STOCHASTIC APPROACH FOR REPLACEMENT STRATEGIES IN MANPOWER PLANNING

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Abstract

Manpower planning is a dynamic process, which manages the flow of labour into, through and out of the organization, to achieve an optimum match. Manpower planning must take into consideration the total corporate plan together with issues like technology, competition, legislation and regulation by the government plus changing social norms and expectations, which affect people employment. To be effective, plans must ensure that critical issues are addressed, appropriate actions taken and the process sustained, all of which gives it credibility and a sense of ownership among the implementers. Statistical tools are indispensable tools in the analysis of the planning of human resources. The formation of human resource, its wastage during the phase of its development, recanalization of the part of the resources into some undesirable sectors, and finally inadequate utilization of the produced human capital due to unforeseen circumstances can only be adequately analyzed by appropriate statistical techniques. In this paper, the application of renewal process and reliability theory can be combined together to decide suitable manpower replacement policies for promotion as well as replacement of manpower for the successful maintenance of the system is discussed.

Key word: Stochastic model, Manpower planning, Replacement strategies, Promotion policy, Reliability

Introduction

Manpower planning assesses current levels and utilization of staff and skills, relates these elements to the market demand for the organizations products and provides alternatives to match manpower resources with anticipated demand. It is a dynamic process, which manages the flow of labour into, through and out of the organization, to achieve an optimum match. Manpower planning is the systematic approach to the utilization of available manpower according to the demands that exist in different sectors of the country. Hence, manpower models are developed taking into consideration the various real life situation. Manpower is a term, which means a group of persons who have acquired some particular skill or expertization to undertake a particular type of job. Scientific methods of approach to economic problems and evolving suitable economic policies are important aspects of efficient administration. The reliability theory and replacement strategies can be combined together to decide suitable manpower policies. The application of this theory is considered in relation to manpower systems and suitable strategies for promotion as well as replacement of manpower for the successful maintenance of the system is discussed. Application of replacement strategies to manpower planning has been discussed by Robinson (1974).

Methodology

It is conceptualized that there are two compartments namely, the grade I, which is called the training grade and there is a grade called grade II known as the grade in the organization is considered in this study. The individual replacement in stage I is more expensive than in stage II. If a person in stage II leaves the organization the vacancy is filled up from stage I. If there is any wastage in stage I due to exit and also transfer from stage I to stage II then recruitment for training grade is done. It may be observed that the hazard rate, which is otherwise known as the force of separation depends upon the completed length of service. It may be observed that the force of separation or the propensity of the individual to leave the organization depends upon completed length of service and it is defined as the probability that a person leaves in a small time interval $(t, t+\delta t)$ having served a period of t and $\lambda(t) = \frac{f(t)}{G(t)}$. Under these assumptions it is proposed to determine the size of grade I or the number of persons in the training grade. The mean time to promotion is also to be determined.

Notations

(i) N_1 and N_2 denote the size of the training grade and the organization respectively. N_2 is assumed to be known and fixed. i.e., $N = N_1 + N_2$
(ii) W_1 : individual loss rate in Grade I
(iii) W_2 : individual loss rate in Grade II
(iv) P : Promotion rate.

The two rules examined here are

(i) Promotion by seniority, i.e., the most senior (and hence the most experienced) member of the training grade is promoted.

(ii) Promotion at random, with respect to length of service. Thus promotion is made on some other, uncorrelated, basis- e.g. ability, personnel qualities and are given in the following figure.

Model

We suppose that all individuals have the same completed length of service (CLS) distribution, with density function f(t), where t denotes the time which has elapsed since the person in question joined the organization. Thus f(t) δt is the probability that a man leaves with t period of service in (t, t+δt). The survivor function G(t) gives the probability that a person remains in the organization for at least time t. Hence G(t) $=\int_t^{\infty} f(u)du$.

We define the further quantity λ(t), known as the 'force of separation at length of service t', as follows. λ(t) δt is the probability that an individual who has been in the organization for a length of time 't', leaves in the interval (t, t+δt). It is easily shown that λ(t) =f(t)/ G(t), for all $t \geq 0$.

Since the input and output of each grade must balance, we have for Grade II

$N_1P=N_2W_2$ … (1)

Moreover, in equilibrium the expected input to the system over unit time is N/μ, where μ the mean length of completed service. Thus for grade I, we have,

$N/\mu = N_1(P + W_1) = N_1 W_1 + N_2 W_2$ … (2)

In general W_1, W_2 and P are functions of time. It shows that P, W_1, W_2 tend to equilibrium values which are independent of the age of the system.

Let μ_1 be the average time spent in Grade I. Then in equilibrium the expected number of vacancies occurring per unit time in this Grade is N_1/μ_1. These can be caused either by promotion or losses, and hence

$N_1/\mu_1 = N_1 (P + W_1)$

$1/\mu_1 = P + W_1$ … (3)

It follows that (2) and (3) that

$N/\mu_1. = N_1/\mu_1.$ … (4)

PROMOTION BY SENIORITY

Let a(t/T) denote the age distribution of the system at time T given that the system was established at T = 0. Thus a(t/T)δt is the

probability that an individual chosen at random at time T has length of service in (t, t+δt). Thus

a(t/T) δt= P{ individual joined in (T-t, T-t+δt) and remained for time t}

= h(t-T)δT G(t)

where h(t) is the renewal density for the whole system, i.e. h(t)δt is the probability of loss in (t, t+δt). Hence a(t/T) = h(T-t) h(t). This is true for t<T, when t=T, We have

a(T/T) = P{Original member of organization is still there at time T} = G(T)

Now $\lim_{T \to \infty} h(T)=1/\mu$, … (5)

whatever the form of the CLS distribution.

Now, since a loss from Grade II is replaced by the most senior member of Grade I, it follows that at any time every individual in Grade II has length of service at least as long as any individual in Grade I and hence that there exists some threshold value t_i such that all individuals with length of service less than t_1 are in Grade I, where t_1 is a random variable. But if the grade sizes are large their expected value can be found from the approximate formula,

$\int_{t_1}^{\infty} a(t)dt = N_2/(N_1 + N_2)$ …(6)

The expected number of promotions per unit time will be the proportion of new recruits whose services to the threshold length of service t_1 is

$N_1 P = G(t_1) N/\mu$ … (7)

PROMOTION AT RANDOM

Let $F_1(t)$ be the probability that an individual remains in the system for a time t without being promoted. Let n_t be the number of promotions in (0,t), then

$F_1(t)$ = Prob{Individual not promoted in (0,t) / doesn't leave in (0,t)} . Prob {doesn't leave in (0,t)}

$= (1 - 1/N_1)^{n_t} G(t)$

Now expected value of n_t is N_1P_1. Thus, as a first approximation we can take

$$F_1(t) = G(t)\, e^{-P_t}$$

It follows that the average time spent in Grade I

$$\mu_1 = \int_0^\infty F_1(t)dt = \int_0^\infty G(t)e^{-P_t}dt$$

But from (4), $\mu_1 = N_1\, \mu / N$, Thus

$$1/\mu \int_0^\infty G(t)e^{-P_t}dt = N_1/N \quad \ldots(8)$$

THE MEAN TIME TO PROMOTION

Let μ_L be the average length of time spent in Grade I, and let μ_P be the average length of time spent in Grade I by those who are eventually promoted to Grade II. As before, let μ_1 be the average sojourn time in Grade I. (This can be terminated by promotion on leaving). Let us consider the problem of choosing N_1 so that μ_p has some predetermined value. The two promotion rules are considered separately.

PROMOTION BY SENIORITY

In this case μ_p is equivalent to the average value of t_1 introduced earlier, and hence equations (6) and (7) hold.

Thus we have

$$\int_{\mu_p}^\infty a(t)dt = N_2/N \quad \ldots (9)$$

$$1/\mu \int_{\mu_p}^\infty G(t)dt = N_2\,/N \quad \ldots (10)$$

Also $N_1 P = G(\mu_p)\frac{N}{\mu}$.

Equation (10) gives $R = N_1/N_2$ as a function of μ_p and hence knowing the size N_2 of the organization, we can determine N_1 for any specified value of μ_p.

PROMOTION AT RANDOM

Let q(t) be the probability density function of the time interval between entry of an individual into Grade I and his promotion to Grade II, given that he is promoted before leaving

Let $Q(t) = \int_t^\infty q(x)dx$, then Q(t)=P{Not promoted in a period of length t / doesn't leave in that period}

$$= (1\text{-}1\,/\,N_1)^{n\,t}$$

where n_t is the number of promotions in an interval of length t. Proceeding as before replace n_t by its expected value, $N_1 P_t$. Thus we obtain the approximate relation, $Q(t) = (1\text{-}1\,/\,N_1)\, N_1\, P_t$

Assuming that N_1 is large, We have $Q(t) \cong e^{-p_t}$

Then $\mu_p = \int_0^\infty Q(t)dt = \int_0^\infty e^{-pt}\, dt = 1/p$

Using this in conjunction with equation (9) gives N_1 in terms of μ_p .

We have $$\frac{N_1}{N} = 1/\mu \int_0^\infty G(t)\, e^{-t/\mu_p}\, dt \quad \ldots (11)$$

Promotion by seniority

Case (i)

If the Completed length of services (CLS) is taken to be mixed exponential distribution

$$f(t) = x\lambda e^{-\lambda_1 t} + (1-x)\lambda_2 e^{-\lambda_2 t}; \quad 0 < x < 1; \lambda_1, \lambda_2 > 0; t \geq 0$$

Then $G(t) = xe^{-\lambda_1 t} + (1-x)e^{-\lambda_2 t}$

$$\mu = x/\lambda_1 + (1-x)/\lambda_2$$

From equation (11) implies

$$\boldsymbol{R} = \frac{x\,(1\text{-}e^{\lambda_1{}^{\mu_p}})/\lambda_1 + (1\text{-}x)\,(1\text{-}e^{\lambda_2{}^{\mu_p}})/\lambda_2}{x\,(e^{-\lambda_1{}^{\mu_p}})/\lambda_1 + (1\text{-}x)\,(e^{-\lambda_2{}^{\mu_p}})/\lambda_2}$$

The values of R corresponding to various values of μ_p for the mixed Exponential CLS distribution with parameters x=0.4, $\lambda_1 = 0.2$ and $\lambda_2 = 2.0$ so that $\mu = 2.3$ are given in table 2.

Case (ii)

Now the CLS taken to be a Pearsonian Type XI distribution and the behavior of R with respect to μ_p ,γ and c is studied.

$$f(t) = \frac{\gamma}{c}\left(1 + \frac{t}{c}\right)^{-(\gamma-1)}$$

$$G(t) = (1 + t/c)^{-\gamma}; t \geq 0, \gamma > 1, c > 0$$

$$\mu = \frac{c}{\gamma - 1}$$

From equation (11) implies

$$\frac{N_2}{N} = \frac{1}{\mu}\int_{\mu_p}^{\infty} G(t).dt$$

$\frac{N_2}{N} = \frac{1}{\mu}\int_{\mu_p}^{\infty}(1+t/c)^{-\gamma}.dt$

The integration is solved by using Trapezoidal rule.

Then $R = \left[\left(\frac{c}{\mu_p+c}\right)^{1-\gamma} - 1\right]$. The value of R for the various values of μ_p , γ and c is given in Table 3.

Table 1: CLS as Mixed Exponential Distribution

μ_p	0.5	1.0	1.5	2.0	2.5	3.0
R	0.196	0.399	0.609	0.828	1.059	1.306

Table 2: CLS as Pearsonian Type XI Distribution

μ \ γ	1.0	1.5	2.0	2.5	3.0
2	0.3333	0.500	0.6667	0.8333	1.00
4	1.3704	2.3750	3.6296	5.1620	7.00
6	3.2140	6.5937	11.8601	19.7113	31.00
8	6.4915	16.0859	34.7224	68.6129	127.00
10	12.3183	37.4433	98.2290	232.9768	511.00

PROMOTION AT RANDOM

Case (iii)

If CLS is taken to be mixed Exponential distribution, then eqn.(11) implies

$$N_1/N = 1/\mu \int_0^{\infty} e^{-t/\mu_p}\left[xe^{-\lambda_1 t} + (1\text{-x})e^{-\lambda_2 t}\right]dt$$
$$= \{x/(\lambda_1 + 1/\mu_p) + (1\text{-x})/(\lambda_2 + 1/\mu_p)\}/\mu$$

Then

$$R = \frac{[x(\lambda_2 + 1/\mu_p) + (1\text{-x})(\lambda_1 + 1/\mu_p)]\lambda_1\,\lambda_2\,\mu_p}{x\lambda_2(\lambda_2 + 1/\mu_p) + (1\text{-x})\,\lambda_1(\lambda_1 + 1/\mu_p)}$$

Case (iv)

Now the CLS is taken to be a Pearsonian Type XI distribution and the behavior of R with respect to μ_p is studied, $f(t) = \frac{\gamma}{c}\left(1+\frac{t}{c}\right)^{-(\gamma-1)}$, $\mu = \frac{c}{\gamma-1}$

and equation (12) implies

$\frac{N_1}{N} = \frac{1}{\mu}\int_0^{\infty} G(t).e^{-t/\mu_p}dt$,

$\frac{N_1}{N} = \frac{1}{\mu}\int_0^{\infty}(1+t/c)^{-\gamma}.e^{-t/\mu_p}.dt$

$$R = \frac{(c^{\gamma-1}(\gamma-1)\int_0^{\infty}(1+t/c)^{-\gamma}.e^{-t/\mu_p})}{1-(c^{\gamma-1}(\gamma-1)\int_0^{\infty}(1+t/c)^{-\gamma}.e^{-t/\mu_p})}$$

The integration is solved by using Trapezoidal rule.

Table 3 : CLS as Mixed Exponential Distribution

μ_p	0.5	1.0	1.5	2.0	2.5	3.0
R	0.172	0.322	0.462	0.596	0.728	0.857

Table 4: CLS as Pearsonian Type XI Distribution

μ_p \ γ	1	1.5	2	2.5	3
2	0.2721	0.4786	0.4872	0.5845	0.6770

4	0.8577	1.2671	1.6015	1.9779	2.3861
6	1.4786	2.1736	2.8607	3.5427	4.2215
8	2.1145	3.1320	4.1433	5.1513	6.1573
10	2.6710	4.1057	5.4461	6.7838	8.1206

DISCUSSION

In the case of Mixed exponential distribution it is found that to have promotion earlier, it is better to have less of recruits at the training grade. In the case of CLS as Pearsonian Type XI distribution, only a small proportion of trainees would survive to promotion and hence a large training grade would be required. In organizations with large number of employees and with different grades in hierarchy, the study of the data relating to completed length of service will be useful to predict the size or the number of persons to be trained at each grade so that the normal routine of functions in the organization will not suffer due to the wastage of personnel in the different grades. This is due to the fact that the prediction of number of leavers for different length of CLS is possible.

CONCLUSION

Table 1 shows the values of R corresponding to various values of μ_p and the particular distribution is fitted to manpower data, it is normally found that x takes a value near ½ and that λ_2 is about ten times the value of λ_1, and hence producing a fairly skew distribution. Then the mixed exponential CLS distribution with parameters $\lambda_1 = 0.2$, $\lambda_1 = 2.0$, $x = 0.4$, so that $\mu = 2.3$.

Table 2 shows values of R calculated for various values of γ and μ_p. The values of R becomes large when γ, μ_p increase. The average time to promotion is greater than the overall mean length of completed service. Thus only a very small proportion of trainees would survive to promotion and hence a large training grade would be required. Table 3. gives the values of R for promotion at random corresponding to the values given in table 1 for promotion by seniority.

$$R = \left(e^{-\lambda\mu_p} - 1\right) = \lambda\mu_p + \left(\lambda\mu_p\right)^2/2! + \ldots$$

Thus for λ, $\mu_p<1$ we have $R \cong \lambda\mu_p$, which in the result obtained for promotion at random. This would suggest that for small R the two promotion rules are equivalent.

A possible explanation is as follows. As $R \to 0$, i.e. the size of the training grade becomes smaller and smaller relative to the size of the organization, the age at which promotions occur will decrease whichever selection rule is used. In the limit, the average time to promotion will be very small in both cases and the differences produced by the two promotion rules will be practically negligible. Table 3. shows that for a given μ_p the value of R required to produce this μ_p is greater in the case of promotion by seniority than when promotion is at random. Table 4. shows values of R calculated for various values of γ and μ_p. The values of R become large, when γ, $\mu_p > 1$, and then increase as (γ, μ_p) increases. This is what one would intuitively expect, since $\gamma,\mu_p > 1$ implies that $\mu_p > 1/\gamma$, i.e. the average time to promotion is greater than the overall mean length of completed service. Thus only a very small proportion of trainees would survive to promotion and hence a large training grade would be required.

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