# Unique method to solve fuzzy transportation problem for multi-purpose objectives

Priyanka swankar[1], Richa Gupta[1]

[1]Department of Mathematic, Sarvepalli Radhakrishnan University, Bhopal, India

## 1. Introduction

This work includes the finding of optimal approximate solution to a special type of optimization problem called a fuzzy transportation problem using pentagonal fuzzy numbers. The values of the cost, supply, and demand for fuzzy transportation problems are taken as pentagonal fuzzy numbers (1-6). The pentagonal fuzzy numbers are converted into crisp values using a novel suggested ranking function. By comparing this with the conventional ranking methods, we can achieve better results with the aid of the proposed new ranking method. Other conventional methods are then applied to calculate the solution of given problem and compare the optimum solution results (5-11).

### 1.1 Fuzzy Set

A fuzzy set $A$ in R (real line) is defined as a set of ordered pairs

A = $\{x_0, (x_0) / x_0 \in A, \mu_A (x_0) \rightarrow [0,1]\}$

Where $(x0)$ is said to be the membership function.

### 1.2 Fuzzy Number

$A$ is a fuzzy set on the real line $R$, must satisfy the subsequent conditions.

- $\mu_A (x_0)$ is piecewise continuous
- There exist at least one $x_0 \in \Re$ with $\mu_A (x_0) = 1$
- A must be regular & convex

### 1.3 Pentagonal Fuzzy Number

A fuzzy number A on R is said to be a pentagonal fuzzy number (PFN) or linear fuzzy number which is named as ($_1$, $a_2$, $a_3$, $a_4$, $a_5$) if its membership function $\mu_A(x)$ has the following characteristic

$$\mu_A(x) = \begin{cases} 0, if\ x < a_1 \\ u_1\left(\frac{x-a_2}{a_3-a_2}\right), if\ a_1 \leq x \leq a_2 \\ 1, if\ x = a_3 \\ 1-(1-u_2)\left(\frac{a_4-x}{a_4-a_3}\right), if\ a_3 \leq x \leq a_4 \\ u_2\left(\frac{a_5-x}{a_5-a_4}\right), if\ a_4 \leq x \leq a_5 \\ 0, if\ x > a_5 \end{cases}$$

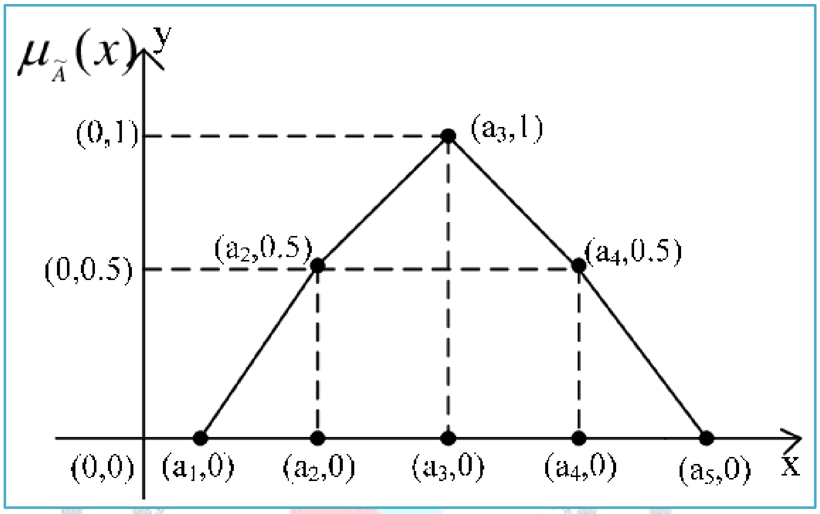


Figure.1 Graphical Representation of Pentagonal fuzzy number

## 2. Arithmetic Operations

Let $\bar{A} = (a_1, b_1, c_1, d_1, e_1)$ and $\bar{B} = (a_2, b_2, c_2, d_2, e_2)$ are two fuzzy numbers. Where $a_1 \leq b_1 \leq c_1 \leq d_1 \leq e_1$ and $a_2 \leq b_2 \leq c_2 \leq d_2 \leq e_2$.

Then the arithmetic operations are defined as

**(i) Addition**

$\bar{A} + \bar{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, e_1 + e_2)$

**(ii) Subtraction**

$\bar{A} - \bar{B} = (a_1 - e_2, b_1 - d_2, c_1 - c_2, d_1 - b_2, e_1 - a_2)$

**(iii) Multiplication**

$\bar{A} * \bar{B} = (a_1/5\ \mu_\theta, b_1/5\ \mu_\theta, c_1/5\ \mu_\theta, d_1/5\ \mu_\theta, e_1/5\ \mu_\theta)$ Where $\mu_\theta = (a_2 + b_2 + c_2 + d_2 + e_2)$

**(iv) Division**

$\bar{A} \div \bar{B} = (5a_1\ \mu_\theta, 5b_1\ \mu_\theta, 5c_1\ \mu_\theta, 5d_1\ \mu_\theta, 5e_1\ \mu_\theta)$ if $\mu_\theta \neq 0$,

Where $\mu_\theta = (a_2 + b_2 + c_2 + d_2 + e_2)$

**(V) Scalar Multiplication**

k$\bar{A}$= {$(ka, kb, kc, kd, ke)$ $if$ $k$ > 0, $(ke, kd, kc, kb, ka)$ $if k$ < 0}

## 3. Mathematical formulation of fuzzy transportation problem

Consider a fuzzy transportation problem with m sources and n destinations with pentagonal fuzzy numbers. Let, ($ai$ ≥0) be the fuzzy availability at source i and $bj$ , ($bj$ ≥0) be the fuzzy requirement at destination j. Let $cij$ be the fuzzy unit transportation cost from source i to destination j. Let $xij$ denote the number of fuzzy units to be transported from source i to destination j. Then the problem is to find the feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimized. The mathematical formulation of the fuzzy transportation whose parameters are pentagonal fuzzy numbers under the case that the total supply is equivalent to the total demand is given by:

Min z = ∑ ∑ $cij$ $n$ $j$=1 $m$ $i$=1 $xij$

Subject to ∑ $xij$ = $n$ $j$=1, i = 1, 2, ……m.

∑ $xij$ $m$ $i$=1, j =1, 2, …….n.

∑ $ai$ $m$ $i$=1 = ∑ $bj$ $n$ $j$=1; i = 1, 2, ……m; j = 1, 2, ……n and

$xij$ ≥0.

The fuzzy transportation problem is explicitly represented by the fuzzy transportation table:

| | **1** | ⋯ | **n** | **Supply** |
|---|---|---|---|---|
| 1 | $c_{11}$ | ⋯ | $c_{1n}$ | $a_1$ |
| ⋮ | ⋮ | ⋯ | ⋮ | ⋮ |
| m | $c_{m1}$ | ⋯ | $c_{mn}$ | $a_m$ |
| Demand | $b_1$ | ⋯ | $b_n$ | |

***Figure 5.1 Fuzzy transportation table***

## 4. MAX-MIN method algorithm

### 4.1 Algorithm:

**Step (1):** Construct the transportation table we examine whether total demand equals total supply then go to step 2

**Step (2):** By using range technique, we convert the fuzzy cost can be converted into crisp values to the given transportation problem

**Step (3):** For the row-wise difference between maximum and minimum of each row, and it is divided by the number of columns of the cost matrix.

**Step (4):** For the column-wise difference between maximum and minimum of each column, and it is divided by the number of rows of the cost matrix.

**Step (5):** We find the maximum of the resultant values and find the corresponding minimum cost value and do the allocation of that particular cell of the given matrix. Suppose we have more than one maximum consequent value. We can select anyone.

**Step (6):** Repeated procedures 1 to 5 until all the allocations are completed.

Consider the fuzzy transportation problem. A Product is produced by four factories Factory1, Factory2, Factory3, Factory4 production capacity of the four factories are 30, 27, 40, and 50 units, respectively. The product is supplied to four stores Store1, Store2, Store3, and Store4, the requirements of Demands, which are 20, 40, 34 and 53, respectively. Here Unit costs of fuzzy transportation are represented as fuzzy pentagonal numbers are given below. Find the fuzzy transportation plan such that the total production and transportation cost is minimum.

| | Store1 | Store2 | Store3 | Store4 | Capacity |
|---|---|---|---|---|---|
| ***Factory1*** | (2,4,6,8,9) | (3,5,7,8,9) | (2,4,5,6,7) | (3,4,6,7,12) | 30 |
| ***Factroy2*** | (0,2,5,6,8) | (4,5,6,8,11) | (2,3,5,7,11) | (1,5,6,9,11) | 27 |
| ***Factory3*** | (1,2,3,4,5) | (2,3,4,6,8) | (4,5,6,8,9) | (6,7,8,9,13) | 40 |
| ***Factory4*** | (3,5,6,7,8) | (1,5,6,7,8) | (2,7,8,9,10) | (3,3,4,5,9) | 50 |
| Demand | 20 | 40 | 34 | 53 | 147 |

**Step 1:** Construct the fuzzy transportation table for the given fuzzy transportation problem and then, convert it into a balanced one, if it is not.

**Step 2:** Using Range technique, we have to convert fuzzy pentagonal numbers into a crisp value.

**Step 3:** Then find the maximum of the resultant values and find the corresponding minimum cost value and allocate the particular cost cell of the given matrix. If we have more than one maximum resultant benefits, we can select anyone.

**Step 4:** Again, find the maximum of the resultant values and find the corresponding minimum cost value and allocate the particular cost cell of the given matrix. If we have more than one maximum resultant benefits, we can select anyone.

| | Store1 | Store2 | Store3 | Store4 | Capacity | (Max-Min) / 4 |
|---|---|---|---|---|---|---|
| ***Factory1*** | 7 | 6 | 5 (30) | 9 | 30 | 4/4 = 1 |
| ***Factroy2*** | 8 | 7 | 9 | 10 | 27 | 3/4 = 0.75 |
| ***Factory3*** | 4 (20) | 6 | 5 | 7 | 40 (20) | 3/4 = 0.75 |
| ***Factory4*** | 5 | 7 | 8 | 6 | 50 | 3/4 = 0.75 |
| Demand | 20 | 40 | 4 | 53 | 147 | |
| (Max-Min) / 4 | 4/3 = 1.3 | 1/3= 0.3 | 4/3 = 1.3 | 4/3 = 1.3 | | |

**Step 5:** Repeat again the above steps.

| | Store1 | Store2 | Store3 | Store4 | Capacity | (Max-Min) / 4 |
|---|---|---|---|---|---|---|
| ***Factory1*** | 7 | 6 | 5 (30) | 9 | 30 | 4/4 = 1 |
| ***Factroy2*** | 8 | 7 | 9 | 10 | 27 | 3/3 = 1 |
| ***Factory3*** | 4 (20) | 6 | 5 | 7 | 20 | 2/3 = 0.67 |
| ***Factory4*** | 5 | 7 | 8 | 6 (50) | 50 | 2/3 = 0.67 |
| Demand | 20 | 40 | 4 | 53 (3) | 147 | |
| (Max-Min) / 4 | 4/3 = 1.3 | 1/3= 0.3 | 4/3 = 1.3 | 4/3 = 1.3 | | |

The same procedure will be followed again and again until we reach the final allocation.

Table 5.3 IBFS by using fuzzy version of Max-Min method

| | Store1 | Store2 | Store3 | Store4 | Capacity |
|---|---|---|---|---|---|
| ***Factory1*** | 7 | 6 | 5 (30) | 9 | 30 |
| ***Factroy2*** | 8 | 7 (27) | 9 | 10 | 27 |
| ***Factory3*** | 4 (20) | 6 (13) | 5 (4) | 7 (3) | 40 |
| ***Factory4*** | 5 | 7 | 8 | 6 (50) | 50 |
| Demand | 20 | 40 | 34 | 53 | 147 |

The Transportation Cost Z = $5*30 + 4*20 + 5*4 + 7*27 + 6*13 + 7*3 + 6*50$

**Z = 838.**

## 5. Russell's method ALGORITHM

### 5.1 Algorithm:

**Step (1):** In the reduced given FTP, identified the row and column difference by considering the least two numbers of that respective row and column.

**Step (2):** Selected the maximum among the difference we attained above (if more than one, then we can select any of them) and then allocated the respective demand/supply to the minimum value of that corresponding row or column.

**Step (3):** We have taken the difference of the corresponding supply and demand of the allocated cell which in turn leads to either of one to zero, thus eliminated corresponding row or column (eradicate both row and column if both demand and supply is attained zero)

**Step (4):** Repeated the steps (i), (ii) and (iii) until all the rows and columns gets eliminated.

**Step (5):** Finally, total minimum cost is going to calculated as sum of the product of the cost and the allocated value.

Considered the same above following fuzzy transportation problem:

| | Store1 | Store2 | Store3 | Store4 | Capacity |
|---|---|---|---|---|---|
| ***Factory1*** | (2,4,6,8,9) | (3,5,7,8,9) | (2,4,5,6,7) | (3,4,6,7,12) | 30 |
| ***Factroy2*** | (0,2,5,6,8) | (4,5,6,8,11) | (2,3,5,7,11) | (1,5,6,9,11) | 27 |
| ***Factory3*** | (1,2,3,4,5) | (2,3,4,6,8) | (4,5,6,8,9) | (6,7,8,9,13) | 40 |
| ***Factory4*** | (3,5,6,7,8) | (1,5,6,7,8) | (2,7,8,9,10) | (3,3,4,5,9) | 50 |
| Demand | 20 | 40 | 34 | 53 | 147 |

***Table 5.4 Crisp Transportation Problem***

| | **Store1** | **Store2** | **Store3** | **Store4** | **Capacity** |
|---|---|---|---|---|---|
| ***Factory1*** | 7 | 6 | 5 | 9 | **30** |
| ***Factroy2*** | 8 | 7 | 9 | 10 | **27** |
| ***Factory3*** | 4 | 6 | 5 | 7 | **40** |
| ***Factory4*** | 5 | 7 | 8 | 6 | **50** |
| **Demand** | **20** | **40** | **34** | **53** | **147** |

| | Store1 | Store2 | Store3 | Store4 | Capacity | Row Difference |
|---|---|---|---|---|---|---|
| ***Factory*1** | 7 | 6 | 5 (30) | 9 | 30 | 1 |
| ***Factroy*2** | 8 | 7 | 9 | 10 | 27 | 1 |
| ***Factory*3** | 4 | 6 | 5 (4) | 7 | 40 (34) | 1 |
| ***Factory*4** | 5 | 7 | 8 | 6 | 50 | 1 |
| Demand | 20 | 40 | 34 (4) | 53 | 147 | |
| Coln Difference | 1 | 1 | 3 | 1 | | |

| | Store1 | Store2 | Store3 | Store4 | Capacity | Row Difference |
|---|---|---|---|---|---|---|
| ***Factory*1** | 7 | 6 | 5 (30) | 9 | 30 | 1 |
| ***Factroy*2** | 8 | 7 (27) | 9 | 10 | 27 | 3 |
| ***Factory*3** | 4 (20) | 6 | 5 (4) | 7 | 40 (14) | 1 |
| ***Factory*4** | 5 | 7 | 8 | 6 | 50 | 1 |
| Demand | 20 | 40 (13) | 34 (4) | 53 | 147 | |
| Coln Difference | 1 | 1 | 3 | 1 | | |

*Table 5.5 IBFS by using fuzzy version of Russell's method*

| | Store1 | Store2 | Store3 | Store4 | Capacity |
|---|---|---|---|---|---|
| ***Factory*1** | 7 | 6 | 5 (30) | 9 | 30 |
| ***Factroy*2** | 8 | 7 (27) | 9 | 10 | 27 |
| ***Factory*3** | 4 (20) | 6 (13) | 5 (4) | 7 (3) | 40 |
| ***Factory*4** | 5 | 7 | 8 | 6 (50) | 50 |
| Demand | 20 | 40 | 34 | 53 | 147 |

Therefore, the total transportation cost using Russell's method is

Minimize Z = $5*30 + 4*20 + 5*4 + 7*27 + 6*13 + 7*3 + 6*50$

Z= 838

## 6. Comparison between existing algorithms

The comparison of the proposed method with the existing process is tabulated below, in which it is clearly shown that the proposed method provides the optimal results.

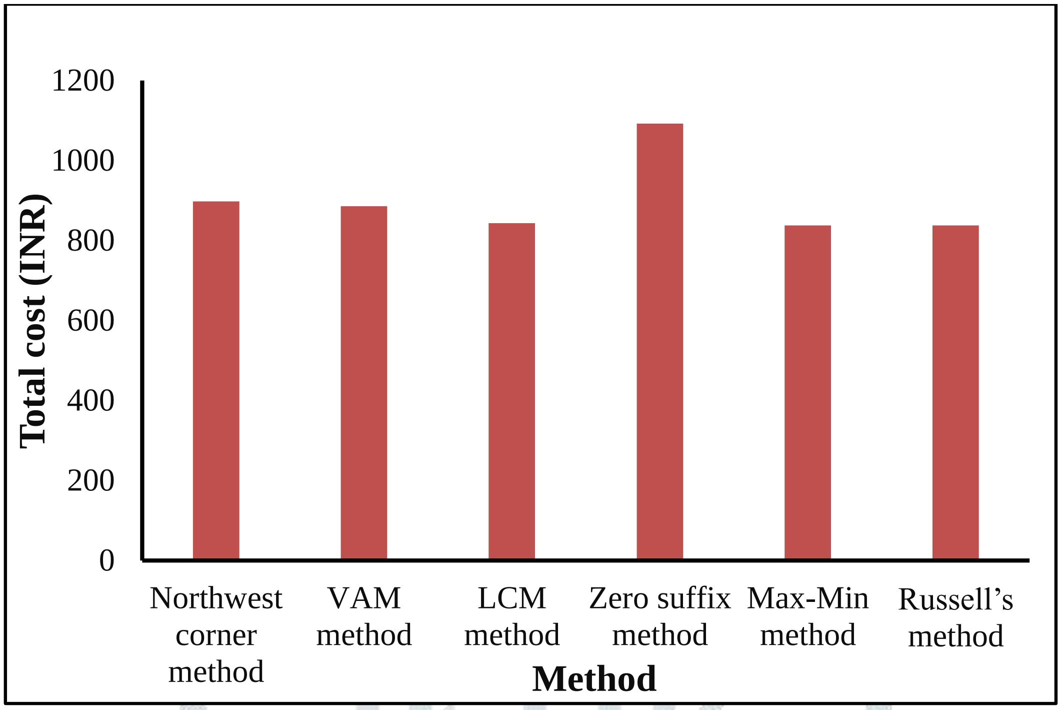


Figure. Shows the comparative total cost calculated through different methods

## 7. Conclusion

This work proposed a new technique called as range technique, which is very tranquil for attaining crisp value, and recommended the max-min method as well as Russell's method which were applied to solve the fuzzy transportation problem, which in turn is very humble and also attained minimum transportation cost to associate all other pre-existing plans.

## References

1. Reinke D. and D. Malarkey, (1996), "Implementing integrated transportation planning in metropolitan planning organization: Procedural and analytical issues", Transportation Research Record: Journal of the Transportation Research Board, 1552, 71-78.
2. Barcelo J., E. Codina, J . Casas, J.L. Ferrer and D. Garcia, (2004), "Microscopic traffic simulation: A tool for the design, analysis and evaluation of intelligent transport systems", Journal of Intelligent and Robotic Systems, 41, 173- 203.
3. Ben-Akiva M., M. Bierlaire, D. Burton, H.N. Koutsopoulos and R Mishalani, (2001), "Network state estimation and prediction for real-time traffic management", Networks and spatial Economics, 1,293- 318.
4. Patankar V.M., R Kumar and G. Tiwari, (2007), "Impacts of bus rapid transit lanes on traffic and commuter mobility", J Urban Plng. and De vel. , 133(2), 99- 106.
5. Ma G., Y Tian, T. Ju and Z. Ren, (2006), "Assessment of traffic noise pollution from 1989 to 2003 in Lanzhou city", Environmental Monitoring and Assessment, 123, 413-430.
6. Cho H.J. and I.C. Hwanc, (2005), "Day-to-day vehicular flow dynamics in intelligent transportation network", Mathematical and Computer Modelling, 41, 501- 522.

**7.** Anderson M.D. and AB. Sandlin, (2001), "A rural transit vehicle management system and condition predictor model", Journal of Public Transportation, 4(1), 1-20.

**8.** Ambak K, R. Atiq and R. Ismail, (2009), "Intelligent transport system for motorcycle safety and issues", European Journal of Scientific Research, 28(4), 600-611.

**9.** Munaka T., T. Yamamoto, M. Kuroda and T. Watanabe, (2003), "Location dependent data multicasting in intelligent transportation system and its evaluation", Transportation Research Part C, 11, 355-373.

**10.** Bouamrane K, C. Tahon and B. Beldjilali, (2005), "A making-decision system for an urban transportation network", JCS&T, 5(3), 144- 149.

**11.** Paulley N., R Balcombe, R Mackett, H. Titheridge, J . Preston, M. Wardman, J. Shires and P . White, (2006), "The demand for public transport: The effects of fares, quality of service, income and car ownership", Transport Policy, 13, 295-306.