Batch Arrival Single Server Queue with Vacation and Repairs

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Abstract : A single server batch arrival queue with two stages of heterogeneous services has been considered. After first stage service, the server must provide second stage service. At the end of each second stage service, the server may take compulsory vacation. The system may breakdown at random and it undergoes two types of repairs. The arrival, service, vacation and repair times are distributed according to some designed statistical distributions. The probability generating functions and expected number of units in the system have been derived.

Keywords: Batch arrival, breakdown, vacation, heterogeneous services, repairs, state probabilities, mean queue size.

I. INTRODUCTION

Queueing models have been studied by using some techniques such as vacation, repair, bulk size, retrial, optional service and so on. Vacation policy on queues has been analysed for over three decades as a very powerful technique for modeling and analysing production systems, inventory systems, banking services, computer systems and communication networks. The vacation time may start when there are no customers for service at the end of a service period. One of the factors of server's vacation is server breakdown which leads to the system must be repaired. Any queueing system has been characterised by a single or multi server with different types of arrivals as well as services. The arrivals may occur one by one or batches. Similarly, the services are performed in one stage or multiple stages.

II. REVIEW OF LITERATURE:

Based on the concepts mentioned in section 1, researchers on queues have analysed queueing models in different angles and achieved many applicable results. Doshi (1986) has summarised vacation policy with wide applications in production and communication systems. Borthakur and Choudhury (1997) have discussed the vacation queue with batch arrival and derived performance measures. Madan and Choudhury (2004) have adopted heterogeneous services and vacation under restricted rule with feedback and obtained explicit expressions for expected queue length, busy period and waiting time. Chodhury and Madhuchanda (2004) have discussed batch arrival queue with two phases of heterogeneous service under Bernoulli schedule vacation. Using imbedded Markov chain technique, levels, waiting time distribution and related performance measures have been derived. Ganesan and Sundarrajan (2008) have analysed breakdown analysis in a bulk arrival queue and derived mathematical expressions for the mean queue length and other performance measures. Maraghi et al (2009) have discussed batch arrival queue with random breakdowns and Bernoulli server vacation. The steady state probabilities and related results have been obtained. Krishnamoorthy and Sreenivasan (2012) have considered a two server Markovian queue with heterogeneous servers in which one server is always available and other goes on vacation when there are no customers in the system. Baba (2012) has constructed a single server bulk arrival Markovian queue with multiple vacations. The transition probability matrixes of Markov chain and matrix analytic method have been applied to solve the stated queueing model. Laplace Stieltjes transform of the stationary length and mean waiting time have been obtained. Ayyappan et al (2013) have studied a single server batch arrival queue in which arrival of each batch follows exponential distribution and service times follow general distribution. Break down and vacation policies are utilised. The mathematical expressions for the queueing performances have been derived. Ayyappan and Shyamala (2013) have investigated batch arrival queue with optional repair. Balamani (2014) has analysed a single server queue by using the concepts such as heterogeneous services with different service time distributions, vacation, random breakdown and repairs. The expressions for the steady state solutions, mean queue length and mean waiting time have been obtained. Ibe (2015) has studied a queue in which the server does not immediately take another vacation on returning from a vacation and waits for a pre-assigned time. He has listed out different service policies and vacation schemes. Analytical results are derived for the mean waiting time of a vacation queue under server timeout. Maragathasundari and Karthikeyan (2015) have considered a single server Markovian arrival queue with general service time and general vacation. At any time the system may breakdown which leads to repair process. The expressions for mean queue size and mean waiting time have been derived explicitly and numerically. Sundarrajan et al (2015) have discussed a batch arrival queue with multistage service. In this system, the server's vacation occurs when the system becomes empty. Suppose the system breakdowns, it is submitted to repair process. If any unit is unsatisfactory on service, the unit again tries to get re service. The probability generating function of the queue size and average queue size have been mathematically obtained. Bharathidass et al (2018) have analysed a single server Erlangian queue with server breakdown and repairs. In this model, the server takes vacation when the system becomes empty and the server's breakdown which leads to repairs. The mathematical expressions for the state probabilities and expected number of units in the system under different positions of the server have been derived.

In this paper, consider a single server batch arrival queue. The services of each unit are performed in two different stages. The server takes compulsory vacation at the end of the second stage service of a unit. The system may breakdown at random and it leads to repair process. The system demands two types of repairs such that the first one is major repair and the second is minor repair. After completing the major repair, the system may opt for minor repair with probability p or continues the working process with probability $(1 - p)$. Suppose the service of any unit interrupted due to breakdown, it goes back to the head of the queue. The mathematical expression for the expected number of units in the system has been derived.

III. DESCRIPTION OF THE MODEL:

- Units arrive at the system in batches of variable size in a compound Poisson process. Let $\lambda c_i \Delta t$ (i = 1,2,…) be the first order probability that a batch of i units arrives at the system during a short interval of time (t, t + Δt), where $0 \le c_i \le 1$ and $\sum_{i=1}^{n} c_i = 1$ and $\lambda > 0$ mean arrival rate of batches.
- The single server provides two stages of heterogeneous service one after the other in succession, defined as the first stage (FS) and second stage (SS) services respectively. The service discipline is assumed to be on a first come first served basis (FCFS). The service time of the two stages follow general distribution function $B_j(v)$ and the density function $b_j(v)$, $j = 1,2$
- Let $\mu_j(x)dx$ be the conditional probability of the j^{th} stage service during the interval $(x, x+dx)$ given that elapsed service time is x, so that

  $$\mu_j(x) = \frac{b_j(x)}{1-B_j(x)}\,;\ j = 1,2 \tag{1}$$

  and therefore,

  $$b_j(v) = \mu_j(v)e^{-\int_0^v \mu_j(x)dx}\,;\ j = 1,2 \tag{2}$$

- Once the second stage service of a unit is complete, the server will take compulsory vacation.
- The vacation times follow exponential distribution with parameter γ.
- On returning from vacation the server instantly starts serving the unit at the head of the queue if any.
- The system may breakdown at random and breakdowns occur according to Poisson stream with mean breakdown rate $\alpha > 0$. The unit receiving service during breakdown returns back to the head of the queue.
- Once the system breakdowns, it is immediately sent for repair wherein the repairman first provides major repair. After finishing the major repair, the server adopts minor repair with probability p or joins the system with probability $(1-p)$ to render service for waiting units.
- The major repair times are distributed according to exponential distribution with parameter β_1.
- The minor repair times follow general distribution with cumulative distribution functions $R^{(2)}(x)$.

  Let $\beta_2(x)\,dx$ be the conditional probability of a completion of minor repair during the interval $(x, x+dx)$ given the elapsed repair time x.

  $$\beta_2(x) = \frac{r(x)}{1-R^{(2)}(x)} \tag{3}$$

  and

  $$r(t) = \beta_2(t)e^{-\int_0^t \beta_2(x)dx} \tag{4}$$

IV. NOTATIONS AND ASSUMPTIONS:

The important notations applied in the text and related assumptions are given below.

λ – The mean arrival rate of each batch and it follows Poisson distribution

$\mu_1(x)$ and $\mu_2(x)$ – the mean service rates and they follow General distributions

γ – the mean vacation times and follow exponential distribution

β_1 – the mean of major repair times which follow exponential distribution

$\beta_2(x)$ – the mean of minor repair times which follow General distribution

$P_n^{(j)}(x,t)$ = Probability that at time t, server is providing j^{th} stage (j = 1,2) service and there are n(≥ 0) units in the queue excluding the one unit in j^{th} stage (j = 1,2) being served and the elapsed service time for this unit is x. Consequently,

$P_n^{(j)}(t) = \int_0^\infty P_n^{(j)}(x,t)dx$

$R_n^{(1)}(t)$ = Probability that at time t, the server is inactive due to breakdown and the system is under major repair while there are n (≥ 0) units in the queue.

$R_n^{(2)}(t)$ = Probability that at time t, the server is inactive due to breakdown and the system is under minor repair while there are n (≥ 0) units in the queue.

$Q(t)$ = Probability that at time t, there are no units in the system and the server is idle but available in the system.

V. SYSTEM OF BASIC EQUATIONS:

According to the descriptions given in sections 3 and 4, the required differentiate difference equations are framed as follows:

$$\frac{\partial}{\partial x}P_n^{(1)}(x) + (\lambda + \mu_1(x) + \alpha + \gamma)P_n^{(1)}(x) = \lambda\sum_{i=1}^{n-1} c_i P_{n-i}^{(1)}(x);\ n \ge 1 \tag{5}$$

$$\frac{\partial}{\partial x}P_0^{(1)}(x) + (\lambda + \mu_1(x) + \alpha + \gamma)P_0^{(1)}(x) = 0 \tag{6}$$

$$\frac{\partial}{\partial x}P_n^{(2)}(x) + (\lambda + \mu_2(x) + \alpha + \gamma)P_n^{(2)}(x) = \lambda\sum_{i=1}^{n-1} c_i P_{n-i}^{(2)}(x);\ n \ge 1 \tag{7}$$

$$\frac{\partial}{\partial x}P_0^{(2)}(x) + (\lambda + \mu_2(x) + \alpha + \gamma)P_0^{(2)}(x) = 0 \tag{8}$$

$$(\lambda + \beta_1)R_n^{(1)} = \lambda\sum_{i=1}^{n} c_i R_{n-i}^{(1)} + \alpha\int_0^\infty P_{n-1}^{(1)}(x)dx + \alpha\int_0^\infty P_{n-1}^{(2)}(x)dx;\ n \ge 1 \tag{9}$$

$$(\lambda + \beta_1)R_0^{(1)} = 0 \tag{10}$$

$$\big(\lambda + \beta_2(x)\big)R_n^{(2)}(x) = \lambda\sum_{i=1}^{n} c_i R_{n-i}^{(2)} + p\beta_1 R_n^{(1)};\ n \ge 1 \tag{11}$$

$$\big(\lambda + \beta_2(x)\big)R_0^{(2)}(x) = p\beta_1 R_0^{(1)} \tag{12}$$

$$(\lambda + \alpha)Q = (1-p)\beta_1 R_0^{(1)} + \int_0^\infty \beta_2(x)R_0^{(2)}(x)dx + \gamma\, V_0 \tag{13}$$

For solving the stated equations, the following boundary conditions are applied.

$$P_n^{(1)}(0) = c_{n+1}\lambda Q + (1-p)\beta_1 R_{n+1}^{(1)} + \int_0^\infty \beta_2(x)R_{n+1}^{(2)}(x)dx + \gamma V_{n+1};\ n \ge 0 \tag{14}$$

$$P_n^{(2)}(0) = \int_0^\infty P_n^{(1)}(x)\mu_1(x)dx;\ n \ge 1 \tag{15}$$

$$V_n = \int_0^\infty P_n^{(2)}(x)\mu_2(x)dx \tag{16}$$

VI. PROBABILITY GENERATING FUNCTIONS:

In this stage, the probability generating functions of the above probabilities are defined.

$$\left.\begin{array}{l} G(x,z)=\sum_{n=0}^{\infty} z^n G_n(x) \\ G(z)=\sum_{n=0}^{\infty} z^n G_n\,;\ G=P^{(1)},P^{(2)} \text{ and } R^{(2)} \\ V(z)=\sum_{n=0}^{\infty} z^n V_n \\ R^{(1)}(z)=\sum_{n=0}^{\infty} z^n R_n^{(1)} \end{array}\right\} \quad (17)$$

Multiply the equations from (5) to (12) by the proper powers of z and apply the expressions given in (17).

$$\frac{\partial}{\partial x}P^{(1)}(x,z)+(\lambda-\lambda\,c(z)+\mu_1(x)+\alpha+\gamma)P^{(1)}(x,z)=0 \quad (18)$$

$$\frac{\partial}{\partial x}P^{(2)}(x,z)+(\lambda-\lambda\,c(z)+\mu_2(x)+\alpha+\gamma)P^{(2)}(x,z)=0 \quad (19)$$

$$(\lambda-\lambda\,c(z)+\beta_1)R^{(1)}(z)=\alpha z\int_0^{\infty}P^{(1)}(x,z)dx+\alpha z\int_0^{\infty}P^{(2)}(x,z)dx \quad (20)$$

$$(\lambda-\lambda\,c(z)+\beta_2(x))R^{(2)}(x,z)=p\beta_1R^{(1)}(z) \quad (21)$$

Following the same procedure in the boundary conditions (14),(15) and (16) we get,

$$zP^{(1)}(0,z)=-(\lambda-\lambda\,c(z)+\alpha)Q+(1-p)\beta_1R^{(1)}(z)+\int_0^{\infty}\beta_2(x)\,R^{(2)}(x,z)dx+\gamma\,V(z) \quad (22)$$

$$P^{(2)}(0,z)=\int_0^{\infty}P^{(1)}(x,z)\,\mu_1(x)dx \quad (23)$$

and

$$R^{(2)}(0,z)=\int_0^{\infty}P^{(2)}(x,z)\mu_2(x)dx \quad (24)$$

Now integrating the equations (18), (19) and (20) over the interval $(0, x)$, we get,

$$P^{(1)}(x,z)=P^{(1)}(0,z)e^{-(\lambda-\lambda\,c(z)+\alpha+\gamma)\,x-\int_0^x\mu_1(t)dt} \quad (25)$$

$$P^{(2)}(x,z)=P^{(2)}(0,z)e^{-(\lambda-\lambda\,c(z)+\alpha+\gamma)x-\int_0^x\mu_2(t)dt} \quad (26)$$

$$R^{(2)}(x,z)=R^{(2)}(0,z)e^{-(\lambda-\lambda\,c(z)+\alpha)x-\int_0^x\beta_2(t)dt} \quad (27)$$

The integration is carried out for the equations (25), (26) and (27) with respect to x.

$$P^{(1)}(z)=P^{(1)}(0,z)\left[\frac{1-B_1^*(\lambda-\lambda\,c(z)+\alpha+\gamma)}{(\lambda-\lambda\,c(z)+\alpha+\gamma)}\right] \quad (28)$$

$$P^{(2)}(z)=P^{(2)}(0,z)\left[\frac{1-B_2^*(\lambda-\lambda\,c(z)+\alpha+\gamma)}{(\lambda-\lambda\,c(z)+\alpha+\gamma)}\right] \quad (29)$$

$$R^{(2)}(z)=R^{(2)}(0,z)\left[\frac{1-R^{(2)*}(\lambda-\lambda\,c(z)+\alpha)}{(\lambda-\lambda\,c(z)+\alpha)}\right] \quad (30)$$

Where $H^*(Q)=\int_0^{\infty}e^{-\theta x}dH(x)$ is the Laplace – Stieltjes transform of H(x) and H = B_1, B_2 and $R^{(2)}$.

Apply the equations (28) and (29) in the equation (20), we obtain,

$$R^{(1)}(z)=\frac{\alpha z\,P^{(1)}(0,z)}{(\lambda-\lambda c(z)+\beta_1)}\left[\frac{1-B_1^*(\lambda-\lambda c(z)+\alpha+\gamma)}{(\lambda-\lambda c(z)+\alpha+\gamma)}\right]+\frac{\alpha z\,P^{(2)}(0,z)}{(\lambda-\lambda c(z)+\beta_1)}\left[\frac{1-B_2^*(\lambda-\lambda c(z)+\alpha+\gamma)}{(\lambda-\lambda c(z)+\alpha+\gamma)}\right] \quad (31)$$

Multiply both sides of the equations (25), (26) and (27) by $\mu_1(x)$, $\mu_2(x)$ and $R^{(2)}(x)$ respectively and integrate with respect to x over the interval (0,∞). The resultant equations are

$$\int_0^{\infty}P^{(1)}(x,z)\mu_1(x)dx=P^{(1)}(0,z)B_1^*(\lambda-\lambda c(z)+\alpha+\gamma) \quad (32)$$

$$\int_0^{\infty}P^{(2)}(x,z)\mu_2(x)dx=P^{(2)}(0,z)B_2^*(\lambda-\lambda c(z)+\alpha+\gamma) \quad (33)$$

and

$$\int_0^{\infty}R^{(2)}(x,z)\beta_2(x)dx=R^{(2)}(0,z)R^{(2)*}(\lambda-\lambda c(z)+\alpha) \quad (34)$$

Let us consider

$(\lambda-\lambda c(z)+\alpha)=k$;

$(\lambda-\lambda\,c(z)+\alpha+\gamma)=m$

and

$(\lambda-\lambda\,c(z)+\beta_1)=n$

Here k, m and n are the functions of z

Apply the equations (32) to (34) in the equations (22) to (24)

$$zP^{(1)}(0,z)=-kQ+(1-p)\beta_1R^{(1)}(z)+\gamma\,V(z)+R^{(2)}(0,z)R^{(2)*}(k) \quad (35)$$

$$P^{(2)}(0,z)=P^{(1)}(0,z)B_1^*(m) \quad (36)$$

$$R^{(2)}(0,z)=P^{(1)}(0,z)B_1^*(m)B_2^*(m) \quad (37)$$

The equations (29) and (31) are rewritten after applying the equation (36) and get,

$$P^{(2)}(z)=\left\{\frac{1-B_1^*(m)}{m}\right\}B_1^*(m)P^{(1)}(0,z) \quad (38)$$

$$R^{(1)}(z)=\frac{\alpha z}{mn}[1-B_1^*(m)B_2^*(m)]P^{(1)}(0,z) \quad (39)$$

Similarly adopt (37) in (30)

$$R^{(2)}(z)=B_1^*(m)B_2^*(m)\left[\frac{1-R^{(2)*}(k)}{k}\right]P^{(1)}(0,z) \quad (40)$$

Also apply the equations (37) and (39) in the equation (35)

$$P^{(1)}(0,z)=\frac{mn\{-kQ+\gamma\,V(z)\}}{A} \quad (41)$$

where $A=mnz-(1-p)\beta_1\alpha z\{1-B_1^*(m)B_2^*(m)\}-mn\,B_1^*(m)B_2^*(m)R^{(2)*}(k)$

Apply the expression for $P^{(1)}(0,z)$ given in (41) in the expressions (28), (38), (39) and (40) and respectively get,

$$P^{(1)}(z)=\frac{n}{A}\{-kQ+\gamma\,V(z)\}\big(1-B_1^*(m)\big) \quad (42)$$

$$P^{(2)}(z)=\frac{n}{A}B_1^*(m)\big(1-B_1^*(m)\big)\{-kQ+\gamma\,V(z)\} \quad (43)$$

$$R^{(1)}(z)=\frac{\alpha z}{A}[1-B_1^*(m)B_2^*(m)]\{-kQ+\gamma\,V(z)\} \quad (44)$$

$$R^{(2)}(z) = B_1^*(m)B_2^*(m)\left[\frac{1-R^{(2)*}(k)}{k}\right]\frac{mn}{A}\{-kQ + \gamma\, V(z)\} \quad (45)$$

The probability generating function of the number of units in the system is easily obtained by adding the probability generating functions given in the expressions from (42) to (45)

$$W(z) = \left[\frac{-kQ+\gamma\, V(z)}{A}\right]\Big[n\big(1 - B_1^*(m)\big)\{1 + B_1^*(m)\} + \alpha z\big(1 - B_1^*(m)B_2^*(m)\big)$$
$$+ mnB_1^*(m)B_2^*(m)\left\{\frac{1-R^{(2)*}(k)}{k}\right\}\Big] \quad (46)$$

The unknown constant Q in the expression (46) is to be determined by using $w\,(1) = 1$.

$$Q = \frac{\alpha(\gamma-p\beta_1)-\gamma\beta_1\big(B_1^*(\alpha+\gamma)\big)^2-\left[\alpha(\gamma+(1-p)\beta_1)-\frac{\beta_1}{\alpha}(\alpha+\gamma)\left\{\gamma+(\alpha-\gamma)R^{(2)^*}(\alpha)\right\}\right]B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)}{\alpha\left[\alpha+\beta_1\left\{1-\big(B_1^*(\alpha+\gamma)\big)^2\right\}\right]-\left\{\alpha^2-(\alpha+\gamma)\beta_1\left(1-R^{(2)^*}(\alpha)\right)\right\}B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)} \quad (47)$$

VII. MEAN NUMBER OF UNITS IN THE SYSTEM:

The mean number of units in the system is obtained by differentiating the expression (46) and letting z = 1,

$$W'(1) = \frac{A(1)\{D(1)E'(1)+E(1)D'(1)\}-D(1)E(1)A'(1)}{[A(1)]^2} \quad (48)$$

Where

$$D(1) = -\alpha Q + \gamma \quad (49)$$
$$D'(1) = \lambda\, c'(1)\gamma\, E(V) \quad (50)$$
$$A(1) = \beta_1\left[\gamma + \alpha p + B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)\left\{(1-p)\alpha - (\alpha+\gamma)R^{(2)^*}(\alpha)\right\}\right] \quad (51)$$
$$A'(1) = \beta_1\left\{(\alpha+\gamma) - (1-p)\,\alpha\big(1 - B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)\big)\right\}$$
$$+\ \lambda c'(1)\Big[\beta_1\left\{(\alpha+\gamma)R^{(2)^*}(\alpha) - (1-p)\alpha\right\}$$
$$\{B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)\ +\ B_2^*(\alpha+\gamma)B_1^*(\alpha+\gamma)\}$$
$$-\ \lambda(\alpha+\gamma+\beta_1) + B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)$$
$$\{\beta_1(\alpha+\gamma)R^{2^*}(\alpha) + (\alpha+\gamma+\beta_1)R^{2^*}(\alpha)\}\Big] \quad (52)$$
$$E(1) = \alpha + \beta_1\left(1 - \big(B_1^*(\alpha+\gamma)\big)^2\right) - \frac{B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)}{\alpha}\left\{\alpha^2 - (\alpha+\gamma)\beta_1\left(1 - R^{2^*}(\alpha)\right)\right\} \quad (53)$$

and

$$E'(1) = \alpha\left(1 - B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)\right)$$
$$+\frac{\lambda c'(1)}{\alpha^2}\Big[2\alpha^2\beta_1B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma) - \alpha^2\left(1 - \big(B_1^*(\alpha+\gamma)\big)^2\right)$$
$$+\alpha\{B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma) + B_2^*(\alpha+\gamma)B_1^*(\alpha+\gamma)\}$$
$$\left\{\alpha - (\alpha+\gamma)\beta_1\left(1 - R^{2^*}(\alpha)\right)\right\}$$
$$-\alpha(\alpha+\gamma+\beta_1)B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)\left(1 - R^{2^*}(\alpha)\right)$$
$$+(\alpha+\gamma)\beta_1B_1^*(\alpha+\gamma)B_2^*(\alpha+\gamma)\left\{\alpha R^{2^{*\prime}}(\alpha) + 1 - R^{2^*}(\alpha)\right\}\Big] \quad (54)$$

VIII. CONCLUSION:

A single server queue with bulk arrival and two stages of service is studied. The breakdown and vacation policies are employed and the system is admitted to repair due to breakdown. This system is analysed through probability generating functions and Laplace Stieltjes transform. The expressions for the probability generating function and the mean number of units in the system are derived explicitly.

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