NORMAL AND HIGHER ORDER QUANTUM SQUEEZING AND SNR IN SPONTANEOUS AND STIMULATED INTERACTIONS DURING TEN WAVE MIXING PROCESS

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Abstract**:** The present paper investigates the quantum squeezing in the fundamental mode in spontaneous and stimulated processes during ten wave mixing up to second order Hamiltonian interaction under a short time approximation based on a fully quantum mechanical approach theoretically. The coupled Heisenberg equations of motion involving real and imaginary parts of field quadrature operator are established. The process involves the absorption of five photons of frequency ω_1 and emission of four photons of same frequency ω_2 and one photon of frequency ω_3.The occurrence of amplitude and amplitude- squared squeezing in one of the quadrature of the field amplitude during spontaneous and stimulated interactions are found to be dependent on the selective phase values of the field amplitude of fundamental mode. The degree of squeezing and signal to noise ratio (SNR) is found to be higher in stimulated interaction than during spontaneous process. The photons statistics in the fundamental mode has also been studied and found to be Sub-Poissonian in nature. Comparisons of results for spontaneous and stimulated interactions are represented graphically.

Index Terms **– Quantum Squeezing, Poissonian Statistics, Wave Mixing**

1. INTRODUCTION

In quantum mechanics, the operators corresponding to canonical conjugate components of a single mode field are non-commutative and obey the Heisenberg uncertainty relation. The uncertainty in electromagnetic field quadrature refers to the quantum noise which limits the sensitivity of laser interferometer for precise measurements and optical communications. The state of the electromagnetic field having less value of noise level in one of the quadrature component as compared with that of the coherent light at the expense of larger noise in the complementary quadrature component is called squeezed quantum state. Due to low noise property, the squeezed light has found its potential applications in optical communication networks, high precision interferometric measurements, high resolution laser spectroscopy, gravitational wave detection, quantum teleportation, quantum images etc. (Yonezawa et.al., 2007, Buonanno, 2004, Caves, 1981, Bernnett et. al. 1993, Kempe, 1999). In the past few decades, quantum squeezing has been found in various non linear processes like higher harmonic generation, multi wave mixing, Raman and hyper Raman spectroscopy, parametric amplification etc. theoretically and experimentally (Caves, 1985, Wen et. al., 2016, Wen et. al., 2017, Verma, 2010, Giri et. al., 2010, Jawahar Lal, 1998, delCoso et. al., 2004, Jan Ranger, 2010, Mandel, 1982, Hillery, 1987, Rani, 2011, Giri, 2004, Kumar, 1995, Kumar, 1996, Wallas, 1983, Antonosyan, 2009, Sizmann, 1990, Gevorkyan, 1997, Jawahar Lal, 2014).
Recently, Wen et.al.(2016, 2017) have reported the triple mode squeezing with dressed six wave mixing and high order squeezing and entanglement of harmonic oscillators in superconducting circuits.
The present paper investigates the field amplitude and amplitude- squared quantum squeezing in the fundamental mode in spontaneous and stimulated interactions during ten wave mixing process. The variations of degree of squeezing and signal to noise ratio (SNR) with number of photons have been shown graphically. The Sub-Poissonian statistics of the photons in the fundamental mode has also been studied.

2. DEFINITION AND CONDITION OF SQUEEZING

Squeezed states of an electromagnetic field are the states with reduced noise below the vacuum limit in one of the canonical conjugate quadrature. Consider a single mode of electromagnetic field of frequency ω. Amplitude squeezing is defined in terms of the operators

$$X_1=1/2\ (A+A^{\dagger}) \tag{2.1}$$

and

$$X_2 =1/2i\ (A-A^{\dagger}) \tag{2.2}$$

where X_1 and X_2 are the real and imaginary parts of the field amplitude.

A and $A^{\dagger}$ are slowly time varying operators defined by

$A=a\exp(i\omega t)$ (2.3)

and

$A^{\dagger}=a^{\dagger}\exp(-i\omega t)$ (2.4)

The operators obey the commutation relation

$[X_1, X_2]=i/2$ (2.5)

which leads to the uncertainty relation

$\Delta X_1 \Delta X_2 \geq 1/4$ (2.6)

A quantum state is said to be squeezed in X_i variable if

$\Delta X_i < 1/2$ for i=1or 2. (2.7)

A coherent state is defined as that for which

$(\Delta X_1) = (\Delta X_2) = 1/4$ (2.8)

Similarly, Amplitude-squared squeezing is said to exist in X_i variable for which

$[\Delta X_{iA}(t)]^2 - \langle N+1/2 \rangle < 0$

where N is the usual photon number operator.

3. SQUEEZING IN FUNDAMENTAL MODE AT FREQUENCY ω_1 DURING SPONTANEOUS INTERACTION

The ten wave process involves the absorption of five pump photons of frequency ω_1 and subsequently emission of four photons of frequency ω_2 and one photon having frequency ω_3 such that the system returns to its original state |1> as shown in fig.1.

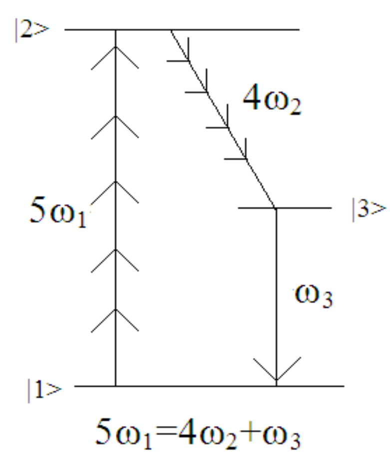

Fig.1 Ten wave interaction model

The Hamiltonian for this process is given as follows ($\hbar=1$)

$H = \omega_1 a^{\dagger}a + \omega_2 b^{\dagger}b + \omega_2 c^{\dagger}c + g(a^{\dagger 5}b^4 c + a^5 b^{\dagger 4} c^{\dagger})$ (3.9)

Here, g is a coupling constant characterizing the properties of the nonlinear medium for ten wave mixing process. $A=a\exp(i\omega_1 t)$, $B=b\exp(i\omega_2 t)$ and $C=c\exp(i\omega_3 t)$, respectively are the slowly time varying operators for the modes of frequencies ω_1,ω_2 and ω_3 .$a(a^{\dagger})$,$b(b^{\dagger})$, $c(c^{\dagger})$ are the usual annihilation (creation) operators with the relation

$5\omega_1=4\omega_2+\omega_3$

The Heisenberg equation of motion for the mode A is

$dA/dt=\partial A/\partial t+i[H,A]$ (3.10)

Using Eq. 3.9 in Eq. 3.10, we obtain

$$\AA = -5igA^{\dagger 4}B^4C \tag{3.11}$$

Similarly, we obtain the relations for dB/dt and dC/dt as

$$\dot{B} = -4igA^5B^{\dagger 3}C^{\dagger} \tag{3.12}$$

$$\dot{C} = -igA^5B^{\dagger 4} \tag{3.13}$$

With the assumption of small interaction time during the process, expanding A(t) in Taylor's series and retaining the terms upto g^2t^2 ($g^2t^2<<1$), we obtain

$$\begin{aligned} A(t)= & A - 5igtA^{\dagger 4}B^4C + 5/2g^2t^2 (-16A^{\dagger 4}A^5B^{\dagger 3}B^3C^{\dagger}C -72A^{\dagger 4}A^5B^{\dagger 2}B^2C^{\dagger}C-96A^{\dagger 4}A^5B^{\dagger}BC^{\dagger}C \\ & -24A^{\dagger 4}A^5C^{\dagger}C -4A^{\dagger 4}A^5B^{\dagger 4}B^4 -16A^{\dagger 4}A^5B^{\dagger 3}B^3-72A^{\dagger 4}A^5B^{\dagger 2}B^2-96A^{\dagger 4}A^5 B^{\dagger}BC^{\dagger}C \\ & -24A^{\dagger 4}A^5 \quad - A^{\dagger 4}A^5 B^{\dagger 4} B^4-16A^{\dagger 4}A^5 B^{\dagger 3} B^3-72A^{\dagger 4}A^5 B^{\dagger 2} B^2-96A^{\dagger 4}A^5 B^{\dagger} B \\ & +20A^{\dagger 3}A^4B^{\dagger 4} B^4C^{\dagger}C+120 A^{\dagger 2} A^3 B^{\dagger 4} B^4 C^{\dagger}C \\ & +240 A^{\dagger} A^2 B^{\dagger 4} B^4 C^{\dagger}C +120 A B^{\dagger 4} B^4 C^{\dagger}C) \end{aligned} \tag{3.14}$$

The real quadrature component for squeezing of field amplitude in the pump mode A is given as

$$X_{1A}(t) = ½[A (t) + A^{\dagger}(t)] \tag{3.15}$$

Initially, we consider the quantum state of the system as a product of coherent state for the mode A

and vacuum states for the modes B and C,

i.e. $\langle\psi|=\langle\alpha|\langle 0|\langle 0|$ (3.16)

where α is the complex field amplitude of the mode A.
Using Equations.3.14 to 3.16, the expectation values for the real quadrature are derived as

$$\begin{aligned} \langle\psi|X^2_{1A}(t)|\psi\rangle= & ¼[\alpha^2+ \alpha^{*2}+ 2|\alpha|^2 +1+5/2g^2t^2(- 48\alpha^2|\alpha|^8- 48\alpha^{*2}|\alpha|^8 \\ & -96 \alpha^2|\alpha|^6- 96 \alpha^{*2}|\alpha|^6-96|\alpha|^{10}] \end{aligned} \tag{3.17}$$

and

$$\langle\psi |X_{1A}(t)|\psi\rangle^2 = ¼[\alpha^2+ \alpha^{*2}+2|\alpha|^2+5/2g^2t^2(-48 \alpha^2|\alpha|^8 - 48 \alpha^{*2}|\alpha|^8 -96|\alpha|^{10}] \tag{3.18}$$

Therefore the variance of the field quadrature A is

$$\begin{aligned} [\Delta X_{1A}(t)]^2 &= \langle X^2_{1A}(t) \rangle - \langle X_{1A}(t) \rangle^2 \\ &=1/4 [1- 240g^2t^2 |\alpha|^6 (\alpha^2 + \alpha^{*2})] \end{aligned} \tag{3.19}$$

$$[\Delta X_{1A}(t)]^2-1/4 = - 120g^2t^2 |\alpha|^8 \cos 2\theta \tag{3.20}$$

where θ is the phase angle with $\alpha = |\alpha| e^{i\theta}$ and $\alpha^* = |\alpha| e^{-i\theta}$.

The right-hand side of the expression (3.20) is negative, indicating that squeezing will occur in the field amplitude of pump mode in the spontaneous ten wave mixing process for which cos2θ >0. Squeezing of light will be maximum, when θ = 0.
In parallel to the spontaneous interaction, the stimulated emission is caused due to the coupling of the atom to other states of the field.

For stimulated interaction, we consider the quantum state of the system as a product of coherent states for the modes A and B and vacuum state for the mode C,

i.e. $\langle\psi|=\langle\alpha|\langle\beta|\langle 0|$ (3.21)

Using Equations 3.14, 3.19 and 3.21, we obtain

$$\begin{aligned} [\Delta X_{1A}(t)]^2-1/4= & - 5g^2t^2 |\alpha|^8 (|\beta|^8 + 16|\beta|^6+72|\beta|^4 \\ & +92|\beta|^2 +24) \cos 2\theta \end{aligned} \tag{3.22}$$

which is negative indicating the amplitude squeezing in stimulated interaction during the process.

4. AMPLITUDE-SQUARED SQUEEZING IN SPONTANEOUS INTERACTION AT FREQUENCY ω_1

For second-order squeezing, the real quadrature component of the field amplitude in the pump mode A is given as

$$Y_{1A}(t) = ½[A^2(t) + A^{\dagger 2}(t)] \tag{4.23}$$

Using Equations 3.14, 3.16 and 4.23, we get the expectation values of the quadrature component $Y_{1A}(t)$ as

$$\langle\psi|Y^2_{1A}(t)|\psi\rangle = ¼[\alpha^4 + \alpha^{*4} + 2|\alpha|^4 + 4|\alpha|^2 + 2 + 5/2g^2t^2(-96\alpha^4|\alpha|^8 - 96\alpha^{*4}|\alpha|^8 -576\alpha^4|\alpha|^6 - 576\alpha^{*4}|\alpha|^6 - 1152\alpha^4|\alpha|^4 - 1152\alpha^{*4}|\alpha|^4 -576\alpha^4|\alpha|^2 - 576\alpha^{*4}|\alpha|^2 - 192|\alpha|^{12} - 576|\alpha|^{10}] \tag{4.24}$$

and

$$\langle\psi|Y_{1A}(t)|\psi\rangle^2 = ¼[\alpha^4 + \alpha^{*4} + 2|\alpha|^4 + 5/2g^2t^2(-96\alpha^4|\alpha|^8 - 96\alpha^{*4}|\alpha|^8 - 192\alpha^4|\alpha|^6 - 192\alpha^{*4}|\alpha|^6 - 384|\alpha|^{10} - 192|\alpha|^{12}] \tag{4.25}$$

The number of photons in the mode A may be expressed as

$$\langle N(t)\rangle = |\alpha|^2 + 5/2\, g^2t^2(-48|\alpha|^{10})$$

Therefore the variance in the amplitude- squared field quadrature is

$$[\Delta Y_{1A}(t)]^2 - \langle N+1/2\rangle = -5/2g^2t^2(96|\alpha|^6 + 288|\alpha|^4 + 144|\alpha|^2)(\alpha^4 + \alpha^{*4}) \tag{4.26}$$

which gives

$$[\Delta Y_{1A}(t)]^2 - \langle N+1/2\rangle = -120g^2t^2(4|\alpha|^{10} + 12|\alpha|^8 + 6|\alpha|^6)\cos 4\Theta \tag{4.27}$$

Equation 4.27 shows the amplitude-squared squeezing in the fundamental mode A in spontaneous interaction for selective values of $\cos 4\Theta$.

For stimulated interaction, we obtain

$$[\Delta Y_{1A}(t)]^2 - \langle N+1/2\rangle = -5g^2t^2(4|\alpha|^{10} + 12|\alpha|^8 + 6|\alpha|^6)(|\beta|^8 + 16|\beta|^6 + 72|\beta|^4 + 92|\beta|^2 + 24)\cos 4\Theta \tag{4.28}$$

which is negative showing the amplitude-squared squeezing for stimulated interaction much greater than squeezing during spontaneous interaction.

5. PHOTONS-STATISTICS

The fluctuations of photons in the pump mode A in ten-wave mixing process can be seen from the difference $\langle\Delta n(t)\rangle^2 - \langle n(t)\rangle$, which measures the departure from the Poisson statistics.

From Equations 3.14 and 3.16, we have

$$\langle \Delta n(t)\rangle^2 - \langle n(t)\rangle$$

$$= \langle n^2(t)\rangle - \langle n(t)\rangle^2 - \langle n(t)\rangle$$

$$= -480g^2t^2|\alpha|^{10} = < 0 \tag{5.29}$$

i.e., $\langle \Delta n\rangle^2 < \langle n\rangle$

The above expression (5.29) indicates that the squeezed light also observes the sub-Poissonian behavior.

6. SIGNAL-TO-NOISE RATIO (SNR)

Signal-to-noise ratio is defined as the ratio of the magnitude of signal to the magnitude of noise. With the approximations $\Theta = 0$ and $|gt|^2 << 1$ the maximum signal-to-noise ratio (in decibels) in field amplitude and higher order is given below

Using Equations 3.18 and 3.20, the SNR in field amplitude is defined as

$$SNR_1 = 20*\log_{10} \langle X_{1A}(t) \rangle^2 / [\Delta X_{1A}(t)]^2$$
$$= 20*\log_{10} |\alpha|^2$$

For amplitude –squared quadrature

$$SNR_2 = 20*\log_{10}\langle Y_{1A}(t) \rangle^2 / [\Delta Y_{1A}(t)]^2$$
$$= 20*\log_{10} (|\alpha|^6 + 2|\alpha|^2 / 2|\alpha|^4 + 6 |\alpha|^2 + 3)$$

The variations of degree of squeezing and SNR with photon number for spontaneous and stimulated interactions are shown as

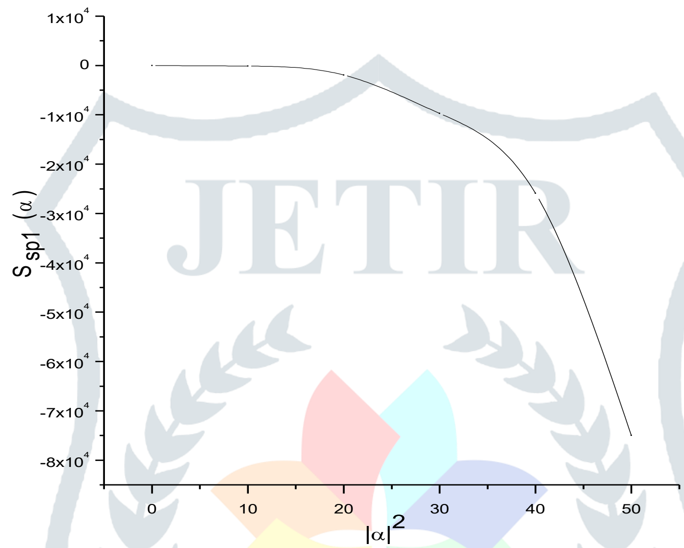

Fig.2 Dependence of first order squeezing S_{sp1} with $|\alpha|^2$ during spontaneous interaction

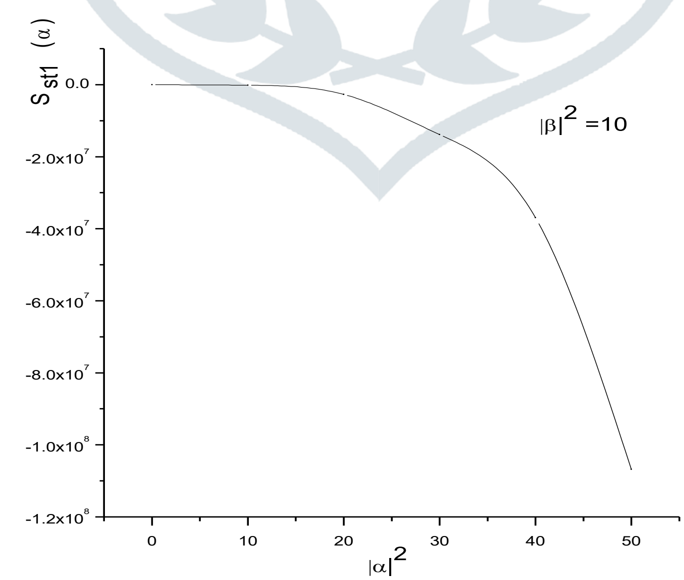

Fig.3 Dependence of first order squeezing S_{st1} with $|\alpha|^2$ during stimulated interaction

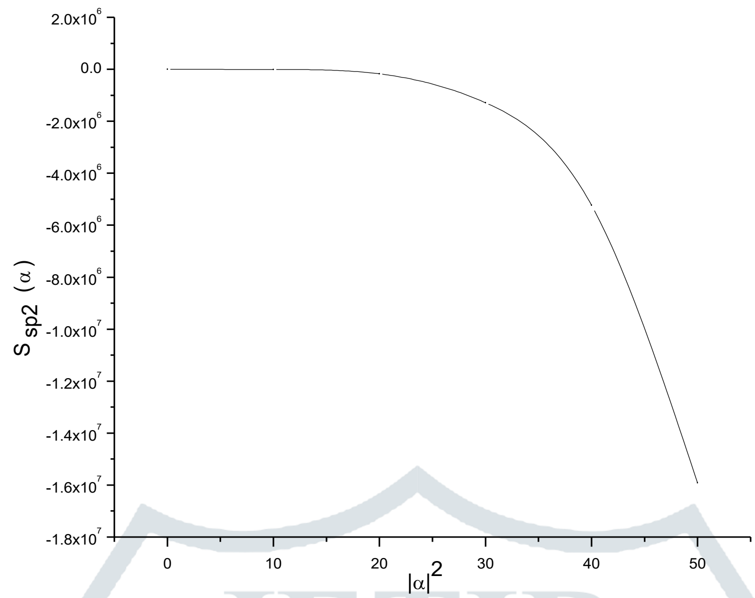

Fig.4 Dependence of second order squeezing S_{sp2} with $|\alpha|^2$ during spontaneous interaction

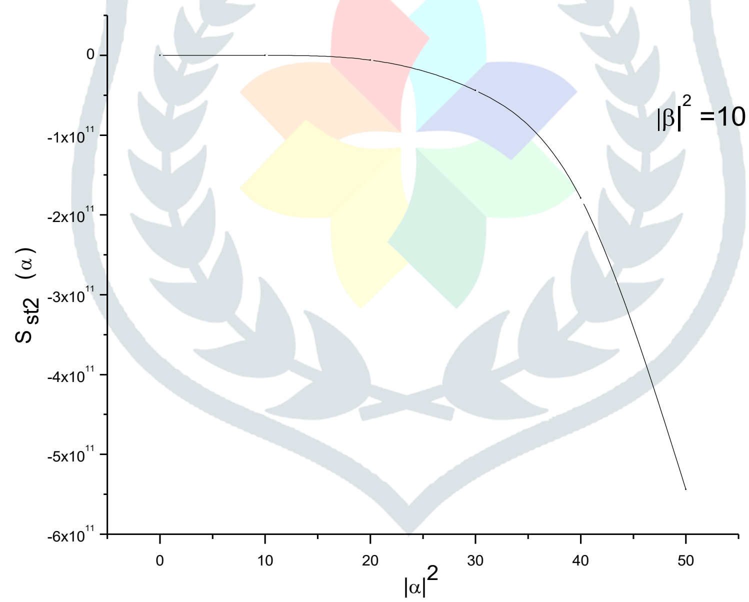

Fig.5 Dependence of second order squeezing S_{st1} with $|\alpha|^2$ during stimulated interaction

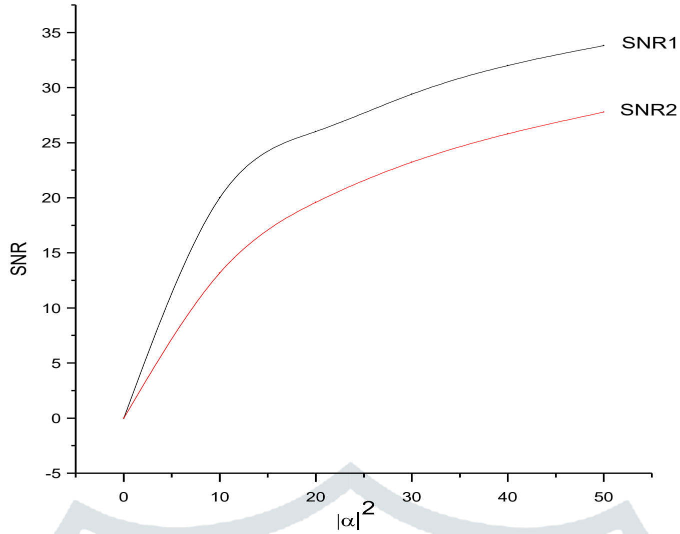

Fig.6 Dependence of signal to noise ratio in amplitude (SNR1) and amplitde- sqaured (SNR2) squeezing with $|\alpha|^2$.

6. RESULTS AND DISCUSSION

The present paper shows one of the possible way of generating amplitude and amplitude-squared quantum squeezed states in fundamental mode in spontaneous and stimulated interactions during ten wave mixing process. The results show that degree of squeezing and SNR depends upon the values of interaction time 't' between modes, coupling constant 'g' for nonlinear medium and selective phase values (Ө) of the field amplitude of fundamental mode. Also, the comparative analysis of graphs shows that quantum squeezing in stimulated interaction is ($|\beta|^8+16|\beta|^6+72|\beta|^4+92|\beta|^2+24$) times greater than the value obtained for spontaneous interaction. It is also found that signal to noise ratio is greater during amplitude squeezing than in amplitude squared squeezing as shown in figure 6. Nonlinear materials with higher order susceptibility values will help in generating low noise quantum states making their applicability in improving the sensitivity of laser interferometry devices and in optical communication channels.

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