The nonlinear Cauchy-Riemann structural transformation equations and the nonlinear Laplace equation

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Abstract :

The purpose of this study is to look into a functional K-transformation p (b) p (b) = p (b) F (b) that is used to reconsider the complex differentiability of a given complex function w, and then to derive structural holomorphic to determine whether a complex function is complex structure differentiable. Because F (b) can be chosen at random, the range of possible implementations has been greatly expanded. We discovered the Carleman-Bers-Vekua equations, which are more straightforward because all coefficients are determined by the structural function (b). At the same time, Wirtinger derivatives are made. This talk includes an examination of the second-order nonlinear Laplace equation.

Keywords: Cauchy-Riemann equation, *K*-transformation, structural holomorphic condition, Carleman-Bers-Vekua equations, nonlinear Laplace equation

1.0.Introduction

1.1.Cauchy-Riemann equations

The Cauchy-Riemann equations are a set of two partial differential equations in complex analysis that, when combined with specific continuity and differentiability criteria, create a necessary and sufficient condition for a complex function to be complex differentiable, i.e. holomorphic. In 1752, Jean le Rond d'Alembert published a paper that included this system of equations. Leonhard Euler later linked this system to analytic functions in 1797. Cauchy then built his theory of functions using these equations in 1814. In 1851[1–3], Riemann published his dissertation on function theory.

Let be a complex-valued function on and be an open set in A. If the quotient converges to a limit when e 0 at the point x0, the function w is holomorphic. The quotient is properly defined in this case since e A and e = 0 with x0 +e The derivative of p at x0 is indicated as p′(x0), which denotes the quotient's limit when it exists:

It should be noted that in the limit above, e is a complex number that can approach 0 from any direction. If w is holomorphic at every point of on, the function p is said to be holomorphic on. We say w is holomorphic on if q is holomorphic in some open set including A if is a closed subset of A. Finally, we say that p is whole if w is holomorphic throughout all of A. Consider the complex plane A S2 = (a, b) | an S, b S | an S, b S | an S, b S | an S, b S | an S, b S | an S, b S | an S, b S | The Wirtinger erivatives are the first-order linear partial differential operators that are defined as follows. The space of A1 functions on a domain S2 is obviously the natural domain of definition for these partial differential operators, but because they are linear and have constant coefficients, they may be easily extended to any space of generalised functions. The Cauchy-Riemann elliptic system of differential equations can be deduced as follows, which is a classical approach of constructing the theory of analytic functions p = s +1r of a complex variable m = a + 1b. The Wirtinger derivative of p with respect to the complex conjugate of is zero in equation (1), which means that p = s + 1r is functionally independent of the complex conjugate of m. The Cauchy-Riemann (CR) equations are formalised as follows: The holomorphic functions are two real-valued functions with continuous first derivatives that solve the Cauchy-Riemann equations, a set of two partial differential equations..

1.2.Carleman-Bers-Vekua (CBV) equation.

The theory of generalised analytic functions was included into the pool of major partial differential equations approaches. The reason for this is that the theory of generalised analytic functions can take advantage of the benefits of complex analysis to solve more general systems of partial differential equations than traditional complex analysis can.

Initially, theory only looked at linear uniformly elliptic systems in the plane for two desired real-valued functions. The theories of I. N. Vekua are now used to solve partial differential equations in higher dimensions. Naturally, a theory with the same level of generality has yet to be created [4]. [5] claimed that a comparable theory could be built using a more extended elliptic system of first order differential equations.,

It is generally known that the system that was first explored [6,7] achieved a fundamental quality of the solutions, namely their uniqueness, under general assumptions about the coefficients. Teodorescu [8] previously investigated a system of the following type and used analytic functions to generate a general description of the solutions. This finding was crucial in the development of the whole theory. where D,E,F,G are the unknown coefficients picked at random. is known as the Carleman-Bers-Vekua (CBV) equation, and it has the form p = s + 1r. The theory of generalised analytic functions connects two aspects of analysis: the theory of complex variable analytic functions and the theory of elliptic type differential equations with two independent variables. Following the publication of I. Vekua's monograph, in which the author's long-term research and some of his students' and followers' results are given, the theory was developed as an independent element of analysis. The theory of extended analytic functions was founded on the foundations of [9.] Bers proposed a generalisation of analytic functions (so-called pseudo-analytic functions) based on a revision of the concept of the derivative around the same time as Vekua. Many writers presented various generalisations, reducing to specific examples until a complete theory of generalised analytic functions developed see [10,11].

When studying the aforementioned differential systems, a natural question arises: how should the solutions be interpreted? (the definition problem). It is obvious that assuming the fulfilment of mentioned differential equalities is insufficient to produce the class of functions with the required structure, even for the simplest and most fundamental case of system. The problem is considerably more complicated in the case of the system, because there are now more coefficients to consider. We'll see how J-transformation can be used to address this problem, making it easier to understand, especially because additional coefficients are given distinct meanings connected with one structural function J.

They investigated the solvability of the Riemann-Hilbert problem for a generalised Cauchy-Riemann system with many singularities and discovered several novel phenomena. Boundary value issues and their generalisations have been studied extensively in recent years with regular coefficients W,X Lp (), (p > 2) where is a bounded domain in the complex plane A. More specific boundary value problems arise in shell theory when elliptic systems of equations with singular coefficients appear. The research into the generalised Cauchy-Riemann system has yielded a slew of important findings, yet more coefficients remain unknown and undetermined..

1.3.Nonlinear Cauchy-Riemann equations

Cauchy-Riemann equations are linear equations that only allow linear Laplace equations to be solved. Carleman-Bers-Vekua (CBV) equation[13] was solved using generalised C-R equations.

The inhomogeneous Cauchy-Riemann equations, according to the theorem 2, are made up of two equations for a pair of unknown functions s(a,b) and r(a,b) of two real variables for some provided functions O(a,b) and T (a,b) defined in an open subset of R2. Most of the time, these equations are incorporated into a single equation[14].

2.0.Conclusions

In general, this article has succeeded in constructing a clear theory mode that unifies all types of theoretic forms by transforming them, which is essentially a unified transformation. -transformation is a subset of K-transformation in terms of mathematics. The generalised Wirtinger derivatives and structural holomorphic condition are then used to build a nonlinear Laplace equation. As a result, we'll go over each point individually.

3.0.References

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