Ψ- DOMINATION NUMBER OF A GRAPH

A.Muthukamatchi
Assistant Professor of Mathematics
Department of Mathematics, Govt. Arts College for women,Nilakkottai-624 208
Tamil Nadu, India

Abstract: Let G be a connected graph. An acyclic graphoidal cover of G is a collection ψ if non-trivial in G such that every edge of G is in exactly one path in ψ and every vertex of G is an internal vertex of at most path in ψ. Let ψ be a acyclic graphoidal cover of G. Let u,v ∈ V(G). The vertices u and v are said to be ψ adjacent if there exists a u-v path P in ψ. A subset D of V is called a ψ-dominating set of G if for every vertex v ∈ V−D, there exists a vertex u in D such that u and v are ψ - adjacent. The minimum cardinality of a minimal ψ-dominating set is called the ψ-domination number of G and is denoted by $\gamma_\psi(G)$.
In this paper we obtain several results on this parameter.

***IndexTerms* -** Acyclic graphoidal cover, domination number, ψ-domination number, upper ψ-domination number.

1. Introduction

Let G = (V,E) be a finite, simple connected graph with at least two vertices. For graph theoretic terminology, we refer to Harary [5]. Acharya and Sampathkumar [2] introduced the concept of graphoidal covers and graphoidal covering number of a graph. Arumugam and Suresh Suseela [4] introduced the concept of acyclic graphoidal cover and acyclic graphoidal covering number of a graph. An elaborate review of results in graphoidal covers with several interesting applications and a collection of unsolved problems is given in [3]. Any graph theoretic concept which depends only on adjacency of vertices can be extended in the context of graphoidally covered graph and ψ = E(G) yields the original concept as a special case. Acharya and Purnima Gupta [1] generalised the concept of domination in graphs. Most of the results presented in [1] deal with infinite graphs. In this paper, we obtain several results on the graphoidal cover domination number of finite graphs. Since loops do not play any role in thestudy of domination in graphs, we assume throughout that ψ is an acyclic graphoidal cover of G.

2. Main Results

Definition 2.1 Let (G,ψ) be a graphoidally covered graph.We define another graph G(ψ) as follows. V(G(ψ)) = V(G) and two vertices u and v are adjacent in G(ψ) if and only if u and v are ψ-adjacent in G.

The ψ-domination number $\gamma_\psi(G)$ and the upper ψ-domination number $\Gamma_\psi(G)$ are respectively, the domination number and the upper domination number of the graph G(ψ).

Example 2.2 Consider the acyclic graphoidal cover ψ ={(1,2,3),(2,4),(4,5)} of the graph G given in figure 2.1(a). Then G(ψ) is given in figure 2.1(b).

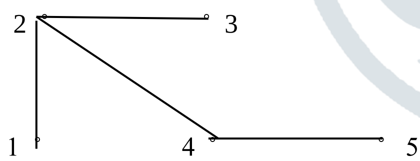

Figure 2.1 (a)

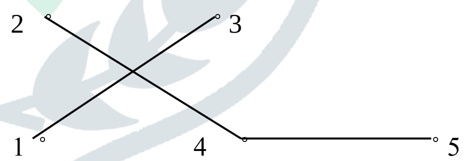

Figure 2.1 (b)

Here $\gamma_\psi(G) = 2$ and $\Gamma_\psi(G) = 3$.

Remark 2.3 In the above example $\gamma_\psi(G) = \gamma(G)$ and $\Gamma_\psi(G) = \Gamma(G)$. Further, we observe that {1,4} and {3,4} are the only γ_ψ-sets and {1,2,5} and {2,3,5} are the only Γ_ψ-sets. Also {2,4} and {2,5} are γ-sets of G and {1,3,5}, {1,3,4} are Γ-sets of G. Thus no γ_ψ-set is a γ-set and no Γ_ψ-set is a Γ-set whereas $\gamma_\psi(G) = \gamma(G)$ and $\Gamma_\psi(G) = \Gamma(G)$.

Remark 2.4 Consider the graph G given in figure 2.2. Then ψ_1 = {(8,1,2,3,4,5),(2,7),(3,6)}, ψ_2 = {(1,8),(1,2,7),(2,3),(3,6),(3,4),(4,5)} and ψ_3 = E(G) are acyclic graphoidal covers of G. The corresponding graphs $G(\psi_1)$, $G(\psi_2)$ and $G(\psi_3)$ are given in figures 2.2(a), 2.2(b) and 2.2(c) respectively.

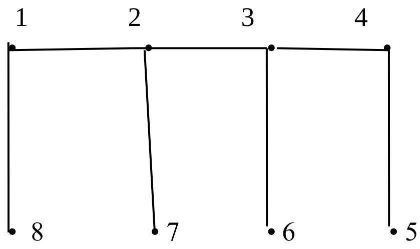

Figure 2.2.

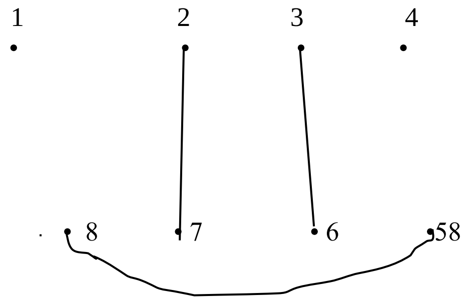

Figure 2.2(a)

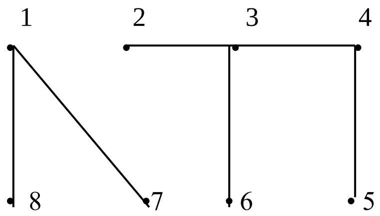

Figure 2.2(b)

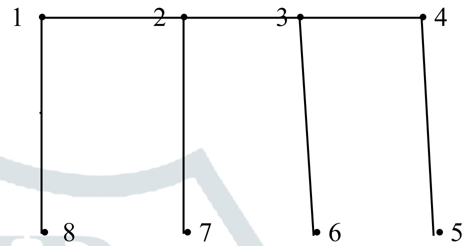

Figure 2.2

clearly $\gamma(G) = 4$, $\gamma_{\psi_1}(G) = 5$, $\gamma_{\psi_2}(G) = 2$ and $\gamma_{\psi_3}(G) = \gamma(G) = 4$. Thus there is no relation between γ_ψ and γ. In the following theorems, we prove that the differences $\gamma - \gamma_\psi$ and $\gamma_\psi - \gamma$ can be made arbitrarily large.

Theorem 2.5 Given any positive integer n, there exists a graph G and an acyclic graphoidal cover ψ of G such that

$$\gamma(G) - \gamma_\psi(G) = n.$$

Proof : We construct a graph G as follows. Consider a cycle $C = (v_1,v_2,\ldots\ldots v_{2n-1},v_{2n},v_1)$ of length 2n. For each i, $1 \le i \le 2n$, attach exactly one pendant vertex w_i if i is odd and attach two pendant vertices x_i and y_i if i is even. Let $P_i = (w_{2i-1},v_{2i-1},v_{2i})$, $i = 1,2,\ldots.n$. Then $\psi = \{P_1,P_2,\ldots\ldots,P_n\} \cup S$ where S is the set of edges of G not covered by the paths $P_i (1 \le i \le n)$ is an acyclic graphoidal cover of G. Further the graph $G(\psi)$ is a disconnected graph with n components, each is isomorphic to a star and hence $\gamma_\psi(G) = n$. Since $\gamma(G) = 2n$, we have $\gamma(G) - \gamma_\psi(G) = n$.

Example 2.6: For the graph G given in figure 2.3 (a), $\psi = \{(w_1,v_1,v_2),(w_3,v_3,v_4),\ldots,(w_9,v_9,v_{10})\} \cup E(G)$ is an acyclic graphoidal cover of G. The graph $G(\psi)$ is given in figure 2.3 (b). Here $\gamma(G) = 10$ and $\gamma_\psi(G) = 5$.

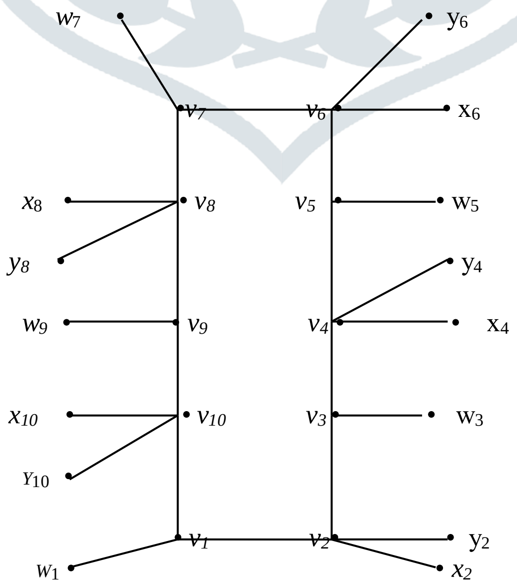

Figure 2.3(a)

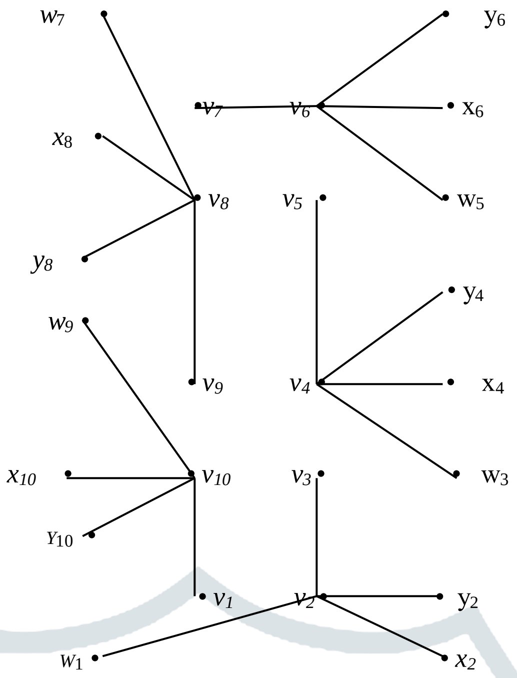

Figure 2.3(b)

Theorem 2.7 : For any positive integer n, there exists a graph G and an acyclic graphoidal cover ψ of G such that

$$\gamma_\psi(G) - \gamma(G) = n.$$

Proof: Let G be the graph obtained from the cycle $C_n = \{v_1, v_2, ...v_n, v_1\}$ by attaching three pendant vertices x_i, y_i and z_i to vertex v_i, $1 \le i \le n$. Let $P_i = (x_i, v_i, y_i)$, $1 \le i \le n$. Then $\psi = \{(P_i : 1 \le i \le n)\} \cup S$, where S is the set of edges of G not covered in the paths P_i, is an acyclic graphoidal cover of G. Now $G(\psi) = (C_n \circ K_1) \cup nK_2$, so that $\gamma_\psi(G) = 2n$. Since $\gamma(G) = n$.
We have $\gamma_\psi(G) - \gamma(G) = n$.

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