

Naqueeb's formula of interpolation

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Abstract :- Interpolation is a way to get value of any function at any value of variable either those values present or not. We have many formulae and methods for it, but Bessel's formula is more approximate than other. As we know that all methods and formulae gives many times an approximate value of function at desired value of variable due to convergence nature of function but Bessel's formula helps to get more nearest value of those function at desired point. But Bessel's formula is not enough and we shouldn't stop with this formula. There is no doubt Bessel's formula is more efficient, effective and approximate but there exists a new formula which is more approximate than Bessel's formula. This formula is derived by me and it is one step ahead and more effective, accurate and easy to apply.

Keywords :- Interpolation, approximation, efficient, convergence nature, mean value, Bessel's formula, Sterling's formula, combination.

Introduction :- We know that as we progressed we get some advance and efficient tools for our life which makes it easy. Also we get many options and suitable ways for facing difficulties. Many years ago we were unable to get value of any function at any point but now it is possible. But calculating exact value for all function is difficult yet specially for convergent function. There are many formulae regarding this derived and all these are giving approximate value finely but we need much exact way to get exact value. Bessel's formula and Sterling's formula are more powerful formula but not enough yet. I derived a new formula which is more powerful than these two formulae and gives much accurate, approximate value of any function.

Explanation :- We have Sterling's formula and Bessel's formula for interpolation.

Bessel's formula is more approximate formula for interpolation yet. But now, I am introducing a new formula for interpolation which is more approximate than

Bessel's formula and I have named it Naqueeb's formula. This formula deals about any kinds of interpolation.

After taking mean of Bessel's and Sterling formula we have

$$y = \frac{1}{4} [(3y_0 + y_1) + \{(3u - 1)\Delta y_0 + u\Delta y_{-1}\} + \left\{ \frac{u(3u-1)\Delta^2 y_{-1} + u(u-1)\Delta^2 y_0}{2} \right\} + \frac{u(u-1)}{3} \{3u\Delta^3 y_{-1} + (u+1)\Delta^3 y_{-2}\} + \dots$$

For example

$$y = \log_{10} x ,$$

Actual value of $y_{25} = 1.398$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	1				
20	1.3010	0.3010			
30	1.4771	0.1761	-0.1249		
40	1.6020	0.1249	-0.0512	0.0737	
50	1.6990	.0670	-0.0579	-0.0067	-0.0804

By Bessel's formula

$$y_{25} = \frac{1}{2} (1.3010 + 1.4771) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!} \cdot \frac{(-0.1761)}{2} + \dots$$

$$= 1.38905 + 0.0110$$

$$= 1.40005$$

But, by this formula

$$\begin{aligned}
 y_{25} &= \frac{1}{4} [(3 \times 1.3010 + 1.4771) + \left\{ \left(\frac{3}{2} - 1 \right) \times 0.1761 + \frac{1}{2} \times 0.3010 \right\} \\
 &+ \left\{ \frac{1}{4} \times \frac{-0.1249 - 0.0512}{2} \right\} \\
 &+ \frac{\left(\frac{1}{2} \right) \left(-\frac{1}{2} \right)}{3} \left\{ \frac{3}{2} \times 0.0067 + \frac{3}{2} \times 0.0737 \right\} + \dots \\
 &= \frac{1}{4} [5.3801 + 0.23855 - 0.02201 - \frac{1}{12} \times 0.1206] \\
 &= \frac{1}{4} \times 5.58659 = \mathbf{1.3966}
 \end{aligned}$$

From above example, it is clear that proposed formula gives most approximate result than other known method.

References:-

1. Bessel's formula
2. Sterling's formula