SOLUTION OF (3×3) LINEAR SYSTEM OF CONGRUENCES BY USING GAUSS ELIMINATION AND GAUSS JORDAN METHODS

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Abstract: The purpose of this paper is to find the solution of 3×3 linear congruence system by using numerical methods. In Gauss elimination and Gauss Jordan methods the system converted into matrix form. The techniques employed to solve linear system of equations using matrices can be adapted nicely to solve higher order linear system (Congruences).

Keywords:* *Congrurnces,* 3×3 *linear system, Gauss Elimination Method and Gauss Jordan Method.

I. INTRODUCTION

Numerical methods play a vital role to fit an approximate value of a system of linear equations. In number theory, the solutions are always an integer. Gauss Elimination and Jordan methods are two direct methods to find the accurate solution of system of linear equations. Gauss Jacobi and Seidel methods are not suitable to solve the congruences system, since these two are iterative methods. Here considering 3×3 linear system (Congruences) instead of linear equations.

Definition: Congruence

If m is a positive integer, we say the integers a and b are congruent modulo m, and write $a \equiv b \pmod m$, if they have the same remainder on division by m.

Definition: 3×3 linear system of congruences

A 3×3 linear system is a system of linear congruences is of the form

$$a_1x + b_1y + c_1z = d_1 \pmod m$$
$$a_2x + b_2y + c_2z = d_2 \pmod m$$
$$a_3x + b_3y + c_3z = d_3 \pmod m$$

II. METHODOLOGY

Gauss Elimination Method:

Gauss elimination is an algorithm for solving systems of linear equations $AX = B$,

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

In which, the augmented matrix $[A, B]$ is transformed into an upper triangular matrix. Then the solution is obtained by using back substitution method.

Gauss Jordan Method:

In this method, the augmented matrix $[A, B]$ is transformed into diagonal matrix or unit matrix. Then the solution is obtained directly from the resultant matrix.

III. EXAMPLE

Solve $2x - 3y + z \equiv 12 \pmod{13}$, $x + 2y - z \equiv 6 \pmod{13}$ and $3x - y + 2z \equiv 2 \pmod{13}$ by Gauss elimination method and Gauss Jordan method.

Solution:

Gauss Elimination Method:

$$(A,B)\equiv\begin{pmatrix}2 & -3 & 1 & 12(\text{mod}13)\\ 1 & 2 & -1 & 6(\text{mod}13)\\ 3 & -1 & 2 & 2(\text{mod}13)\end{pmatrix}$$

$$\equiv\begin{pmatrix}2 & -3 & 1 & 12(\text{mod}13)\\ 0 & 7 & -3 & 0(\text{mod}13)\\ 0 & 7 & 1 & 7(\text{mod}13)\end{pmatrix} \qquad (\because R_2\to 2R_2-R_1 \quad and \quad R_3\to 2R_3-3R_1)$$

$$\equiv\begin{pmatrix}2 & -3 & 1 & 12(\text{mod}13)\\ 0 & 7 & -3 & 0(\text{mod}13)\\ 0 & 0 & 4 & 7(\text{mod}13)\end{pmatrix} \qquad (\because R_3\to R_3-R_2)$$

By back Substitution method,

From R_3, $4z\equiv 7(\text{mod}13)$, i.e., $z\equiv 5(\text{mod}13)$ ....(1)

From R_2, $7y-3z\equiv 0(\text{mod}13)$, i.e., $y\equiv 4(\text{mod}13)$ (by (1)) ....(2)

From R_1, $2x-3y+z\equiv 12(\text{mod}13)$, i.e., $x\equiv 3(\text{mod}13)$ (by (1) & (2)) ....(3)

Therefore the solution is

$x\equiv 3(\text{mod}13)$

$y\equiv 4(\text{mod}13)$

$z\equiv 5(\text{mod}13)$

Gauss Jordan Method:

$$(A,B)\equiv\begin{pmatrix}2 & -3 & 1 & 12(\text{mod}13)\\ 1 & 2 & -1 & 6(\text{mod}13)\\ 3 & -1 & 2 & 2(\text{mod}13)\end{pmatrix}$$

$$\equiv\begin{pmatrix}2 & -3 & 1 & 12(\text{mod}13)\\ 0 & 7 & -3 & 0(\text{mod}13)\\ 0 & 7 & 1 & 7(\text{mod}13)\end{pmatrix} \qquad (\because R_2\to 2R_2-R_1 \quad and \quad R_3\to 2R_3-3R_1)$$

$$\equiv\begin{pmatrix}2 & -3 & 1 & 12(\text{mod}13)\\ 0 & 7 & -3 & 0(\text{mod}13)\\ 0 & 0 & 4 & 7(\text{mod}13)\end{pmatrix} \qquad (\because R_3\to R_3-R_2)$$

$$\equiv\begin{pmatrix}7 & 0 & -1 & 3(\text{mod}13)\\ 0 & 7 & -3 & 0(\text{mod}13)\\ 0 & 0 & 4 & 7(\text{mod}13)\end{pmatrix} \qquad (\because R_1\to 7R_1+3R_2)$$

$$\equiv\begin{pmatrix}14 & 0 & 0 & 3(\text{mod}13)\\ 0 & 4 & 0 & 3(\text{mod}13)\\ 0 & 0 & 4 & 7(\text{mod}13)\end{pmatrix} \qquad \left(\because R_1\to 4R_1+R_3 \quad and \quad R_2\to \frac{4R_2+3R_3}{7}\right)$$

Then from R_1, $14x\equiv 3(\text{mod}13)$, i.e., $x\equiv 3(\text{mod}13)$

From R_2, $4y\equiv 3(\text{mod}13)$, i.e., $y\equiv 4(\text{mod}13)$

From R_3, $4z\equiv 7(\text{mod}13)$, i.e., $z\equiv 5(\text{mod}13)$

Hence the solution is

$x\equiv 3(\text{mod}13)$

$y\equiv 4(\text{mod}13)$

$z\equiv 5(\text{mod}13)$

Result:

$x \equiv 3 \pmod{13}$

$y \equiv 4 \pmod{13}$

$z \equiv 5 \pmod{13}$

IV. CONCLUSION

Therefore the solution of 3×3 linear system on congruences is unique by using Gauss elimination and gauss Jordan methods.

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