DOMINATION PARAMETERS OF STAR, DUPLICATION OF STAR AND ITS TRANSFORMATION

[1]Stephen John. B and [2]Annie Subitha. M.P

[1]Associate Professor,PG and Research Department of Mathematics, Annai Velankanni College, Tholayavattam & [2]Research Scholar (Reg.No.: 18123152092022), PG and Research Department of Mathematics, S.T.Hindu College (Nagercoil, Kanniyakumari District), **Affiliated By Manonmaniam Sundaranar University**, Abishekapatti, Tirunelveli-627 012, TamilNadu, India;

Abstract :

Domination is one of the recent research area in graph theory. Many domination parameters have been studied by authors depend upon the situation and its application. Graph transformation is one of the fastest developing area in graph theory. In this paper, we have studied the domination parameters of star ,duplication of star graphs and its transformation. Also we have analyze the relation between these parameters.

Keywords : Domination, Transformation, Star Graph, Duplication

Introduction : 1

Graph theory is one of the florescent area to find the solution for some unsolved problems in real life which are motivated by objects and relation between them. Wu and Meng have studied the concept of Graph transformation and many applications have been studied in this topic. It is used in cell biology and cancer treatment.

Definition :2.1

A **graph** G = (V, E), where V is a finite set of elements called vertices and E is a set of unordered pairs of distinct vertices of G called edges.

Definition :2.2

The **degree** of a vertex *v* in G is the number of edges incident on it. A graph G is said to be **k-regular** if all its vertices are of degree k.

Definition :2.3

Every pair of its vertices are adjacent in G, is said to be **complete**, the complete graph on 'n' vertices is denoted by K_n .

Definition : 2.4

A graph G is said to be **bipartite** if the vertex set of V(G) can be partitioned in to two subsets X and Y such that every edge of G has one end in X and the other end in Y. A bipartite graph G with | X | = m and | Y| = n is said to be **complete** if every element in one partition is adjacent with all elements of the other partition and is denoted by $K_{m, n}$.

$\mathbf{K_{1, n}}$ is called **a Star graph.**

Definition :2.5

A set D ⊆ V is a **dominating set** of G if every vertex $v \in V - D$ is adjacent to at least one vertex of D. We call a dominating set D is a **minimal** if there is no dominating set D' ⊆ V(G) with D′⊂ D and D′≠ D. Further we call a dominating set D is minimum if these is no dominating set D′⊆V(G) with | D' | < | D |. The cardinality of a minimum dominating set is called the **domination number** denoted by γ(G) and the minimum dominating set D of G is also called a **γ- set**.

Definition: 2.6

Let $G = (V(G), E(G))$ be a graph and x, y,z be three variables taking values + or -. The **transformation graph G^{xyz}** is the graph having $V(G) \cup E(G)$ as the vertex set and for $\propto, \beta \in V(G) \cup E(G), \propto$ *and* β are adjacent in G^{xyz} if and only if the following holds:

(i) $\propto, \beta \in V(G), \propto$ *and* β are adjacent in G if x = +; $\propto$ *and* β are not adjacent in G if x = -

(ii) $\propto, \beta \in E(G). \propto$ *and* β are adjacent in G if y = +; $\propto$ *and* β are not adjacent in G if y = -

(iii) $\propto \in V(G), \beta \in E(G), \propto$ *and* β are incident in G if z = +; $\propto$ *and* β are not incident in G if z = - .

Definition: 2.7

Duplication of a vertex *v* of a graph G produces a new graph G′ by adding a new vertex *v* ′such that *N* (*v*)′ = *N*(*v*) . In other words, a vertex *v*′ is said to be duplication of *v* if all the vertices which are adjacent to *v* in G are also adjacent to *v*′ in G′. For a graph G, the graph obtained by duplication of all the vertices of G is denoted by $\boldsymbol{D(vG)}$.

Lemma : 3.1

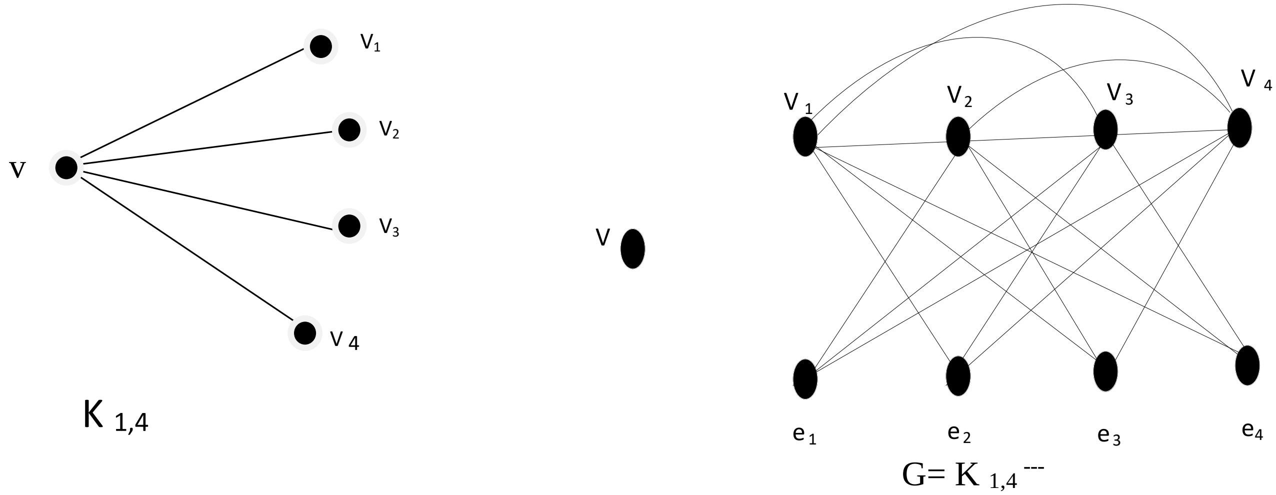
Let G be any star graph with n+1 vertices ($K_{1,n}$) then γ(G)=1 and γ_t(G) =2.

Lemma : 3.2

Let ($K_{1,n}$) be any star graph with n+1 vertices; G is a transformation of ($K_{1,n}$), that is G = $K_{1,n}^{---}$ then γ (G^{---}) =3 .

Proof :

For example, $K_{1,4}$ and G = $K_{1,4}^{---}$ is given in Figure : 1



(Figure: 1)

Let $G = K_{1,n}^{---}$. The vertices of G is denoted by V(G) = { v_i ,e_i ,v / i = 1,2,3….n }

In G, $d(v_i) = 2(n-1)$, $d(e_i) = n-1$, $d(v) = 0$.

That is, $N(v_i) = \{ v_i, e_j / i \neq j , j = 1,2,3\ldots n \}$

$N(e_i) = \{ v_j / i \neq j , j = 1,2,3\ldots n\}$

$N(v) = \{ \}$

Hence, every element $v_i \in V(G)$ is adjacent with all elements of V(G) other than v, e_i for all i. Therefore, each set { v_i ,e_i ,v / i = 1,2,3….n } is a dominating set of G.

Hence, $|D(G)| = 3$.

Also, every set { v_i ,e_i ,v / i = 1,2,3….n } are independent in G.

Therefore, { v_i ,e_i ,v / i = 1,2,3….n } is an independent dominating set with minimum cardinality. Hence, γ(G) = 3 . Thus, $\gamma(K_{1,n}) \leq \gamma(G)$ where $G = K_{1,n}^{---}$.

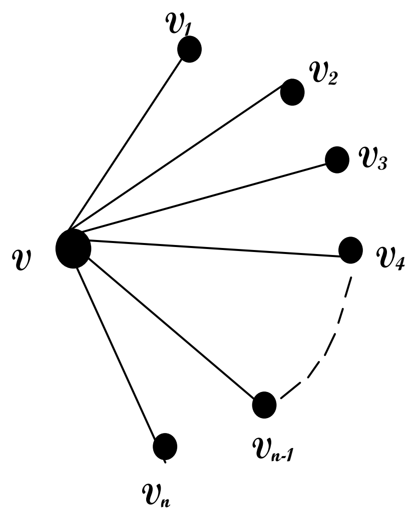
Hence the theorem.

Theorem: 3.3

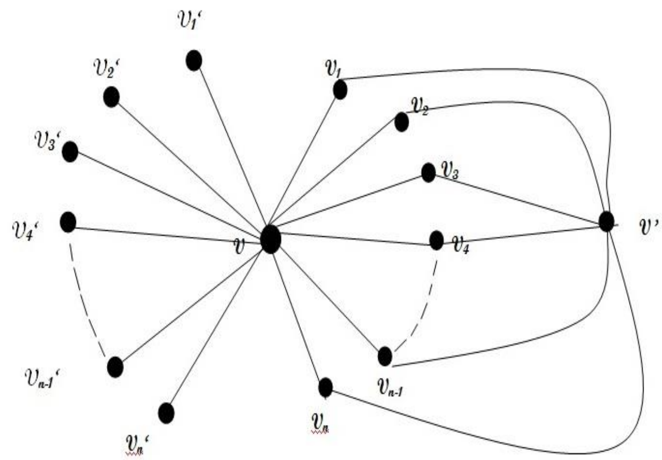
Let $K_{1,n}$ be any star graph with n+1 vertices. G is a graph obtained from $K_{1,n}$ by duplicating all the vertices , then γ (G) = 2.

Proof:

Let $K_{1,n}$ be a star graph with n+1 vertices. G is a graph obtained from $K_{1,n}$ by duplicating all the vertices. Then the graph $K_{1,n}$ and G is given below in Figure : 2.



K $_{1,4}$



G= D(K $_{1,4}$)

(Figure : 2)

Let V(K $_{1,n}$) be the vertex set of K $_{1,n}$ and V(G) be the vertex set of G, the duplication of K $_{1,n}$.

Therefore, V(K $_{1,n}$) = { v, v_i / i = 1,2,3,…n}

V(G) = { v, v', v_i, v_i' / i = 1,2,3,…n}

In G, N(v) = { v_i, v_i'/ I = 1,2,…n}

That is, all vertices other than v' is adjacent with v and v' is independent to v.

Hence D = { v, v'} is the minimum dominating set and independent dominating set of G.

Therefore, γ (G)= γ_i (G) = 2.

Hence γ (K $_{1,n}$) ≤ γ (G).

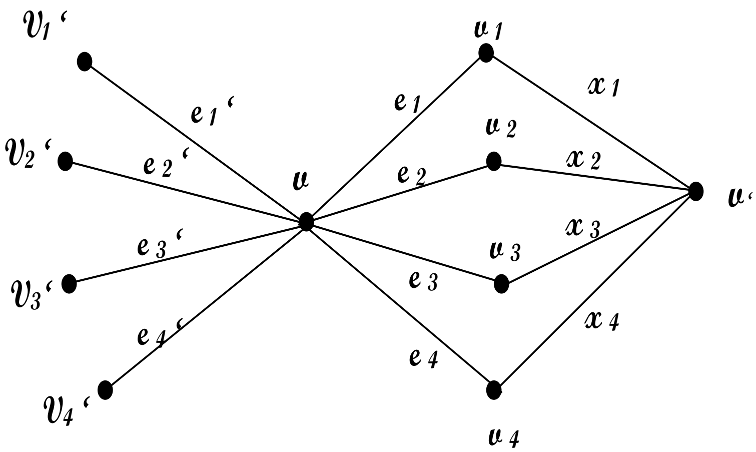
Theorem : 3.4

Let G be a duplication of all vertices of K $_{1,n}$, G $^{+--}$ is the transformation of G, then

γ (G) = γ (G $^{+--}$)

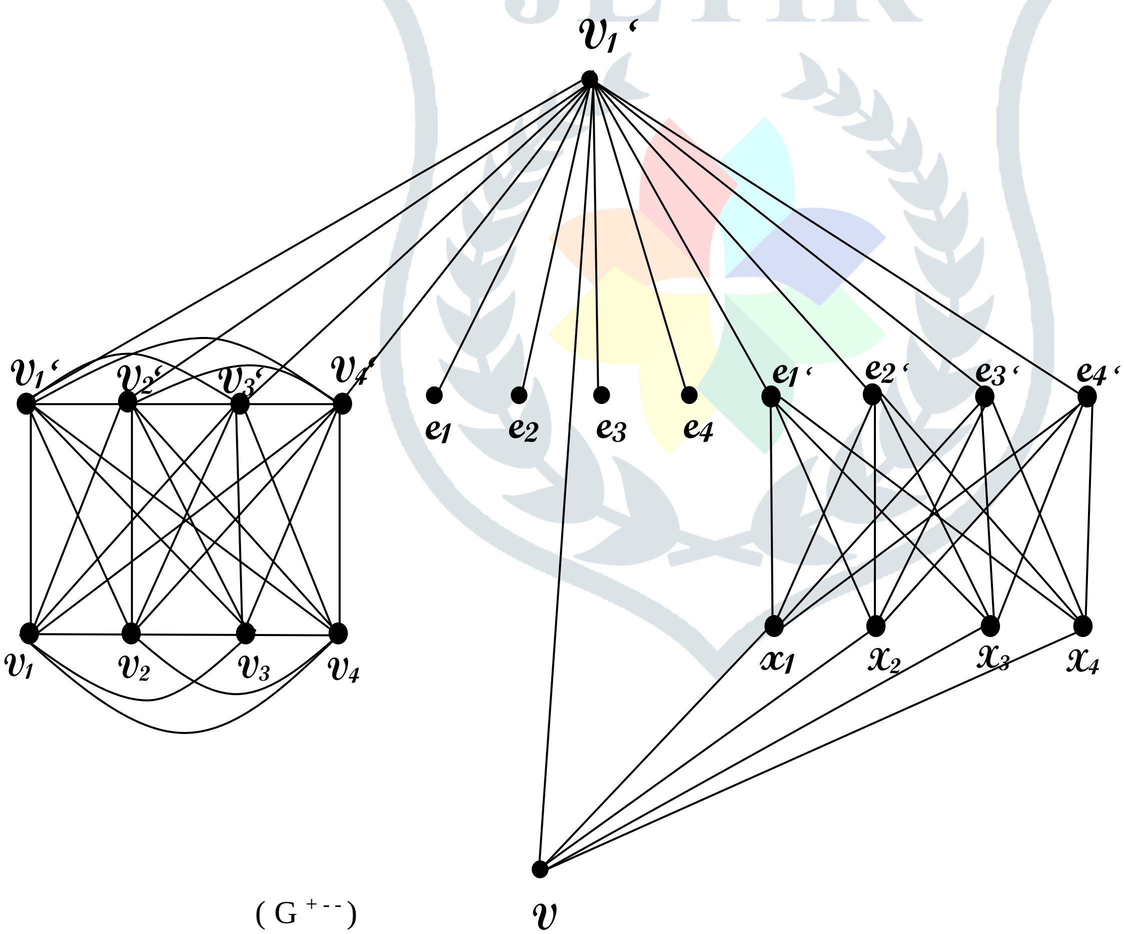
Proof :

For example ,the star graph K $_{1,4}$ and its duplication of K $_{1,4}$ (i.e, G = D(v K $_{1,4}$)) is given in Figure : 3 .



G = D (v $K_{1,4}$)

(Figure : 3)



(G^{+--})

(Figure : 4)

Let $V(G^{+--})$ be the vertex set of G^{+--} where $G = D(v\, K_{1,n})$

Let $S_1 = \{ v_i, v_i' \,/\, i = 1,2,\ldots n\}$; $S_2 = \{ e_i, e_i' / i = 1,2,\ldots n\}$

$S_3 = \{ x_i \,/\, i = 1,2,\ldots n\}$; $S_4 = \{ v, v' \}$

$N(v') = \{ v_i, v_i', e_i, e_i' / i = .1,2,\ldots n\}$

$N(v_i') = \{ v', v_j', v_i / i = 1,2,\ldots n,\ j = 1,2,\ldots n,\ i \neq j\}$

$N(v) = \{ v', x_i \,/\, i = 1,2,\ldots n\}$

Hence, all the elements of $V(G^{+--})$ are adjacent with every set of the form

$\{ v, v', v_i' / i = 1,2,\ldots n\}$. Hence every ordered triple $\{ v, v', v_i' / i = 1,2,\ldots n\}$ is a dominating

set of G^{+--}. Also these three elements are connected in G^{+--}. Hence, $\{ v, v', v_i' / i=1,2,\ldots n\}$ are the minimum connected dominating sets of G^{+--}. Hence, $\gamma(G^{+--}) = 3$.
Therefore, $\gamma(G) \leq \gamma(G^{+--})$.

Theorem : 3.5

Let G be the duplication of $K_{1,n}$. G^{+++} is a transformation of G then $\gamma(G^{+++}) = 2$.

Proof:

Let $S_1 = \{ v_i, v_i' / i = 1,2,\ldots n\}$; $S_2 = \{ e_i, e_i' / i = 1,2,\ldots n\}$

$S_3 = \{ x_i \,/\, i = 1,2,\ldots n\}$; $S_4 = \{ v, v'\}$

By the definition of G^{+++},

$N(v) = \{ v_i, v_i', e_i, e_i' / i = 1,2,\ldots n\}$

$N(v') = \{ v_i, x_i' \,/\, i = 1,2,\ldots n,\ j = 1,2,\ldots n\}$.

Hence all elements of G^{+++} are adjacent with v and v' in G^{+++}.

Therefore, $D = \{v, v'\}$ is the dominating set with minimum cardinality which implies

$\gamma(G^{+++}) = 2$.

Theorem : 3.6

Let G be the duplication of 2n vertices of $K_{1,n}$, G^{---} be a transformation of G,
then $\gamma(G) \leq \gamma(G^{---})$.

Proof :

Let the graph G is given in Figure : 2. Let $V(G^{---})$ can be partitioned into four sets S_1, S_2, S_3, S_4 such that

$S_1 = \{ v_i, v_i' / i = 1,2,\ldots n\}$; $S_2 = \{ e_i, e_i' / i = 1,2,\ldots n\}$

$S_3 = \{ x_i \,/\, i = 1,2,\ldots n\}$; $S_4 = \{ v, v' \}$

Since the elements of S_1 are independent in G, by the definition of G^{---}, the elements of S_1 forms a clique of 2n vertices in G^{---} . $v'$is independent to every element of S_2 in G.

Therefore, all the elements of S_2 are adjacent with v'in G^{---} .
Similarly each element of S_3 are independent to v in G, therefore every element of S_3 are the neighbours of v in G^{---}.

Therefore ,$S_2 \subseteq N(v')$; $S_3 \subseteq N(v)$ and $S_1 \cup S_2$ forms a clique of order 2n in G.
Hence ,every element of G^{---} are adjacent with either $\{ v, v', v_i / i = 1,2,\ldots n\}$ or $\{ v, v', v_i' / i = 1,2,\ldots n\}$.Hence each sets $\{ v, v', v_i / i = 1,2,\ldots n\}$ or $\{ v, v', v_i' / i = 1,2,\ldots n\}$ is a dominating set of G^{---}. Hence $\gamma(G^{---}) = 3$.

Result :

If G is the duplication of $K_{1,n}$, then $\gamma(G^{+++}) \leq \gamma(G^{---})$. By the theorem 3.5 and 3.6

the result is trivial.

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