

Evaluation of Reliability of a power plant by using Boolean Function Expansion Algorithm

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Abstract:

In this paper, research was performed with the assistance of the Boolean function expansion algorithm to determine the reliability behavior of a power plant. Three power generators in a power house make up the dynamic structure under consideration. The aim of the system is to supply power from a power house (generated by generators) to critical consumers fed from a main output switch, thus calculating the reliability of the power supply by taking into account the failure times for different components, i.e. Arbitrary delivery is followed by wires, generators and primary switch boards, etc. System mean time to failure (M.T.T.F.) was also determined for the distribution of exponential failure time, in particular. To illustrate the utility of the model, some graphs have also been plotted.

Key words:

Boolean function expansion algorithm, Reliability, M.T.T.F, Weibull Distribution, Exponential Distribution.

1. INTRODUCTION:

Reliability is far from an abstract idea nowadays and it ranks at the same level as equipment efficiency. In addition, for all reliability studies, assessment of reliability is a basic prerequisite. It is, however, an open secret that when complexities increase in a system, reliability assessment becomes much too difficult. The derivation of a symbolic expression of reliability for a complex system in a condensed and compact form is therefore of vital importance.

The problem of ensuring engineering systems' reliability is highly complex and applies to all stages of a system's service life. There are a large number of problems today which, for example, are still solved only on the basis of rational reasoning and experience and not with the help of reliability calculations in the design of marine power plants.

A number of studies were carried out by previous researchers [1-7] to evaluate symbolic reliability expressions for complex systems in different reliability systems. The reliability behavior of power plants with the help of the orthogonalization algorithm was considered by Gupta and Sharma. The reliability of the power supply from a power house to a vital user was considered by Gupta and Agarwal, given that all the cables used in the power house are 100 % perfect reliable, but the cables need not be 100 percent reliable in operation.

Therefore, holding the above facts in mind, we considered a complex structure consisting of three parallel related sub-systems. And in a power house, these three sub-systems are nothing but three power generators G_1 , G_2 and G_3 . The

generators G_1, G_2 and G_3 are paired with two-way main switches MSB_1, MSB_2 and MSB_3 are perfectly secure cables, respectively. A connection cable C_4 connects MSB_1, MSB_2 and a connecting cable C_5 connects MSB_2, MSB_3 .

Further cables C_1, C_2 and C_3 connect the two-way main switch MSB_1 to the main output switch $OPMS_4$ and the two-way switch MSB_2 to the main output switch $OPMS_4$ and the two-way switch MSB_3 to the main output switch $OPMS_4$. Thus, nothing but a power plant is the complex system under consideration.

The purpose of the system is to supply power generated by G_1, G_2 and G_3 to critical consumers and consequently the reliability of the power supply fed from $OPMS_4$ has been estimated with the aid of Boolean function expansion algorithm by considering that failure times for various components of the system follow arbitrary time distribution.

Moreover, an important parameter, viz. M.T.T.F., has also been calculated for exponential failure time distribution for various components of the system. Some numerical examples, along with graphs, have also been appended to highlight the important results.

2 ASSUMPTIONS:

- (i) The reliabilities of all constituent components of the system are known in advance.
- (ii) The states of all components are statically independent.
- (iii) The state of each component and of the whole system is either good (operating) or bad (failed)
- (iv) There is no standby or switched redundancy.
- (v) The failure times for all the components are arbitrary.
- (vi) There is no repair facility.
- (vii) The system can fail, i.e., the supply of power can fail, only if :
 - (i) All the three generators fail
 - (ii) At least one component (switch or cable) in the routes of the power supply fails.

3. NOTATIONS:

x_1, x_2, x_3	States of sub-systems (generators) G_1, G_2 and G_3
x_4, x_5, x_6	States of MSB_1, MSB_2 and MSB_3

$x_7, x_8, x_9, x_{11}, x_{12}$ States of the cables C_1, C_2, C_3, C_4 and C_5

x_{10} States of $OPMS_4$

x_k^1 Negation of x_k ($k = 1-12$)

\wedge Conjunction

\vee Disjunction

$$x_i = \begin{cases} 0, & \text{in badstate,} \\ 1, & \text{in good state,} \end{cases} \quad i = (1-12)$$

$\Pr(f = 1)$ The probability of the successful operation of the function f .

The complex system under consideration is shown in Fig.1.

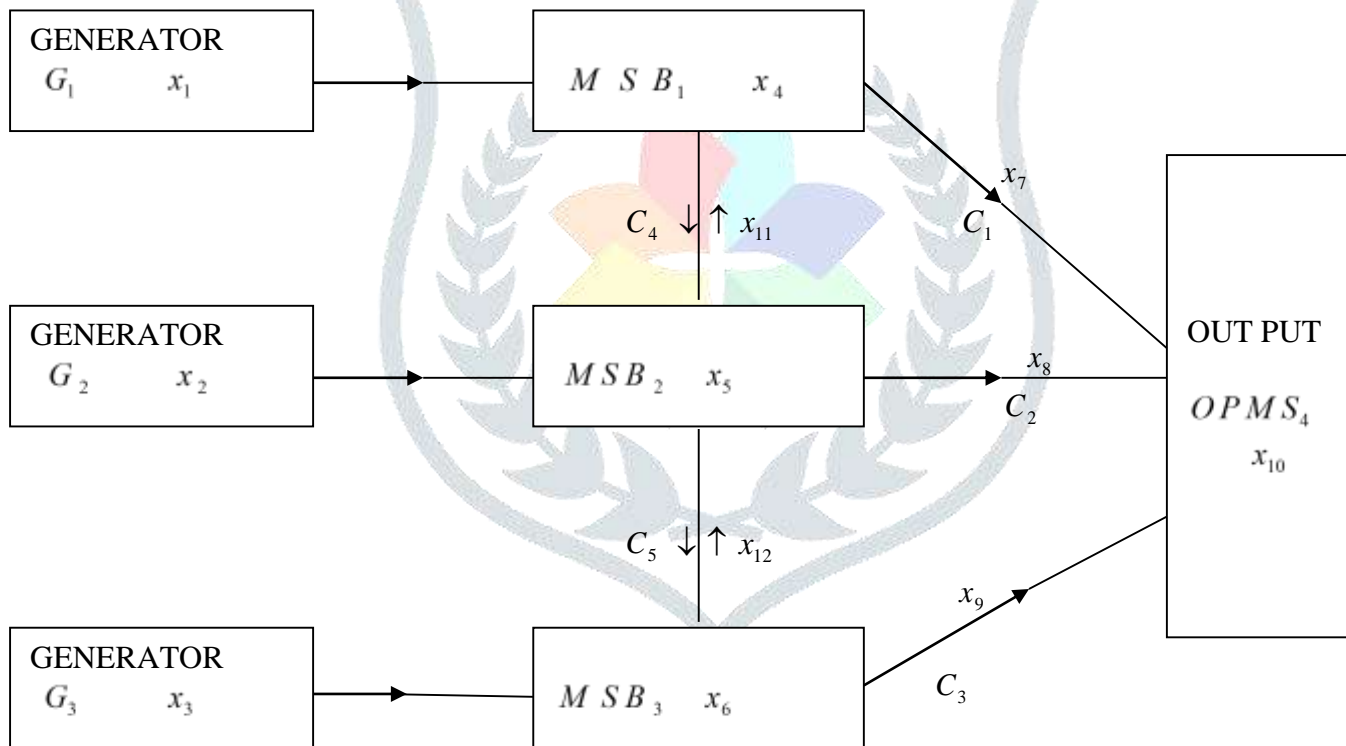


Fig.1 System Configuration

4. FORMULATION OF MATHEMATICAL MODEL:

By using Boolean function technique, the conditions of capability for the successful operation of the complex system in terms of logical matrix are expressed as

$$f(x_1, x_2, x_3, \dots, x_{12}) = \begin{vmatrix} x_1 & x_4 & x_7 & x_{10} \\ x_1 & x_4 & x_{11} & x_5 & x_8 & x_{10} \\ x_1 & x_4 & x_{11} & x_5 & x_{12} & x_6 & x_9 & x_{10} \\ x_2 & x_5 & x_8 & x_{10} \\ x_2 & x_5 & x_{11} & x_4 & x_7 & x_{10} \\ x_2 & x_5 & x_{12} & x_6 & x_9 & x_{10} \\ x_3 & x_6 & x_9 & x_{10} \\ x_3 & x_6 & x_{12} & x_5 & x_8 & x_{10} \\ x_3 & x_6 & x_{12} & x_5 & x_{11} & x_4 & x_7 & x_{10} \end{vmatrix} \tag{1}$$

5. SOLUTION OF THE MODEL:

By the application of algebra of logic equation (1) may be written as

$$f(x_1, x_2, x_3, \dots, x_{12}) = x_{10} \wedge g(x_1, x_2, x_3, \dots, x_9, x_{11}, x_{12}) \tag{2}$$

$$g(x_1, x_2, x_3, \dots, x_9, x_{11}, x_{12}) = \begin{vmatrix} x_1 & x_4 & x_7 \\ x_5 & x_8 & x_{11} \\ x_5 & x_6 & x_9 & x_{11} & x_{12} \\ x_2 & x_5 & x_8 \\ x_4 & x_7 & x_{11} \\ x_6 & x_9 & x_{12} \\ x_3 & x_6 & x_9 \\ x_5 & x_8 & x_{12} \\ x_4 & x_5 & x_7 & x_{11} & x_{12} \end{vmatrix} \tag{3}$$

Now there are two arguments (x_{11}, x_{12}) entering into equation (3) four times, therefore, any of them may be taken to perform the first expansion. Let us take x_{11} and break the complex event into incompatible events as follows

$$g(x_1, x_2, x_3, \dots, x_9, x_{11}, x_{12}) = x_{11}^1 y_0 \vee x_{11} y_1 \tag{4}$$

Where

$$y_0 = \begin{vmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_8 \\ & & x_6 & x_9 & x_{12} \\ x_3 & x_6 & x_9 \\ & & x_5 & x_8 & x_{12} \end{vmatrix} \tag{5}$$

Now, expand the function y_{010} by argument x_9 (say) as follows

$$y_{010} = x_9^1 y_{0100} \vee x_9 y_{0101} \quad (13)$$

Where

$$y_{0100} = \begin{vmatrix} x_1 & x_4 & x_7 \end{vmatrix} \quad (14)$$

$$y_{0101} = \begin{vmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_6 \\ x_3 & x_6 & \end{vmatrix} \quad (15)$$

Since all the letters occur in equation (14) only once, it implies that y_{0100} is non-iterated.

Now, expand the function y_{0101} by argument x_6 (say) as follows

$$y_{0101} = x_6^1 y_{01010} \vee x_6 y_{01011} \quad (16)$$

Where

$$y_{01010} = \begin{vmatrix} x_1 & x_4 & x_7 \end{vmatrix} \quad (17)$$

$$y_{01011} = \begin{vmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & \\ x_3 & & \end{vmatrix} \quad (18)$$

Since all the letters occur in equations (17) and (18) only once, it implies that y_{01010} and y_{01011} are non-iterated.

Now, we expand the function y_{011} by argument x_9 (say) as follows:

$$y_{011} = x_9^1 y_{0110} \vee x_9 y_{0111} \quad (19)$$

Where

$$y_{0110} = \begin{vmatrix} x_1 & x_4 & x_7 \\ x_3 & x_6 & x_5 \end{vmatrix} \quad (20)$$

$$y_{0111} = \begin{vmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_6 \\ x_3 & x_6 & x_5 \end{vmatrix} \quad (21)$$

Since all the letters occur in equation (20) only once, it implies that y_{0110} is non-iterated.

Now, expand the function y_{0111} by argument x_6 (say) as follows:

$$y_{0111} = x_6^1 y_{01110} \vee x_6 y_{01111} \quad (22)$$

$$y_{11} = \begin{vmatrix} x_1 & x_4 & x_7 \\ & & x_5 & x_8 \\ & & x_5 & x_6 & x_9 \\ x_2 & x_5 & x_8 \\ & & x_4 & x_7 \\ & & x_6 & x_9 \\ x_3 & x_6 & x_9 \\ & & x_5 & x_8 \\ & & x_4 & x_5 & x_7 \end{vmatrix} \quad (30)$$

Now expand the function y_{10} by argument x_8 (say) as follows

$$y_{10} = x_8^1 y_{100} \vee x_8 y_{101} \quad (31)$$

Where

$$y_{100} = \begin{vmatrix} x_1 & x_4 & x_7 \\ x_2 & x_5 & x_4 & x_7 \\ x_3 & x_6 & x_9 \end{vmatrix} \quad (32)$$

$$y_{101} = \begin{vmatrix} x_1 & x_4 & x_7 \\ & & x_5 \\ x_2 & x_5 & x_4 & x_7 \\ x_3 & x_6 & x_9 \end{vmatrix} \quad (33)$$

Now, expand y_{100} by argument x_4 (say) as follows

$$y_{100} = x_4^1 y_{1000} \vee x_4 y_{1001} \quad (34)$$

Where

$$y_{1000} = \begin{vmatrix} x_3 & x_6 & x_9 \end{vmatrix} \quad (35)$$

$$y_{1001} = \begin{vmatrix} x_1 & x_7 \\ x_2 & x_5 & x_7 \\ x_3 & x_6 & x_9 \end{vmatrix} \quad (36)$$

Since all the letters occur in equation (35) only once, it implies that y_{1000} is non-iterated.

Now, expand the function y_{1001} by argument x_7 (say) as follows:

$$y_{1001} = x_7^1 y_{10010} \vee x_7 y_{10011} \quad (37)$$

Where

$$y_{10010} = \begin{vmatrix} x_3 & x_6 & x_9 \end{vmatrix} \quad (38)$$

$$y_{10011} = \begin{vmatrix} x_1 \\ x_2 & x_5 \\ x_3 & x_6 & x_9 \end{vmatrix} \quad (39)$$

Since all the letters occur in equation (38), (39) only once, it implies that y_{10010} and y_{10011} are non-iterated.

Now, expand the function y_{101} by argument x_4 (say) as follows:

$$y_{101} = x_4^1 y_{1010} \vee x_4 y_{1011} \quad (40)$$

Where

$$y_{1010} = \begin{vmatrix} x_3 & x_6 & x_9 \end{vmatrix} \quad (41)$$

$$y_{1011} = \begin{vmatrix} x_1 & x_7 \\ & x_5 \\ x_2 & x_5 & x_7 \\ x_3 & x_6 & x_9 \end{vmatrix} \quad (42)$$

Since all the letters occur in equation (41) only once, it implies that y_{1010} is non-iterated.

Now, expand the function y_{1011} by argument x_7 (say) as follows:

$$y_{1011} = x_7^1 y_{10110} \vee x_7 y_{10111} \quad (43)$$

Where

$$y_{10110} = \begin{vmatrix} x_1 & x_5 \\ x_3 & x_6 & x_9 \end{vmatrix} \quad (44)$$

$$y_{10111} = \begin{vmatrix} x_1 & x_5 \\ x_2 & x_5 \\ x_3 & x_6 & x_9 \end{vmatrix} \quad (45)$$

Since all the letters occur in equation (44) only once, it implies that y_{10110} is non-iterated.

Now, expand the function y_{10111} by argument x_5 (say) as follows:

$$y_{10111} = x_5^1 y_{101110} \vee x_5 y_{101111} \quad (46)$$

Where

$$y_{101110} = \begin{vmatrix} x_3 & x_6 & x_9 \end{vmatrix} \quad (47)$$

$$y_{101111} = \begin{vmatrix} x_1 \\ x_2 \\ x_3 & x_6 & x_9 \end{vmatrix} \tag{48}$$

Since all the letters occur in equations (47) and (48) only once, it implies that y_{101110} and y_{101111} is non-iterated.

Now, expand the function y_{11} by argument x_5 (say) as follows

$$y_{11} = x_5^1 y_{110} \vee x_5 y_{111} \tag{49}$$

Where

$$y_{11} = \begin{vmatrix} x_1 & x_4 & x_7 \\ & x_5 & x_8 \\ x_2 & x_5 & x_8 \\ & x_5 & x_6 & x_9 \\ & x_4 & x_7 \\ & x_6 & x_9 \\ x_3 & x_6 & x_9 \\ & x_5 & x_8 \\ & x_4 & x_5 & x_7 \end{vmatrix} \tag{50}$$

$$y_{110} = \begin{vmatrix} x_1 & x_4 & x_7 \\ x_3 & x_6 & x_9 \end{vmatrix} \tag{50}$$

$$y_{111} = \begin{vmatrix} x_1 & x_4 & x_7 \\ & x_8 \\ & x_6 & x_9 \\ x_2 & & x_8 \\ & x_4 & x_7 \\ & x_6 & x_9 \\ x_3 & x_6 & x_9 \\ & x_8 \\ & x_4 & x_7 \end{vmatrix} \tag{51}$$

Since all the letters occur in equation (50) only once, it implies that y_{110} is non-iterated.

Now, expand the function y_{111} by argument x_6 (say) as follows:

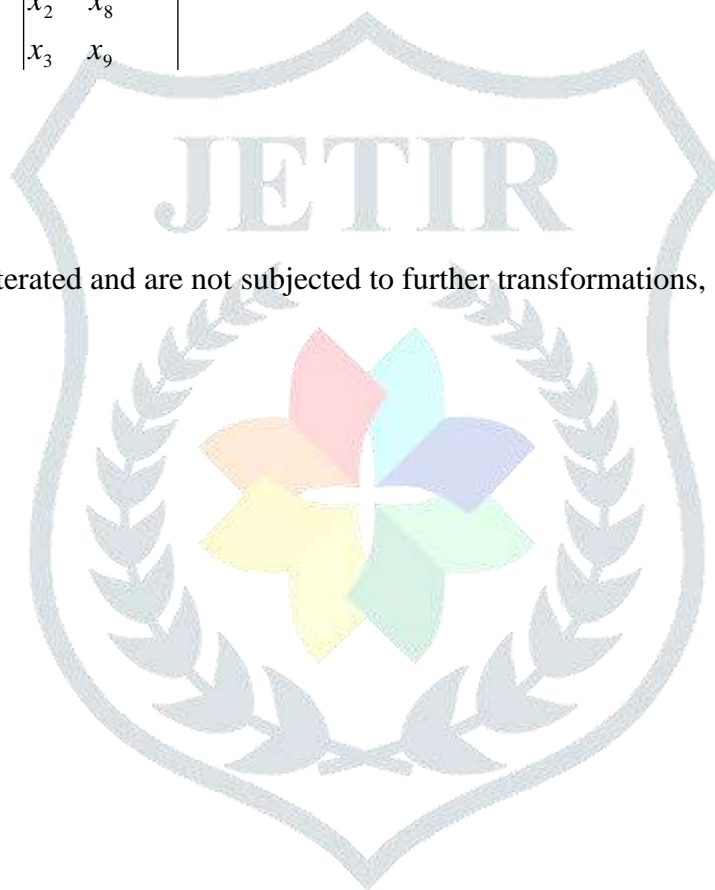
$$y_{111} = x_6^1 y_{1110} \vee x_6 y_{1111} \tag{52}$$

Where

$$y_{1110} = \left| \begin{array}{cc|c} x_1 & x_4 & x_7 \\ & & x_8 \\ x_2 & & x_8 \\ & & x_4 & x_7 \end{array} \right| = \left| \begin{array}{ccc} x_1 & x_8 & \\ x_2 & x_4 & x_7 \end{array} \right| \quad (53)$$

$$y_{1111} = \left| \begin{array}{cc|c} x_1 & x_4 & x_7 \\ & & x_8 \\ & & x_9 \\ x_2 & & x_8 \\ & & x_4 & x_7 \\ & & x_9 \\ x_3 & & x_9 \\ & & x_8 \\ & & x_4 & x_7 \end{array} \right| = \left| \begin{array}{ccc} x_1 & x_4 & x_7 \\ x_2 & x_8 & \\ x_3 & x_9 & \end{array} \right| \quad (54)$$

Now, all the functions are non-iterated and are not subjected to further transformations, so making use of equations (4) to (54), we get



$$g(x_1, x_2, x_3, \dots, x_9, x_{11}, x_{12}) =$$

x_{11}^1	x_{12}^1	x_1	x_4	x_7				
		x_2	x_5	x_8				
		x_3	x_6	x_9				
	x_{12}	x_8^1	x_9^1	x_1	x_4	x_7		
			x_9	x_6^1	x_1	x_4	x_7	
				x_6	x_1	x_4	x_7	
					x_2	x_5		
					x_3			
		x_8	x_9^1	x_1	x_4	x_7		
				x_3	x_6	x_5		
			x_9	x_6^1	x_1	x_4	x_7	
				x_6	x_5^1	x_1	x_4	x_7
					x_5	x_1	x_4	x_7
						x_2		
						x_3		
x_{11}	x_{12}^1	x_8^1	x_4^1	x_3	x_6	x_9		
			x_4	x_7^1	x_3	x_6	x_9	
				x_7	x_1	x_5		
					x_2	x_5		
					x_3	x_6	x_9	
		x_8	x_4^1	x_3	x_6	x_9		
			x_4	x_7^1	x_1	x_5		
				x_7	x_3	x_6	x_9	
				x_7	x_5^1	x_3	x_6	x_9
					x_5	x_1		
						x_2		
						x_3	x_6	x_9
	x_{12}	x_5^1	x_1	x_4	x_7			
			x_3	x_6	x_9			
		x_5	x_6^1	x_1	x_8			
				x_2	x_4	x_7		
			x_6	x_1	x_4	x_7		
				x_2	x_8			
				x_3	x_9			

$$g(x_1, x_2, x_3, \dots, x_9, x_{11}, x_{12}) = \begin{array}{l} k_1 \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_5 \quad x_8 \\ x_3 \quad x_6 \quad x_9 \end{array} \right. \\ k_2 \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_5 \end{array} \right. \\ k_3 \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_5 \\ x_3 \end{array} \right. \\ k_4 \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_3 \quad x_6 \quad x_9 \end{array} \right. \\ k_5 \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_3 \quad x_6 \quad x_9 \end{array} \right. \\ k_6 \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_3 \quad x_6 \quad x_9 \end{array} \right. \\ k_7 \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_5 \end{array} \right. \\ k_8 \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_5 \\ x_3 \end{array} \right. \\ k_9 \left| \begin{array}{l} x_3 \quad x_6 \quad x_9 \\ x_1 \quad x_4 \quad x_7 \end{array} \right. \\ k_{10} \left| \begin{array}{l} x_3 \quad x_6 \quad x_9 \\ x_1 \quad x_4 \quad x_7 \end{array} \right. \\ k_{11} \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_5 \\ x_3 \quad x_6 \quad x_9 \end{array} \right. \\ k_{12} \left| \begin{array}{l} x_3 \quad x_6 \quad x_9 \\ x_1 \quad x_4 \quad x_7 \end{array} \right. \\ k_{13} \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_5 \\ x_3 \quad x_6 \quad x_9 \end{array} \right. \\ k_{14} \left| \begin{array}{l} x_3 \quad x_6 \quad x_9 \\ x_1 \quad x_4 \quad x_7 \end{array} \right. \\ k_{15} \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_5 \\ x_3 \quad x_6 \quad x_9 \end{array} \right. \\ k_{16} \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_3 \quad x_6 \quad x_9 \end{array} \right. \\ k_{17} \left| \begin{array}{l} x_1 \quad x_8 \\ x_2 \quad x_4 \quad x_7 \end{array} \right. \\ k_{18} \left| \begin{array}{l} x_1 \quad x_4 \quad x_7 \\ x_2 \quad x_8 \\ x_3 \quad x_9 \end{array} \right. \end{array}$$

Where $k_1 = x_{11}^1 x_{12}^1$; $k_2 = x_{11}^1 x_{12}^1 x_8^1 x_9^1$; $k_3 = x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6^1$; $k_4 = x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6^1$;
 $k_5 = x_{11}^1 x_{12}^1 x_8^1 x_9^1$; $k_6 = x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6^1$; $k_7 = x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6^1 x_5^1$; $k_8 = x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6^1 x_5^1$;
 $k_9 = x_{11}^1 x_{12}^1 x_8^1 x_4^1$; $k_{10} = x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7^1$; $k_{11} = x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7^1$; $k_{12} = x_{11}^1 x_{12}^1 x_8^1 x_4^1$;
 $k_{13} = x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7^1$; $k_{14} = x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7^1 x_5^1$; $k_{15} = x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7^1 x_5^1$;

$$k_{16} = x_{11} x_{12} x_5^1 ; k_{17} = x_{11} x_{12} x_5 x_6^1 ; k_{18} = x_{11} x_{12} x_5 x_6$$

Now, using Bayes 'formula, the probability of successful operation of the function g is given by

$$\begin{aligned} \Pr(g=1) &= \Pr(k_1) \Pr(g/k_1) + \Pr(k_2) \Pr(g/k_2) + \Pr(k_3) \Pr(g/k_3) + \Pr(k_4) \Pr(g/k_4) + \Pr(k_5) \Pr(g/k_5) \\ &+ \Pr(k_6) \Pr(g/k_6) + \Pr(k_7) \Pr(g/k_7) + \Pr(k_8) \Pr(g/k_8) + \Pr(k_9) \Pr(g/k_9) + \Pr(k_{10}) \Pr(g/k_{10}) \\ &+ \Pr(k_{11}) \Pr(g/k_{11}) + \Pr(k_{12}) \Pr(g/k_{12}) + \Pr(k_{13}) \Pr(g/k_{13}) + \Pr(k_{14}) \Pr(g/k_{14}) + \Pr(k_{15}) \Pr(g/k_{15}) \\ &+ \Pr(k_{16}) \Pr(g/k_{16}) + \Pr(k_{17}) \Pr(g/k_{17}) + \Pr(k_{18}) \Pr(g/k_{18}) \end{aligned}$$

$$\begin{aligned} \Pr(g=1) &= \Pr(x_{11}^1 x_{12}^1) \Pr(y_{00}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_9^1) \Pr(y_{0100}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6^1) \Pr(y_{01010}) \\ &+ \Pr(x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6) \Pr(y_{01011}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_9^1) \Pr(y_{0110}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6^1) \Pr(y_{01110}) \\ &+ \Pr(x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6 x_5^1) \Pr(y_{011110}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_9^1 x_6 x_5) \Pr(y_{011111}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_4^1) \Pr(y_{1000}) \\ &+ \Pr(x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7^1) \Pr(y_{10010}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7) \Pr(y_{10011}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_4^1) \Pr(y_{1010}) \\ &+ \Pr(x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7^1) \Pr(y_{10110}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7 x_5^1) \Pr(y_{101110}) + \Pr(x_{11}^1 x_{12}^1 x_8^1 x_4^1 x_7 x_5) \Pr(y_{101111}) \\ &+ \Pr(x_{11}^1 x_{12}^1 x_5^1) \Pr(y_{110}) + \Pr(x_{11}^1 x_{12}^1 x_5 x_6^1) \Pr(y_{1110}) + \Pr(x_{11}^1 x_{12}^1 x_5 x_6) \Pr(y_{1111}) \end{aligned} \quad (55)$$

If R_i is the reliability of the component of the complex system corresponding to state x_i and Q_i is the corresponding unreliability, then from equation (55) we get

$$\begin{aligned} \Pr(g=1) &= Q_{11} Q_{12} \{1 - (1 - R_1 R_4 R_7)(1 - R_2 R_5 R_8)(1 - R_3 R_6 R_9)\} + Q_{11} R_{12} Q_8 Q_9 R_1 R_4 R_7 + Q_{11} R_{12} Q_8 R_9 Q_6 R_1 R_4 R_7 \\ &+ Q_{11} R_{12} Q_8 R_9 R_6 R_1 R_4 R_7 R_2 R_5 R_3 + Q_{11} R_{12} R_8 Q_9 R_1 R_4 R_7 R_3 R_6 R_5 + Q_{11} R_{12} R_8 R_9 Q_6 R_1 R_4 R_7 \\ &+ Q_{11} R_{12} R_8 R_9 R_6 Q_5 R_1 R_4 R_7 + Q_{11} R_{12} R_8 R_9 R_6 R_5 R_1 R_4 R_7 R_2 R_3 + R_{11} Q_{12} Q_8 Q_4 R_3 R_6 R_9 \\ &+ R_{11} Q_{12} Q_8 R_4 Q_7 R_3 R_6 R_9 + R_{11} Q_{12} Q_8 R_4 R_7 R_1 R_2 R_5 R_3 R_6 R_9 + R_{11} Q_{12} R_8 Q_4 R_3 R_6 R_9 \\ &+ R_{11} Q_{12} R_8 R_4 Q_7 R_3 R_6 R_9 R_1 R_5 + R_{11} Q_{12} R_8 R_4 R_7 Q_5 R_3 R_6 R_9 + R_{11} Q_{12} R_8 R_4 R_7 R_5 R_3 R_6 R_9 \\ &+ R_{11} R_{12} Q_5 R_4 R_2 R_3 R_6 R_9 R_7 + R_{11} R_{12} R_5 Q_6 R_1 R_2 R_8 R_4 R_7 \\ &+ R_{11} R_{12} R_5 R_6 (1 - Q_1 Q_2 Q_3)(1 - Q_4 Q_8 Q_9)(1 - Q_7) \end{aligned} \quad (56)$$

Finally, the probability of the successful operation (i.e., reliability) of the complex system is given by

$$\begin{aligned} R_S &= \Pr(f=1) = \Pr(x_{10}) \Pr(g=1) \\ &= (1 - R_{11})(1 - R_{12}) R_{10} \{1 - (1 - R_1 R_4 R_7)(1 - R_2 R_5 R_8)(1 - R_3 R_6 R_9)\} \\ &+ (1 - R_{11}) R_{12} R_1 (1 - R_8)(1 - R_9) R_{10} R_4 R_7 + (1 - R_{11})(1 - R_8)(1 - R_6) R_{12} R_1 R_9 R_4 R_7 R_{10} \\ &+ (1 - R_{11})(1 - R_8) R_{12} R_9 R_6 R_1 R_4 R_7 R_2 R_5 R_3 R_{10} + (1 - R_{11})(1 - R_9) R_{12} R_8 R_1 R_4 R_7 R_3 R_6 R_5 R_{10} \\ &+ (1 - R_{11})(1 - R_6) R_{12} R_8 R_9 R_1 R_4 R_7 R_{10} + (1 - R_{11})(1 - R_5) R_{12} R_8 R_9 R_6 R_1 R_4 R_7 R_{10} \\ &+ (1 - R_{11}) R_{12} R_8 R_9 R_6 R_5 R_1 R_4 R_7 R_2 R_3 R_{10} + (1 - R_{12})(1 - R_8)(1 - R_4) R_{11} R_3 R_6 R_9 R_{10} \\ &+ (1 - R_{12})(1 - R_8)(1 - R_7) R_{11} R_4 R_3 R_6 R_9 R_{10} + (1 - R_{12})(1 - R_8) R_{11} R_4 R_7 R_1 R_2 R_5 R_3 R_6 R_9 R_{10} \\ &+ (1 - R_{12})(1 - R_4) R_{11} R_8 R_3 R_6 R_9 R_{10} + (1 - R_{12})(1 - R_7) R_{11} R_8 R_4 R_3 R_6 R_9 R_1 R_5 R_{10} \end{aligned}$$

$$\begin{aligned}
& + (1 - R_{12})(1 - R_5)R_{11} R_8 R_4 R_7 R_3 R_6 R_9 R_{10} + (1 - R_{12})R_{11} R_8 R_4 R_7 R_5 R_3 R_6 R_9 R_{10} \\
& + (1 - R_5)R_4 R_2 R_3 R_6 R_9 R_7 R_{11} R_{12} R_{10} + (1 - R_6)R_{11} R_{12} R_5 R_1 R_2 R_8 R_4 R_7 R_{10} \\
& + \{ (1 - (1 - R_1)(1 - R_2)(1 - R_3))(1 - (1 - R_4)(1 - R_8)(1 - R_9)) (1 - (1 - R_7)) R_{11} R_{12} R_5 R_6 R_{10} \} \quad (57)
\end{aligned}$$

$$\begin{aligned}
R_5 = & R_3 R_6 R_9 R_{10} + R_2 R_5 R_8 R_{10} - R_2 R_3 R_5 R_6 R_8 R_9 R_{10} + R_1 R_4 R_7 R_{10} - R_2 R_5 R_8 R_{10} R_{12} \\
& - R_1 R_3 R_4 R_6 R_7 R_9 R_{10} - R_1 R_2 R_4 R_5 R_7 R_8 R_{10} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} - R_2 R_5 R_8 R_{10} R_{11} \\
& + R_2 R_3 R_5 R_6 R_8 R_9 R_{10} R_{11} - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} + R_1 R_3 R_4 R_6 R_7 R_9 R_{10} R_{11} \\
& + R_1 R_2 R_4 R_5 R_7 R_8 R_{10} R_{11} + R_1 R_4 R_7 R_8 R_9 R_{10} R_{12} - 2R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} \\
& - R_3 R_6 R_9 R_{10} R_{12} + R_2 R_3 R_5 R_6 R_8 R_9 R_{10} R_{12} + R_1 R_3 R_4 R_6 R_7 R_9 R_{10} R_{12} + R_1 R_2 R_4 R_5 R_7 R_8 R_{10} R_{12} \\
& - R_1 R_4 R_7 R_{10} R_{11} + R_2 R_5 R_8 R_{10} R_{11} R_{12} - R_2 R_3 R_5 R_6 R_8 R_9 R_{10} R_{11} R_{12} + R_1 R_4 R_7 R_{10} R_{11} R_{12} \\
& - R_1 R_3 R_4 R_6 R_7 R_9 R_{10} R_{11} R_{12} - R_1 R_2 R_4 R_5 R_7 R_8 R_{10} R_{11} R_{12} - R_1 R_4 R_7 R_8 R_{10} R_{12} \\
& + 3R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{12} \\
& + R_1 R_4 R_7 R_8 R_{10} R_{11} R_{12} - R_1 R_4 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_1 R_4 R_6 R_7 R_9 R_{10} R_{12} \\
& + R_1 R_4 R_6 R_7 R_9 R_{10} R_{11} R_{12} - R_1 R_4 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_1 R_4 R_7 R_{10} R_{11} R_{12} \\
& - 2R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} + R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{12} - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} \\
& - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} + 2R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_1 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{12} \\
& + 2R_1 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_3 R_6 R_8 R_9 R_{10} R_{11} + R_3 R_4 R_6 R_8 R_9 R_{10} R_{11} \\
& + R_3 R_4 R_6 R_8 R_9 R_{10} R_{11} R_{12} - R_3 R_4 R_6 R_7 R_9 R_{10} R_{11} + 2R_3 R_4 R_6 R_7 R_8 R_9 R_{10} R_{11} \\
& + R_3 R_4 R_6 R_7 R_9 R_{10} R_{11} R_{12} - 2R_3 R_4 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} + R_1 R_3 R_4 R_5 R_6 R_8 R_9 R_{10} R_{11} \\
& - R_1 R_3 R_4 R_5 R_6 R_8 R_9 R_{10} R_{11} R_{12} - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} - R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} \\
& + 2R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} + R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} - R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} \\
& + R_2 R_3 R_4 R_6 R_7 R_9 R_{10} R_{11} R_{12} + R_1 R_2 R_4 R_5 R_7 R_8 R_{10} R_{11} R_{12} + R_1 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} \\
& + R_1 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} + R_1 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} - R_1 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} \\
& - R_1 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_1 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} + R_2 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12}
\end{aligned}$$

$$\begin{aligned}
& + R_2 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} + R_2 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} - R_2 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} \\
& - R_2 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_2 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} + R_2 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} \\
& + R_3 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} + R_3 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} + R_3 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} \\
& - R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} - R_3 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} \\
& - R_1 R_2 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} - R_1 R_2 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} - R_1 R_2 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} \\
& + R_1 R_2 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} + R_1 R_2 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} - R_1 R_2 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} \\
& - R_2 R_3 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} - R_2 R_3 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} - R_2 R_3 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} \\
& + R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} + R_2 R_3 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} - R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} \\
& - R_1 R_3 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} - R_1 R_3 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} - R_1 R_3 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} \\
& + R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} + R_1 R_3 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} + R_1 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} \\
& - R_1 R_3 R_4 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} + R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_{10} R_{11} R_{12} + R_1 R_2 R_3 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} \\
& + R_1 R_2 R_3 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12} - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_8 R_{10} R_{11} R_{12} - R_1 R_2 R_3 R_5 R_6 R_7 R_8 R_9 R_{10} R_{11} R_{12} \\
& - R_1 R_2 R_3 R_4 R_5 R_6 R_7 R_9 R_{10} R_{11} R_{12}
\end{aligned} \tag{58}$$

6. PARTICULAR CASES:

CASE 1:

If the reliability of each component of the complex system is R , equation (58) yields

$$R_s = R^4(3R^8 - 9R^7 + 13R^6 - 20R^5 + 17R^4 - R^3 - R^2 - 4R + 3) \tag{59}$$

CASE 2: When failure rates follow Weibull distribution

Let the failure rates of the subsystem (generators) G_1, G_2 and $G_3, M SB_1, M SB_2, M SB_3, O P M S_4$, cables C_1, C_2, C_3, C_4, C_5 are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}, \lambda_{11}, \lambda_{12}$ respectively, then from equation (58) reliability of the complex system at an instant t is given by

$$\begin{aligned}
R_{sw}(t) = & \exp(-a_1 t^p) + \exp(-a_2 t^p) - \exp(-a_3 t^p) + \exp(-a_4 t^p) - \exp(-a_5 t^p) - \exp(-a_6 t^p) \\
& - \exp(-a_7 t^p) + \exp(-a_8 t^p) - \exp(-a_9 t^p) + \exp(-a_{10} t^p) - \exp(-a_{11} t^p) + \exp(-a_{12} t^p) \\
& + \exp(-a_{13} t^p) + \exp(-a_{14} t^p) - 2 \exp(-a_{15} t^p) - \exp(-a_{16} t^p) + \exp(-a_{17} t^p) + \exp(-a_{18} t^p) \\
& + \exp(-a_{19} t^p) - \exp(-a_{20} t^p) + \exp(-a_{21} t^p) - \exp(-a_{22} t^p) + \exp(-a_{23} t^p) - \exp(-a_{24} t^p)
\end{aligned}$$

$$\begin{aligned}
& -\exp(-a_{25} t^p) - \exp(-a_{26} t^p) + 3 \exp(-a_{27} t^p) + \exp(-a_{28} t^p) + \exp(-a_{29} t^p) - \exp(-a_{30} t^p) \\
& - \exp(-a_{31} t^p) + \exp(-a_{32} t^p) - \exp(-a_{33} t^p) - \exp(-a_{34} t^p) - 2 \exp(-a_{35} t^p) - \exp(-a_{36} t^p) \\
& - \exp(-a_{37} t^p) + 2 \exp(-a_{38} t^p) - \exp(-a_{39} t^p) + 2 \exp(-a_{40} t^p) - \exp(-a_{41} t^p) + \exp(-a_{42} t^p) \\
& - \exp(-a_{43} t^p) - \exp(-a_{44} t^p) + 2 \exp(-a_{45} t^p) + \exp(-a_{46} t^p) - 2 \exp(-a_{47} t^p) + \exp(-a_{48} t^p) \\
& - \exp(-a_{49} t^p) - \exp(-a_{50} t^p) - \exp(-a_{51} t^p) + 2 \exp(-a_{52} t^p) + \exp(-a_{53} t^p) - \exp(-a_{54} t^p) \\
& + \exp(-a_{55} t^p) + \exp(-a_{56} t^p) + \exp(-a_{57} t^p) + \exp(-a_{58} t^p) + \exp(-a_{59} t^p) - \exp(-a_{60} t^p) \\
& - \exp(-a_{61} t^p) - \exp(-a_{62} t^p) + \exp(-a_{63} t^p) + \exp(-a_{64} t^p) + \exp(-a_{65} t^p) - \exp(-a_{66} t^p) \\
& - \exp(-a_{67} t^p) - \exp(-a_{68} t^p) + \exp(-a_{69} t^p) + \exp(-a_{70} t^p) + \exp(-a_{71} t^p) + \exp(-a_{72} t^p) \\
& - \exp(-a_{73} t^p) - \exp(-a_{74} t^p) - \exp(-a_{75} t^p) - \exp(-a_{76} t^p) - \exp(-a_{77} t^p) - \exp(-a_{78} t^p) \\
& + \exp(-a_{79} t^p) + \exp(-a_{80} t^p) - \exp(-a_{81} t^p) - \exp(-a_{82} t^p) - \exp(-a_{83} t^p) - \exp(-a_{84} t^p) \\
& + \exp(-a_{85} t^p) + \exp(-a_{86} t^p) - \exp(-a_{87} t^p) - \exp(-a_{88} t^p) - \exp(-a_{89} t^p) - \exp(-a_{90} t^p) \\
& + \exp(-a_{91} t^p) + \exp(-a_{92} t^p) + \exp(-a_{93} t^p) - \exp(-a_{94} t^p) + \exp(-a_{95} t^p) + \exp(-a_{96} t^p) \\
& + \exp(-a_{97} t^p) - \exp(-a_{98} t^p) - \exp(-a_{99} t^p) - \exp(-a_{100} t^p) + \exp(-a_{101} t^p) + \exp(-a_{102} t^p)
\end{aligned} \tag{60}$$

Where p is a positive parameter and a_i are given by

$$a_1 = \lambda_3 + \lambda_6 + \lambda_9 + \lambda_{10}; a_2 = \lambda_2 + \lambda_5 + \lambda_8 + \lambda_{10}; a_3 = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10}; a_4 = \lambda_1 + \lambda_4 + \lambda_7 + \lambda_{10}$$

$$a_5 = \lambda_2 + \lambda_5 + \lambda_8 + \lambda_{10} + \lambda_{12}; a_6 = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10}; a_7 = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_{10}$$

$$a_8 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10}; a_9 = \lambda_2 + \lambda_5 + \lambda_8 + \lambda_{10} + \lambda_{11};$$

$$a_{10} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11}; a_{11} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$$

$$a_{12} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11}; a_{13} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11}$$

$$a_{14} = \lambda_1 + \lambda_4 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}; a_{15} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11}$$

$$a_{16} = \lambda_3 + \lambda_6 + \lambda_9 + \lambda_{10} + \lambda_{12}; a_{17} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}; a_{18} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{12}$$

$$a_{19} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{12}; a_{20} = \lambda_1 + \lambda_4 + \lambda_7 + \lambda_{10} + \lambda_{11}; a_{21} = \lambda_2 + \lambda_5 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{68} = \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{69} = \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{70} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{71} = \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{72} = \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{73} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{74} = \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{75} = \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{76} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{77} = \lambda_1 + \lambda_2 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{78} = \lambda_1 + \lambda_2 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{79} = \lambda_1 + \lambda_2 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{80} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{81} = \lambda_1 + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{82} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{83} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{84} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{85} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{86} = \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{87} = \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{88} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{89} = \lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{90} = \lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{91} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{92} = \lambda_1 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{93} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{94} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{95} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{96} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{97} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{98} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{99} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}$$

$$a_{100} = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_9 + \lambda_{10} + \lambda_{11} + \lambda_{12}; a_{101} = \lambda_1 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_{10} + \lambda_{12}$$

$$a_{102} = \lambda_1 + \lambda_4 + \lambda_6 + \lambda_7 + \lambda_8 + \lambda_9 + \lambda_{10} + \lambda_{12}$$

CASE 3: When failure rates follow Exponential distribution:

Exponential distribution is nothing but a particular case of Weibull distribution for $p = 1$ and is very useful in numerous practical problems, thus the reliability of the complex system in this case at an instant t is given by

$$\begin{aligned} R_{SE}(t) &= [R_{SW}(t)] \text{ at } p=1 \\ &= \exp(-a_1 t) + \exp(-a_2 t) - \exp(-a_3 t) + \exp(-a_4 t) - \exp(-a_5 t) - \exp(-a_6 t) - \exp(-a_7 t) + \exp(-a_8 t) \\ &\quad - \exp(-a_9 t) + \exp(-a_{10} t) - \exp(-a_{11} t) + \exp(-a_{12} t) + \exp(-a_{13} t) + \exp(-a_{14} t) - 2 \exp(-a_{15} t) \end{aligned}$$

$$\begin{aligned}
 & - \exp(-a_{16} t) + \exp(-a_{17} t) + \exp(-a_{18} t) + \exp(-a_{19} t) - \exp(-a_{20} t) + \exp(-a_{21} t) - \exp(-a_{22} t) \\
 & + \exp(-a_{23} t) - \exp(-a_{24} t) - \exp(-a_{25} t) - \exp(-a_{26} t) + 3 \exp(-a_{27} t) + \exp(-a_{28} t) \\
 & + \exp(-a_{29} t) - \exp(-a_{30} t) - \exp(-a_{31} t) + \exp(-a_{32} t) - \exp(-a_{33} t) - \exp(-a_{34} t) - 2 \exp(-a_{35} t) \\
 & - \exp(-a_{36} t) - \exp(-a_{37} t) + 2 \exp(-a_{38} t) - \exp(-a_{39} t) + 2 \exp(-a_{40} t) - \exp(-a_{41} t) + \exp(-a_{42} t) \\
 & - \exp(-a_{43} t) - \exp(-a_{44} t) + 2 \exp(-a_{45} t) + \exp(-a_{46} t) - 2 \exp(-a_{47} t) + \exp(-a_{48} t) - \exp(-a_{49} t) \\
 & - \exp(-a_{50} t) - \exp(-a_{51} t) + 2 \exp(-a_{52} t) + \exp(-a_{53} t) - \exp(-a_{54} t) + \exp(-a_{55} t) + \exp(-a_{56} t) \\
 & + \exp(-a_{57} t) + \exp(-a_{58} t) + \exp(-a_{59} t) - \exp(-a_{60} t) - \exp(-a_{61} t) - \exp(-a_{62} t) + \exp(-a_{63} t) \\
 & + \exp(-a_{64} t) + \exp(-a_{65} t) - \exp(-a_{66} t) - \exp(-a_{67} t) - \exp(-a_{68} t) + \exp(-a_{69} t) + \exp(-a_{70} t) \\
 & + \exp(-a_{71} t) + \exp(-a_{72} t) - \exp(-a_{73} t) - \exp(-a_{74} t) - \exp(-a_{75} t) - \exp(-a_{76} t) - \exp(-a_{77} t) \\
 & - \exp(-a_{78} t) + \exp(-a_{79} t) + \exp(-a_{80} t) - \exp(-a_{81} t) - \exp(-a_{82} t) - \exp(-a_{83} t) - \exp(-a_{84} t) \\
 & + \exp(-a_{85} t) + \exp(-a_{86} t) - \exp(-a_{87} t) - \exp(-a_{88} t) - \exp(-a_{89} t) - \exp(-a_{90} t) + \exp(-a_{91} t) \\
 & + \exp(-a_{92} t) + \exp(-a_{93} t) - \exp(-a_{94} t) + \exp(-a_{95} t) + \exp(-a_{96} t) + \exp(-a_{97} t) - \exp(-a_{98} t) \\
 & - \exp(-a_{99} t) - \exp(-a_{100} t) + \exp(-a_{101} t) + \exp(-a_{102} t)
 \end{aligned} \tag{61}$$

7 EVALUATION OF M.T.T.F:

The expression for *M.T.T.F* in this case is given by

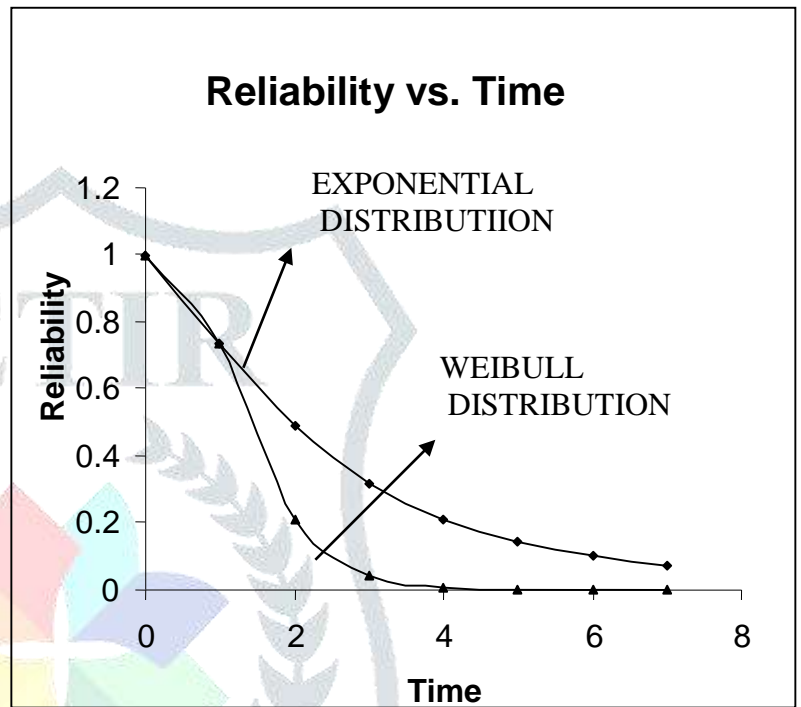
$$\begin{aligned}
 M.T.T.F &= \int_0^{\infty} R_{SE}(t) dt \\
 &= \frac{1}{a_1} + \frac{1}{a_2} - \frac{1}{a_3} + \frac{1}{a_4} - \frac{1}{a_5} - \frac{1}{a_6} - \frac{1}{a_7} + \frac{1}{a_8} - \frac{1}{a_9} + \frac{1}{a_{10}} - \frac{1}{a_{11}} + \frac{1}{a_{12}} + \frac{1}{a_{14}} - \frac{1}{a_{15}} - \frac{1}{a_{16}} + \frac{1}{a_{17}} + \frac{1}{a_{18}} + \frac{1}{a_{19}} \\
 & - \frac{1}{a_{20}} + \frac{1}{a_{21}} - \frac{1}{a_{22}} + \frac{1}{a_{23}} - \frac{1}{a_{24}} - \frac{1}{a_{25}} - \frac{1}{a_{26}} + \frac{1}{a_{27}} + \frac{1}{a_{28}} + \frac{1}{a_{29}} - \frac{1}{a_{30}} - \frac{1}{a_{31}} + \frac{1}{a_{32}} - \frac{1}{a_{33}} - \frac{1}{a_{34}} - \frac{1}{a_{35}} - \frac{1}{a_{36}} \\
 & - \frac{1}{a_{37}} + \frac{1}{a_{38}} - \frac{1}{a_{39}} + \frac{1}{a_{40}} - \frac{1}{a_{41}} + \frac{1}{a_{42}} - \frac{1}{a_{43}} - \frac{1}{a_{44}} + \frac{1}{a_{45}} + \frac{1}{a_{46}} - \frac{1}{a_{47}} + \frac{1}{a_{48}} - \frac{1}{a_{49}} - \frac{1}{a_{50}} - \frac{1}{a_{51}} + \frac{1}{a_{52}} + \frac{1}{a_{53}} \\
 & - \frac{1}{a_{54}} + \frac{1}{a_{55}} + \frac{1}{a_{56}} + \frac{1}{a_{57}} + \frac{1}{a_{58}} + \frac{1}{a_{59}} - \frac{1}{a_{60}} - \frac{1}{a_{61}} - \frac{1}{a_{62}} + \frac{1}{a_{63}} + \frac{1}{a_{64}} + \frac{1}{a_{65}} - \frac{1}{a_{66}} - \frac{1}{a_{67}} - \frac{1}{a_{68}} + \frac{1}{a_{69}} + \frac{1}{a_{70}} \\
 & + \frac{1}{a_{71}} + \frac{1}{a_{72}} - \frac{1}{a_{73}} - \frac{1}{a_{74}} - \frac{1}{a_{75}} - \frac{1}{a_{76}} - \frac{1}{a_{77}} - \frac{1}{a_{78}} + \frac{1}{a_{79}} + \frac{1}{a_{80}} - \frac{1}{a_{81}} - \frac{1}{a_{82}} - \frac{1}{a_{83}} - \frac{1}{a_{84}} + \frac{1}{a_{85}} + \frac{1}{a_{86}} - \frac{1}{a_{87}} \\
 & - \frac{1}{a_{88}} - \frac{1}{a_{89}} - \frac{1}{a_{90}} + \frac{1}{a_{91}} + \frac{1}{a_{92}} + \frac{1}{a_{93}} - \frac{1}{a_{94}} + \frac{1}{a_{95}} + \frac{1}{a_{96}} + \frac{1}{a_{97}} - \frac{1}{a_{98}} - \frac{1}{a_{99}} - \frac{1}{a_{100}} + \frac{1}{a_{101}} + \frac{1}{a_{102}}
 \end{aligned} \tag{62}$$

8. NUMERICAL COMPUTATION FOR RELIABILITY:

Setting $\lambda_i = 0.1$ for $i=1-12$ and $p=2$ in equations (60) and (61), one can compute table 1.

TABLE - 1

S.NO	Time (t)	System reliability	
		$R_{SE}(t)$	$R_{SW}(t)$
1	0	1.00000	1.00000
2	1	0.73681	0.73681
3	2	0.48922	0.21159
4	3	0.31865	0.03901
5	4	0.21159	0.00359
6	5	0.14511	0.00012
7	6	0.10248	1.61×10^{-6}
8	7	0.07378	0.9×10^{-8}

**Fig 2**

9 NUMERICAL COMPUTATION FOR M.T.T.F.:

Setting $\lambda_i = \lambda$ one can compute table -2 from equation (62).

TABLE -2

S.NO.	λ	<i>M.T.T.F</i>
1	0.0	∞
2	0.1	3.59235
3	0.2	1.79617
4	0.3	1.19745
5	0.4	0.89808
6	0.5	0.71847
7	0.6	0.59872
8	0.7	0.51319
9	0.8	0.44904

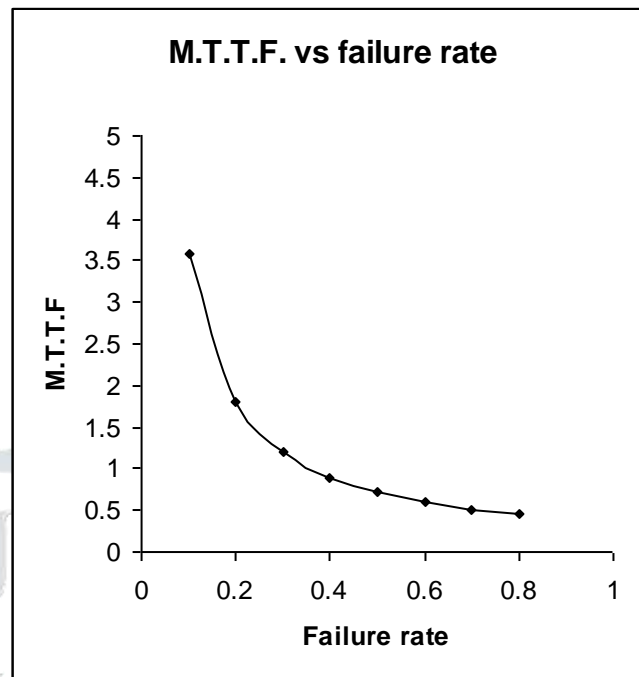
**Fig. 3****10. INTERPRETATION OF THE RESULTS:**

Table 1 represents the reliability of the system at any time t , when failure follows either exponential or Weibull distribution. A critical examination of the graph reliability vs. time (Fig. 2) indicates that the reliability of the complex system decreases approximately at a uniform rate in the case of exponential distribution, whereas it decreases very rapidly when failure follows Weibull distribution.

Further, Table 2 computes the mean time to system failure for different values of failure rates. An inspection of the graph M.T.T.F vs. failure rate (Fig. 3) reveals that M.T.T.F in the beginning decreases catastrophically, but later on it decreases approximately at uniform rate. Thus, for a given set of failure rates of various components of the complex system, one can estimate the reliability of the power supply and mean time to system failure of such a complex system.

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