^{th} penalty criterion. The penalty could represent transportation cost, delivery time, under used capacity, quantity of goods delivered etc from the i^{th} supply point to the j^{th} destination of k^{th} objective Z_k. A variable x_{ij} represents the unknown quantity to be transported from origin O_i to destination D_j.

The mathematical model for the multiobjective transportation problem with mixed constraints is given as follows:

$$\text{Minimize } Z_k=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{k} x_{ij} \qquad k = 1,2,....,K \tag{2.1}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i \in I_1 = \{1,2,...\ m_1\} \tag{2.2}$$

$$\sum_{j=1}^{n} x_{ij} \geq a_i, \quad i \in I_2 = \{m_1 + 1,...,m_2\} \tag{2.3}$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i, \quad i \in I_3 = \{m_2 + 1,..., m\} \tag{2.4}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j \in J_1 = \{1,2,...,n_1\} \tag{2.5}$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j, \quad j \in J_2 = \{n_1 + 1,..., n_2\} \tag{2.6}$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j, \quad j \in J_3 = \{n_2 + 1,...,n\} \tag{2.7}$$

$$x_{ij} \geq 0 \tag{2.8}$$

where, $I_1 \cup I_2 \cup I_3 = \{1,2,....,m\}$ = I= the index set for supply points

and $J_1 \cup J_2 \cup J_3 = \{1,2,.....,n\}$ = J=the index set for destinations;

with $a_i > 0, \forall i \in I$; $b_j > 0, \forall j \in J$ and $c^k_{ij} \geq 0, \forall i \in I, \forall j \in J$ and k = 1, 2,...., K

If k = 1, in (2.1), it gives the single objective transportation problem with mixed constraints. The feasibility conditions are as per Brigden [7]. The model is useful for many practical purposes such as investigating the effect of increasing and decreasing the availability at various origins and/or increasing and decreasing the requirements at various destinations.

3 Algorithm for obtaining the best compromise solution of multiobjective transportation problem with mixed constraints

Bit et al [6] developed fuzzy programming technique with linear membership function for the solution of multiobjective transportation problem which gives efficient solution as well as the best compromise solution. We propose here the fuzzy programming technique with new exponential membership function to solve multi-objective transportation problem with mixed constraints. It also gives the best compromise solution. The proposed algorithm is as follows:

Step 1: Solve the multiobjective transportation problem with mixed constraints as a single objective transportation problem with mixed constraints using only one objective (ignore all others) each time by any method given by (Adlakha et al [3], Pandian and Natrajan [12, 13] and Mondal et al [10]. Let x_i^* be the optimal solution for the i^{th} single objective transportation problem with mixed constraints, where i = 1, 2, 3,..., K.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From the results of step 2, determine the lower bound value (L_k) and upper bound value (U_k) for each objective from the following pay off matrix.(The diagonal of the pay off matrix constitutes the individual ideal solution for the each objective).
The pay off matrix is:

	x_1^*	x_2^*	……	x_K^*
Z_1	$Z_1(x_1^*)$	$Z_1(x_2^*)$	……	$Z_1(x_K^*)$
Z_2	$Z_2(x_1^*)$	$Z_2(x_2^*)$	……	$Z_2(x_K^*)$
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
Z_K	$Z_K(x_1^*)$	$Z_K(x_1^*)$	…….	$Z_K(x_K^*)$

where, Lower bound = $L_t = \{Z_t(x_t^*)\}$;
and Upper bound = $U_t = Max\{ Z_t(x_1^*), Z_t(x_2^*), ..., Z_t(x_K^*)\}$
$\forall$ t=1,2,...,K

Step 4: From Step 3, we find the lower bound (L_k) and the upper bound (U_k) corresponding to the sets of k, k = 1, 2,...., K solutions. Each column of pay off matrix represents a nondominated solution. Thus we get K nondominated solutions from the pay off matrix. The initial fuzzy model of multiobjective transportation problem with mixed constraint [(2.1) to (2.8)] is given as:

Find x_{ij} $\{\forall i \in I$ and $\forall j \in J\}$
such that
$Z_k \lesssim L_k$ k = 1, 2,....,K
and constraints (2.2)-(2.8)

Step 5: A new exponential membership function (Paratane and Bit [14]) $\mu_k(Z_k(x))$ for k^{th} objective function is defined as:

$$\mu_k(Z_k(x)) = \begin{cases} 1 & ; \ if\ Z_k(x) \le L_k \\ e^{-\alpha\left(\frac{Z_k(x)-L_k}{U_k-L_k}\right)^n} & ; \ if\ L_k < Z_k(x) < U_k \\ 0 & ; \ if\ Z_k(x) \ge U_k\ and\ \alpha \to \infty \end{cases}$$

where α is the non zero parameter prescribed by the decision maker, n = 2,4,6,…,12. (Here $U_k \neq L_k$, because if $U_k = L_k$, then $\mu_k(Z_k(x)) = 1$)

Step 6: Then the equivalent nonlinear programming model for multiobjective transportation problem with mixed constraints (2.1)-(2.8) is as follows:

Maximize λ

subject to

$\lambda \le e^{-\alpha\left(\frac{Z_k(x)-L_k}{U_k-L_k}\right)^n}$ for all k=1, 2, ..., K

and constraints (2.2)-(2.8)

$\lambda \ge 0$

where $\lambda = \min\{\mu_1(Z_1(x)), \mu_2(Z_2(x)), \ldots\ldots, \mu_K(Z_K(x))\}$

We can solve this crisp model by existing nonlinear programming algorithm or LINGO software. We get the values of decision variables and λ after solving this nonlinear programming problem. Then substituting these values of decision variables in each $Z_k(x)$, k=1,2,...,K, we get the best compromise values of the objectives.

Step 7: Further in step 6, we consider $x_{ij} \ge 0$, $\forall\ i \in I$, $\forall\ j \in J$ and integers as the values of a_i's, $\forall\ i \in I$ and b_j's, $\forall\ j \in J$ are positive integers and then we apply fuzzy mixed integer nonlinear programming technique to get integer values of x_{ij}.

4 Numerical Example

Consider the multiobjective transportation problem with mixed constraints [P1]:

$$\text{Min } Z_1 = 10x_{11} + x_{12} + 7x_{13} + 5x_{21} + 7x_{22} + x_{23} + 8x_{31} + 9x_{32} + 2x_{33}$$

$$\text{Min } Z_2 = 2x_{11} + 5x_{12} + 4x_{13} + 6x_{21} + 3x_{22} + x_{23} + 8x_{31} + 9x_{32} + 2x_{33}$$

subject to (4.1)

$$\sum_{j=1}^{3} x_{1j} = 5; \quad \sum_{j=1}^{3} x_{2j} \ge 6; \quad \sum_{j=1}^{3} x_{3j} \le 9 \qquad (4.2)$$

$$\sum_{i=1}^{3} x_{i1} = 8; \quad \sum_{i=1}^{3} x_{i2} \ge 10; \quad \sum_{i=1}^{3} x_{i3} \le 5 \qquad (4.3)$$

$$x_{ij} \ge 0; \quad i = 1, 2, 3; \quad j = 1, 2, 3 \qquad (4.4)$$

Step 1: Considering each objective separately, we get value of each objective as:

$Z_1(x_1^*) = 80$; where x_1^* : $x_{12} = 5$; $x_{21} = 8$; $x_{22} = 5$ (V. Adlakha et al [3])

$Z_2(x_2^*) = 58$; where x_2^* : $x_{11} = 5$; $x_{21} = 3$; $x_{22} = 10$ (Adlakha et al [3],Mondal et al [10])

Step 2: Substituting x_1^* in $Z_2(x)$ and x_2^* in $Z_1(x)$, we get $Z_2(x_1^*) = 88$ and $Z_1(x_2^*) = 135$.

Step 3: The pay off matrix is:

	x_1^*	x_2^*
Z_1	80	135
Z_2	88	58

Step 4: The upper and lower bound values for $Z_1(x)$ and $Z_2(x)$ are:

For $Z_1(x)$: $U_1 = 135$; $L_1 = 80$; and $U_1 - L_1 = 55$

For $Z_2(x)$: $U_2 = 88$; $L_2 = 58$; and $U_2 - L_2 = 30$

From this table, we get the non dominated solutions of [P1] as (80,88) and (135,58).

Step 5: Let the values of α and n be $\alpha = 2$ and $n = 4$, Then the membership function value for each objective is given as:

$$\mu_1(Z_1(x)) = e^{-2\left(\frac{Z_1(x)-80}{55}\right)^4}$$ and $$\mu_2(Z_2(x)) = e^{-2\left(\frac{Z_2(x)-58}{30}\right)^4}$$

Step 6: Then the equivalent nonlinear programming problem for [P1] is :

Maximize λ

subject to

$$\lambda \le e^{-2\left(\frac{Z_1(x)-80}{55}\right)^4}$$

$$\lambda \le e^{-2\left(\frac{Z_2(x)-58}{30}\right)^4}$$

and constraints (4.2)-(4.4)

$\lambda \ge 0$

Solving this nonlinear programming problem, we get the solution as:

x^*: $x_{11} = 2.5$, $x_{12} = 2.5$, $x_{21} = 5.5$, $x_{22} = 7.5$,

which gives the best compromise solution as $Z_1(x^*) = 107.5$, $Z_2(x^*) = 73$ and $\lambda = 0.8824$.

Also the membership function values of $Z_1(x^*)$ and $Z_2(x^*)$ are $\mu_1(Z_1(x^*)) = 0.8824$ and $\mu_2(Z_2(x^*)) = 0.8824$ respectively.

Step 7: By applying fuzzy mixed integer nonlinear programming technique, we get the integer solution as:

x^*: $x_{11} = 3$, $x_{12} = 2$, $x_{21} = 5$, $x_{22} = 8$ and $Z_1(x^*) = 113$, $Z_2(x^*) = 70$, $\lambda = 0.7716$

And the membership function values are $\mu_1(Z_1(x^*)) = 0.7716$ and $\mu_2(Z_2(x^*)) = 0.95$.

5 The best compromise solution of multiobjective transportation problem with mixed constraints using different membership functions

5.1 Linear membership function:

The linear membership function $\mu_k(Z_k(x))$ (Bit el al [6]) for k^{th} objective function is defined as:

$$\mu_k(Z_k(x)) = \begin{cases} 1 & ; \quad if\ Z_k(x) \le L_k \\ \dfrac{U_k - Z_k(x)}{U_k - L_k} & ; \quad if\ L_k < Z_k(x) < U_k \\ 0 & ; \quad if\ Z_k(x) \ge U_k \end{cases}$$

According to step 6, the equivalent linear programming problem for [P1] using linear membership function is as follows:

Maximize λ

subject to

$$\lambda \le \frac{135 - Z_1(x)}{55}$$

$$\lambda \le \frac{88 - Z_2(x)}{30}$$

with constraints (4.2)-(4.4)

$\lambda \ge 0$

The solution is:

x^*: $x_{11} = 2.5$, $x_{12} = 2.5$, $x_{21} = 5.5$, $x_{22} = 7.5$ which gives the best compromise solution as $Z_1(x^*) = 107.5$, $Z_2(x^*) = 73$ and $\lambda = 0.5$

By applying fuzzy mixed integer linear programming technique, we get:

x^*: $x_{11} = 3$, $x_{12} = 2$, $x_{21} = 5$, $x_{22} = 8$ and $Z_1(x^*) = 113$, $Z_2(x^*) = 70$, $\lambda = 0.4$.

5.2 Exponential membership function:

The exponential membership function $\mu_k(Z_k(x))$ (Zangiabadi and Maleki [15]) for k^{th} objective function is defined as:

$$\mu_k(Z_k(x)) = \begin{cases} 1 & ; \quad if\ Z_k(x) \le L_k \\ \dfrac{e^{-s\psi_k(x)} - e^{-s}}{1 - e^{-1}} & ; \quad if\ L_k < Z_k(x) < U_k \\ 0 & ; \quad if\ Z_k(x) \ge U_k \end{cases}$$

where

$$\psi_k(x) = \frac{Z_k(x) - L_k}{U_k - L_k}$$

According to step 6, the equivalent nonlinear programming problem for [P1] using exponential membership function is as follows:

Let s=1

Maximize λ

subject to

$$\lambda \leq \frac{e^{\left(\frac{80-Z_1(x)}{55}\right)}-e^{-1}}{1-e^{-1}}$$

$$\lambda \leq \frac{e^{\left(\frac{58-Z_2(x)}{30}\right)}-e^{-1}}{1-e^{-1}}$$

with constraints (4.2)-(4.4)

λ ≥ 0

The solution is:

x^*: x_{11} = 2.5, x_{12} = 2.5, x_{21} = 5.5, x_{22} = 7.5 which gives the best compromise solution as $Z_1(x^*)$ = 107.5, $Z_2(x^*)$ = 73 and λ = 0.3775

By applying fuzzy mixed integer linear programming technique, we get:

x^*: x_{11} = 3, x_{12} = 2, x_{21} = 5, x_{22} = 8 and $Z_1(x^*)$ = 113, $Z_2(x^*)$ = 70, λ= 0.2862

5.3 Hyperbolic membership function:

The hyperbolic membership function $\mu_k(Z_k(x))$ (Bit [5]) for k^{th} objective function is defined as:

$$\mu_k(Z_k(x)) = \begin{cases} 1 & ; \ if\ Z_k(x) \leq L_k \\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(\frac{U_k+L_k}{2}-Z_k(x)\right)\alpha_k\right\} & ; \ if\ L_k < Z_k(x) < U_k \\ 0 & ; \ if\ Z_k(x) \geq U_k \end{cases}$$

where

$$\alpha_k = \frac{6}{U_k - L_k}$$

According to step 6, the equivalent nonlinear programming problem for [P1] using hyperbolic member-ship function is as follows:

Maximize = λ

subject to

$$\lambda \leq \frac{1}{2}+\frac{1}{2}\tanh\left\{(107.5 - Z_1(x))\left(\frac{6}{55}\right)\right\}$$

$$\lambda \leq \frac{1}{2}+\frac{1}{2}\tanh\left\{(73 - Z_2(x))\left(\frac{6}{30}\right)\right\}$$

with constraints (4.2)-(4.4)

λ ≥ 0

The solution is:

x^*: x_{11} = 2.5, x_{12} = 2.5, x_{21} = 5.5, x_{22} = 7.5 which gives the best compromise solution as $Z_1(x^*)$ = 107.5, $Z_2(x^*)$ = 73 and λ = 0.5

By applying fuzzy mixed integer linear programming technique, we get:

x^*: x_{11} = 3, x_{12} = 2, x_{21} = 5, x_{22} = 8 and $Z_1(x^*)$ = 113, $Z_2(x^*)$ = 70, λ= 0.23

The ideal solution for this numerical example [P1] is $Z_1(x)$ = 80; $Z_2(x)$ = 58. The membership function values corresponding to this ideal solution are $\mu_1(Z_1(x))$= 1; $\mu_2(Z_2(x))$= 1 in the membership space. Similarly, the membership function value of $Z_1(x)$ and $Z_2(x)$ corresponding to best compromise solution x^* are $\mu_1(Z_1(x^*))$ and $\mu_2(Z_2(x^*))$ respectively in the membership space. The solution is acceptable if the distance d between the points $(\mu_1(Z_1(x^*)),\mu_2(Z_2(x^*)))$ and (1,1) is minimum, where

$$d = [(1-\mu_1(Z_1(x^*)))^2 + (1-\mu_2(Z_2(x^*)))^2]^{\frac{1}{2}}$$

The non dominated solutions of [P1] are (80,88) and (135,58). The distance between the ideal solution and the nondominated solution (80,88) is d = 0.8646. Also the distance between the ideal solution and the nondominated solution (135,58) is d = 0.8646. The best compromise solution of numerical example [P1] obtained by our algorithm is $Z_1(x^*)$ = 107.5, $Z_2(x^*)$ = 73 and the membership function values are $\mu_1(Z_1(x^*))$ = 0.8824 and $\mu_2(Z_2(x^*))$= 0.8824. Thus the distance between the ideal solution and our best compromise solution is d = 0.1661. Also, by applying fuzzy mixed integer nonlinear programming technique, the best compromise values are $Z_1(x^*)$ = 113; $Z_2(x^*)$ = 70 and the distance d = 0.2337. In both of the cases, the distance obtained by our algorithm d = 0.1661 or d = 0.2337 which is less than the distance d = 0.8646. It shows that the values of $Z_1(x)$ and $Z_2(x)$ obtained by our proposed algorithm can be considered as the best values.

6 Conclusion

In this paper, we have shown the application of fuzzy programming technique with new exponential membership function and obtained the best compromise solution for multiobjective transportation problem with mixed constraints. We have obtained noninteger as well as integer solution of the numerical example. In both of the cases, the solution is acceptable by decision maker. And also it is observed that the best compromise solution obtained by our proposed algorithm is same as the best compromise solution obtained using different membership functions in fuzzy programming technique. It shows that the representation of membership function is not unique. The value of membership function of an objective represents the satisfaction level of the objective.

References

[1] D. Acharya, M. Basu, and A. Das. More-for-less paradox in a transportation problem under fuzzy environments. *J Appl Computat Math*, 4(1):1-4, 2015.

[2] V. Adlakha and K. Kowalski. A heuristic method for 'more-for-less' in distribution-related problems. *International Journal of Mathematical Education in Science and Technology*, 32(1):61-71, 2001.

[3] V. Adlakha, K. Kowalski, and B. Lev. Solving transportation problems with mixed constraints. *International journal of management Science and Engineering management,* 1(1):47-52, 2006.

[4] G. M. Appa. The transportation problem and its variants. *Operational Research Quarterly*, pages 79-99, 1973.

[5] A. K. Bit. Fuzzy programming with hyperbolic membership functions for multi-objective capacitated solid transportation problem. *Journal of Fuzzy Mathematics,* 13(1):1-13, 2005.

[6] A. K. Bit, M. P. Biswal, and S. S. Alam. Fuzzy programming approach to multicriteria decision making transportation problem. *Fuzzy sets and Systems*, 50(2):135-141, 1992.

[7] M. E. B. Brigden. A variant of the transportation problem in which the constraints are of mixed type. *Operational Research Quarterly,* pages 437-445, 1974.

[8] H. Isermann. Solving the transportation problem with mixed constraints. Zeitschrift für Operations Research, 26(1):251{257, 1982.

[9] D. Klingman and R. Russell. The transportation problem with mixed constraints. *Operational Research Quarterly,* pages 447-455, 1974.

[10] R. N. Mondal, R. Hossain, and M. K. Uddin. Solving transportation problem with mixed constraints. *In Proceedings of the 2012 International Conference on Industrial Engineering and Operations Management Istanbul, Turkey*, July, volume 3, pages 1927-1932, 2012.

[11] P. Pandian and D. Anuradha. Path method for finding a more-for-less optimal solution to transportation problems. *In International Conference on Mathematical Computer Engineering*, volume 1, pages 331-337, 2013.

[12] P. Pandian and G. Natarajan. A new approach for solving transportation problems with mixed constraints. *Journal of Physical Sciences,* 14:53-61, 2010.

[13] P. Pandian and G. Natarajan. A new method for finding an optimal more-for-less solution of transportation problems with mixed constraints. *Int. J. Contemp. Math. Sciences,* 5(19):931-942, 2010.

[14] P.Paratane and A. K. Bit. Fuzzy programming technique with new exponential membership function for the solution of multiobjective transportation problem. *Journal of Fuzzy Mathematics*, 27(1):259-270, 2019.

[15] M. Zangiabadi and H. R. Maleki. Fuzzy goal programming for multiobjective transportation problems. *Journal of applied mathematics and Computing,* 24(1):449-460, 2007.