PAIRWISE FUZZY σ-BAIRE SETS

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Abstract : In this paper, the concept of pairwise fuzzy σ-Baire set is introduced and studied. Several characterizations of pairwise fuzzy σ -Baire sets, conditions for fuzzy bitopological spaces to become pairwise fuzzy Baire spaces by means of pairwise fuzzy σ-Baire sets and pairwise fuzzy σ -first category sets, are obtained.

Keywords : Pairwise fuzzy open set, pairwise fuzzy F_σ-set, pairwise fuzzy G_δ -set, Pairwise fuzzy σ-nowhere dense set, pairwise fuzzy σ-residual set, pairwise fuzzy σ-Baire spaces.

I. INTRODUCTION

In order to deal with uncertainties, the notions of fuzzy sets and fuzzy set operations were introduced by L.A.Zadeh [11] in his classical paper in the year 1965, describing fuzziness mathematically for the first time. In 1968, C.L.Chang [1] defined fuzzy topological spaces by using fuzzy sets.

In 1989, A.Kandil [2] introduced and studied fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces. The concept of pairwise fuzzy σ-residual sets in fuzzy bitopological spaces is introduced and studied by the authors in [6], [7], [8] and [9]. The purpose of this paper is to introduce the concept of pairwise fuzzy σ-Baire sets. Besides characterizations of these sets, several properties of these sets are studied.

II. PRELIMINARIES

In order to make the exposition self-contained, some basic notations and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X. Let X be a non-empty set and I the unit interval [0,1]. A fuzzy set λ in X is a mapping from X into I.

Definition 2.1 [4] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ (i = 1, 2). The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set in (X, T_1, T_2).

Definition 2.2 [4] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)' s are pairwise fuzzy open sets in (X,T_1,T_2).

Definition 2.3 [4] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy F_σ-set if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are pairwise fuzzy closed sets in (X,T_1,T_2).

Definition 2.4 [3] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy dense set if $cl_{T_1} cl_{T_2} (\lambda) = cl_{T_2} cl_{T_1} (\lambda) = 1$, in (X,T_1,T_2).

Definition 2.5 [5] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1} cl_{T_2} (\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$, in (X,T_1,T_2).

Definition 2.6 [4] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $int_{T_1} int_{T_2}(\lambda) = int_{T_2} int_{T_1}(\lambda) = 0$.

Definition 2.7 [6] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X,T_1,T_2) is called a pairwise fuzzy σ -first category set if $\lambda = \vee_{k=1}^{\infty} (\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy σ-second category set in (X,T_1,T_2).

Definition 2.8 [6] If λ is a pairwise fuzzy σ-first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy σ-residual set in (X, T_1, T_2).

III. Pairwise Fuzzy σ –Baire Sets

Definition 3.1. Let (X,T_1,T_2) be a fuzzy bitopological space. A fuzzy set λ defined on X is called a pairwise fuzzy σ -Baire set if $\lambda = \mu \wedge \delta$, where μ is a pairwise fuzzy open set and δ is a pairwise fuzzy σ -residual set in (X,T_1,T_2).

Example.3.1.

Let X = { a, b, c}. The fuzzy sets α,β,δ and μ are defined on X as follows :

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.2$; $\alpha(b) = 0.4$; $\alpha(c) = 0.7$.

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.2$; $\beta(b) = 0.2$; $\beta(c) = 0.6$.

$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.1$; $\delta(b) = 0.3$; $\delta(c) = 0.5$.

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.4$; $\mu(b) = 0.3$; $\mu(c) = 0.5$.

Clearly $T_1 = \{0, \alpha, \beta, \delta, \alpha\vee\beta, \beta\vee\delta, \alpha\wedge\beta, \alpha\wedge\delta, \beta\wedge\delta, \alpha\wedge[\beta\vee\delta], 1\}$ and $T_2= \{0, \alpha, \beta, \mu, \alpha\vee\beta, \alpha\vee\mu, \beta\vee\mu, \alpha\wedge\beta, \alpha\wedge\mu, \beta\wedge\mu, \beta\vee[\alpha\wedge\mu], \alpha\wedge[\beta\vee\mu], \mu\wedge[\alpha\vee\beta], \alpha\wedge\beta\wedge\mu, 1\}$ are fuzzy topologies on X. The fuzzy sets $\{\alpha, \beta, \alpha\vee\beta, \alpha\wedge\beta, \beta\vee\delta, \alpha\wedge[\beta\vee\mu], 1\}$ are pairwise fuzzy open sets in (X,T_1,T_2). Now the fuzzy sets 1-β and 1-(α∧β) are pairwise fuzzy F_σ-sets in (X,T_1,T_2). Also $int_{T1}int_{T2}(1-\beta) = int_{T2}int_{T1}(1-\beta) = 0$ and $int_{T1}int_{T2}(1-(\alpha\wedge\beta)) = int_{T2}int_{T1}(1-(\alpha\wedge\beta)) = 0$. Hence 1-β and 1-(α∧β) are pairwise fuzzy σ-nowhere dense sets in (X,T_1,T_2). The fuzzy set $\gamma = [(1-\beta)\vee[1-(\alpha\wedge\beta)]$ is a pairwise fuzzy σ -first category set in (X,T_1,T_2). Let $\eta = 1- \gamma$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2). Thus, $\beta = \alpha \wedge (1- \gamma)$ is a pairwise fuzzy σ -Baire set in (X,T_1,T_2).

Proposition 3.1.

If λ is a pairwise fuzzy σ-Baire set in a fuzzy bitopological space (X,T_1,T_2), then $1 - \lambda = \alpha \vee \beta$ where α is a pairwise fuzzy closed set in (X,T_1,T_2) and β is a pairwise fuzzy σ -first category set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy σ -Baire set in (X,T_1,T_2). Then $\lambda = \mu \wedge \delta$, where μ is a pairwise fuzzy open set and δ is a pairwise fuzzy σ -residual set in (X,T_1,T_2). Then $1- \lambda = 1 - (\mu \wedge \delta) = (1-\mu) \vee (1-\delta)$. Since μ is a pairwise fuzzy open set, 1-μ is a pairwise fuzzy closed set in (X,T_1,T_2). Also since δ is a pairwise fuzzy σ -residual set, 1-δ is a pairwise fuzzy σ -first category set in (X,T_1,T_2). Let α=1-μ and β=1-δ. Hence $1-\lambda=\alpha \vee \beta$, where α is a pairwise fuzzy closed set in (X,T_1,T_2) and β is a pairwise fuzzy σ -first category set in (X,T_1,T_2).

Proposition 3.2.

If $int_{Ti}int_{Tj}(\alpha)=0$ (i,j=1,2 and $i \neq j$), for any pairwise fuzzy σ -residual set α in (X,T_1,T_2) and if λ is a pairwise fuzzy σ -Baire set in (X,T_1,T_2), then $int_{Ti}int_{Tj}(\lambda)=0$ in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy σ -Baire set in (X,T_1,T_2). Then $\lambda = \mu \wedge \delta$, where μ is a pairwise fuzzy open set and δ is a pairwise fuzzy σ-residual set in (X,T_1,T_2). Now $int_{Ti}int_{Tj}(\lambda) = int_{Ti}int_{Tj}(\mu\wedge\delta) = int_{Ti}int_{Tj}(\mu) \wedge int_{Ti}int_{Tj}(\delta) = \mu \wedge int_{Ti}int_{Tj}(\delta)$. [Since μ is a pairwise fuzzy open set, $int_{Ti}int_{Tj}(\mu) = \mu$ in (X,T_1,T_2)]. By hypothesis, $int_{Ti}int_{Tj}(\delta)=0$ for the pairwise fuzzy σ -residual set δ in (X,T_1,T_2). Then, $int_{Ti}int_{Tj}(\lambda) = \mu \wedge 0 = 0$ in (X,T_1,T_2).

Proposition 3.3.

If each pairwise fuzzy σ -first category set is a pairwise fuzzy dense set in a fuzzy bitopological space (X,T_1,T_2) and if λ is a pairwise fuzzy σ -Baire set in (X,T_1,T_2), then 1-λ is a pairwise fuzzy dense set in (X,T_1,T_2).

Proof. Let λ be a pairwise fuzzy σ -Baire set in (X,T_1,T_2). Then by proposition 3.1, $1-\lambda=\alpha \vee \beta$, where α is a pairwise fuzzy closed set and β is a pairwise fuzzy σ -first category set in (X,T_1,T_2). By hypothesis, the pairwise fuzzy σ -first category set is a pairwise fuzzy dense set in (X,T_1,T_2). That is $cl_{Ti}cl_{Tj}(\beta) = 1$ in (X,T_1,T_2). Now, $cl_{Ti}cl_{Tj} (1-\lambda) = cl_{Ti}cl_{Tj} (\alpha \vee \beta) = cl_{Ti}cl_{Tj} (\alpha) \vee cl_{Ti}cl_{Tj} (\beta) = cl_{Ti}cl_{Tj} (\alpha) \vee 1 =1$. Then 1-λ is a pairwise fuzzy dense set in (X,T_1,T_2).

Theorem 3.1. [9]

If (λ_k)'s are pairwise fuzzy dense sets and pairwise fuzzy G_δ-sets in a fuzzy bitopological space (X,T_1,T_2), then $\wedge_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ -residual set in (X,T_1,T_2).

Proposition 3.4.

If $\lambda = \mu \wedge [\wedge_{k=1}^{\infty}\delta_k]$, where μ is a pairwise open set and (δ_k)'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in a fuzzy bitopological space (X,T_1,T_2), then λ is a pairwise fuzzy σ-Baire set in (X,T_1,T_2).

Proof. Let (δ_k)'s (k=1 to ∞) be pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X,T_1,T_2). Then by theorem 3.1, $\wedge_{k=1}^{\infty}(\delta_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2). Let $\lambda = \mu \wedge [\wedge_{k=1}^{\infty}(\delta_k)]$, where μϵT. Then $\lambda = \mu \wedge [\wedge_{k=1}^{\infty}(\delta_k)]$, where μ is a pairwise fuzzy open set and $\wedge_{k=1}^{\infty}(\delta_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2), implies that λ is a pairwise fuzzy σ-Baire set in (X,T_1,T_2).

Proposition 3.5.

If each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set in a fuzzy bitopological space (X,T_1,T_2) and if (λ_k)'s are pairwise fuzzy σ-Baire and pairwise fuzzy F_σ-sets in (X,T_1,T_2), then $\wedge_{k=1}^{\infty}(1-\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2).

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire and pairwise fuzzy F_σ-sets in (X,T_1,T_2). Then by proposition 3.3, $(1-\lambda_k)$'s are pairwise fuzzy dense sets in (X,T_1,T_2). Since (λ_k)'s are pairwise fuzzy F_σ-sets, $(1-\lambda_k)$'s are pairwise fuzzy G_δ-sets in (X,T_1,T_2). Hence, $(1-\lambda_k)$'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X,T_1,T_2). Then by theorem 3.1, $\wedge_{k=1}^{\infty}(1-\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2).

Proposition. 3.6.

If each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set in a fuzzy bitopological space (X,T_1,T_2) and if (λ_k)'s are pairwise fuzzy σ-Baire and pairwise fuzzy F_σ-sets in (X,T_1,T_2), then $\vee_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy dense sets in (X,T_1,T_2). Then by proposition 3.5, $\wedge_{k=1}^{\infty}(1-\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2). Since $\wedge_{k=1}^{\infty}(1-\lambda_k) = 1 - \vee_{k=1}^{\infty}(\lambda_k)$, $1 - \vee_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2) and then $\vee_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2).

IV. Pairwise Fuzzy σ –Baire Sets and Fuzzy Bitopological Spaces.

Definition 4.1 [4] A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy σ-Baire space if $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, (i = 1, 2) ,where (λ_k)'s are pairwise fuzzy σ -nowhere dense sets in (X, T_1, T_2).

Theorem 4.1 [6]

Let (X,T_1,T_2) be a fuzzy bitopological space. Then the following are equivalent:

(1). (X,T_1,T_2) is a pairwise fuzzy σ-Baire space.

(2). $int_{T_i}(\lambda) = 0$, (i = 1,2), for every pairwise fuzzy σ-first category set λ in (X,T_1,T_2).

(3) .$cl_{T_i}(\mu) = 1$, (i = 1,2) , for every pairwise fuzzy σ-residual set μ in (X,T_1,T_2).

Proposition 4.1.

If $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j), where (λ_k)'s are pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in a fuzzy bitopological space (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set, then (X,T_1,T_2) is a pairwise fuzzy σ-Baire spaces.

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set. Then by proposition 3.6, $\vee_{k=1}^{\infty}(\lambda_k)$ is a pairwise fuzzy σ-first category set in (X,T_1,T_2). From the hypothesis, $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$. Let $\mu = \vee_{k=1}^{\infty}(\lambda_k)$,Then $int_{T_i}(\mu) = 0$, where μ is the pairwise fuzzy σ-first category set in (X,T_1,T_2). Then by theorem 4.1, (X,T_1,T_2) is a pairwise fuzzy σ-Baire spaces.

Theorem. 4.2. [6]

If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy σ -Baire space, then (X,T_1,T_2) is a pairwise fuzzy σ -second category space.

Proposotion 4.2.

If $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j), where (λ_k)'s are pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in a fuzzy bitopological space (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set, then (X,T_1,T_2) is a pairwise fuzzy σ-second category space.

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets such that $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j) in (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set. Then by proposition 4.1, (X,T_1,T_2) is a pairwise fuzzy σ-Baire spaces. Hence by theorem 4.2, the pairwise fuzzy σ-Baire space in (X,T_1,T_2) is a pairwise fuzzy σ-second category space.

Proposition 4.3.

If $cl_{T_i}(\wedge_{k=1}^{\infty}(1-\lambda_k)) = 1$ (i,j=1,2 and i≠j), where (λ_k)'s are pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in a fuzzy bitopological space (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set, then (X,T_1,T_2) is a pairwise fuzzy σ-Baire space.

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set. Then by proposition 3.5, $\wedge_{k=1}^{\infty}(1-\lambda_k)$ is a pairwise fuzzy σ-residual set in (X,T_1,T_2). From the hypothesis, $cl_{T_i}(\wedge_{k=1}^{\infty}(1-\lambda_k)) = 1$. Let $\eta = \wedge_{k=1}^{\infty}(1-\lambda_k)$,Then $cl_{T_i}(\eta) = 1$, where η is the pairwise fuzzy σ-residual set in (X,T_1,T_2). Then by theorem 4.1,(X,T_1,T_2) is a pairwise fuzzy σ-Baire space.

Proposition 4.4.

If $cl_{T_i}(\wedge_{k=1}^{\infty}(1-\lambda_k)) = 1$ (i,j=1,2 and i≠j), where (λ_k)'s are pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in a fuzzy bitopological space (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set, then (X,T_1,T_2) is a pairwise fuzzy σ-second category space.

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets such that $cl_{T_i}(\wedge_{k=1}^{\infty}(1-\lambda_k)) = 1$ (i,j=1,2 and i≠j) in (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set. Then by proposition 4.3, (X,T_1,T_2) is a pairwise fuzzy σ-Baire space. Hence by theorem 4.2, the pairwise fuzzy σ-Baire space in (X,T_1,T_2) is a pairwise fuzzy σ-second category space.

Definition 4.2. [5] A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy almost resolvable space if $\vee_{k=1}^{\infty}(\lambda_k)=1$, where the fuzzy sets (λ_k)'s in (X,T_1,T_2) are such that $int_{T_1}int_{T_2}(\lambda_k)=0=int_{T_2}int_{T_1}(\lambda_k)$. Otherwise (X,T_1,T_2) is called a pairwise fuzzy almost irresolvable space.

Theorem 4.3. [8]

If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy σ -Baire space, then (X,T_1,T_2) is a pairwise fuzzy almost irresolvable space.

Remark 4.1. *The following propositions give conditions for fuzzy bitopological spaces to become pairwise fuzzy Baire space by means of pairwise fuzzy σ-Baire sets and pairwise fuzzy σ-first category sets.*

Proposition 4.5.

If $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j), where (λ_k)'s are pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in a fuzzy bitopological space (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set, then (X,T_1,T_2) is a pairwise fuzzy almost irresolvable space.

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets such that $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j) in (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set. By proposition 4.1, (X,T_1,T_2) is a pairwise fuzzy σ-Baire space. Hence by theorem 4.3, the pairwise fuzzy σ-Baire space in (X,T_1,T_2) is a pairwise fuzzy almost irresolvable space.

Definition 4.3 [5] A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy P-space if every non-zero pairwise fuzzy G_δ-set in (X,T_1,T_2) is pairwise fuzzy open in (X,T_1,T_2).

Definition 4.4 [5] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, (i= 1,2) where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X,T_1,T_2).

Theorem 4.4. [7]

If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy σ -Baire space and pairwise fuzzy P-space, then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proposition 4.6.

If $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j), where (λ_k)'s are pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in a pairwise fuzzy P-space (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set, then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets such that $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j) in (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set. By proposition 4.1, (X,T_1,T_2) is a pairwise fuzzy σ-Baire space. By hypothesis, (X,T_1,T_2) is a pairwise fuzzy P-space. Thus (X,T_1,T_2) is a pairwise fuzzy σ-Baire space and pairwise fuzzy P-space. By theorem 4.4, the pairwise fuzzy σ-Baire space and pairwise fuzzy P-space (X,T_1,T_2) is a pairwise fuzzy Baire space.

Definition 4.5 [5] A fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy submaximal space if each pairwise fuzzy dense set in (X,T_1,T_2), is a pairwise fuzzy open set in (X,T_1,T_2). That is., if λ is a pairwise fuzzy dense set in a fuzzy bitopological space (X,T_1,T_2), then $\lambda \epsilon T_i$ (i=1,2).

Theorem. 4.5. [7]

If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy σ -Baire space and pairwise fuzzy submaximal space, then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proposition 4.7.

If $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j), where (λ_k)'s are pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in a pairwise fuzzy submaximal space (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set, then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets such that $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j) in (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set. By proposition 4.1, (X,T_1,T_2) is a pairwise fuzzy σ-Baire space. By hypothesis, (X,T_1,T_2) is a pairwise fuzzy submaximal space. Thus (X,T_1,T_2) is a pairwise fuzzy σ-Baire space and pairwise fuzzy submaximal space. By theorem, 4.5, the pairwise fuzzy σ-Baire space and pairwise fuzzy submaximal space (X,T_1,T_2) is a pairwise fuzzy Baire space.

Definition 4.6 [10] A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy open hereditarily irresolvable space if $int_{T_1}cl_{T_2}(\lambda)\neq 0 \neq int_{T_2}cl_{T_1}(\lambda)$, then $int_{T_1}int_{T_2}(\lambda)\neq 0 \neq int_{T_2}int_{T_1}(\lambda)$ for any non-zero fuzzy set in (X,T_1,T_2).

Theorem 4.6. [10]

If the fuzzy bitopological space (X,T_1,T_2) is a pairwise fuzzy σ-Baire space and pairwise fuzzy open hereditarily irresolvable space, Then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proposition 4.8.

If $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j), where (λ_k)'s are pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets in a pairwise fuzzy open hereditarily irresolvable space (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set, then (X,T_1,T_2) is a pairwise fuzzy Baire space.

Proof. Let (λ_k)'s (k=1 to ∞) be pairwise fuzzy σ-Baire sets and pairwise fuzzy F_σ-sets such that $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$ (i,j=1,2 and i≠j) in (X,T_1,T_2) in which each pairwise fuzzy σ-first category set is a pairwise fuzzy dense set. By proposition 4.1, (X,T_1,T_2) is a pairwise fuzzy σ-Baire space. By hypothesis (X,T_1,T_2) is a pairwise fuzzy open hereditarily irresolvable space. Thus (X,T_1,T_2) is a pairwise fuzzy σ-Baire space and pairwise fuzzy open hereditarily irresolvable space. By theorem, 4.6, the pairwise fuzzy σ-Baire space and pairwise fuzzy open hereditarily irresolvable space (X,T_1,T_2) is a pairwise fuzzy Baire space.

V. CONCLUSION

In this paper, the concept of pairwise fuzzy σ -Baire sets is introduced and studied. Besides characterizations of these sets, several properties of these sets are studied. The condition under which fuzzy bitopological spaces to become fuzzy σ –Baire spaces, σ-second category spaces are obtained by means of pairwise fuzzy σ-Baire sets. The condition under which fuzzy bitopological spaces to become fuzzy σ-Baire spaces are obtained by means of pairwise fuzzy σ-first category set.

REFERENCES

[1] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24 ,1968, 182 – 190.
[2] A.Kandil, Biproximities and fuzzy bitopological spaces, Simon Stevin, 63,1989, 45– 66.
[3] G.Thangaraj, On pairwise fuzzy resolvable and fuzzy irresolvable spaces, Bull. Cal. Math.Soc., 101 ,2010, 59 – 68.
[4] G.Thangaraj and V.Chandiran, On pairwise fuzzy Volterra spaces, Ann.Fuzzy Math. Inform, 7(6), 2014, 1005 – 1012.
[5] G.Thangaraj and S.Sethuraman, On pairwise fuzzy Baire bitopological spaces, Gen.Math. Notes, 20(2),2014, 12 –21.
[6] G.Thangaraj and A.Vinothkumar, On pairwise fuzzy σ -Baire Spaces, Ann. Fuzzy Math. Inform,9(4), 2015, 529 – 536.
[7] G.Thangaraj and A.Vinothkumar, A note on pairwise fuzzy σ-Baire Spaces, Ann.Fuzzy Math. Inform,11(5), 2016, 729 – 736.
[8] G.Thangaraj and A.Vinothkumar, A Short note on pairwise fuzzy σ-Baire Spaces, IOSR Journal of Mathematics,12(3), 2016, 27 – 30.
[9] G.Thangaraj and A.Vinothkumar, Pairwise fuzzy σ-first category sets, International Journal of Applied Engineering Research,13(21), 2018, 14846 – 14848.
[10] A.Vinothkumar and G.Thangaraj, More on pairwise fuzzy σ-Baire Spaces, International Journal of Research and Analytical Reviews, Special Issue, 2019, 62 – 65.
[11] L.A.Zadeh, Fuzzy Sets , Information and Control, 8 ,1965, 338 – 353.