A review on consolidation of poroelastic media due to surface loads

Sunita Rani

Guru Jambheshwar University of Science & Technology, Hisar, Haryana

email-id: sunit.moni@gmail.com

Abstract

Poroelasticity is a continuum mechanics concept. In Poroelasticity, a diffusing pore fluid filters an elastic solid skeleton in an inhomogeneous medium. The subject of poroelastic media consolidation under surface loads has been extensively studied in the literature. The consolidation study benefits geophysical applications such as mine collapse and fluid-driven fissures. The current work provides an overview of the problem of consolidation in poroelastic media.

Keywords: Consolidation; poroelasticity; surface loads

Introduction

Consolidation is the process of a saturated soil being compressed under a constant static pressure (Terzaghi, 1923). The compressibility of soil refers to how much the value of saturated soil decreases when compressed. The process of compression depicts the reduction in soil volume as a result of an external load. A change in shape or size of an object caused by applied force is known as deformation (Wang, 2000). Strain is a common term for deformation. When an object deforms, intermolecular forces oppose the applied forces; if the applied forces are not too great, these forces may be sufficient to totally resist the applied force and allow the object to regain its original shape and size when the forces are removed. Elastic deformations are the name given to such deformations. Irreversible deformations, on the other hand, persist even after the forces have been withdrawn.

Porosity, permeability, and the properties of fluid and solid constituents are all characteristics of a poroelastic medium (Cheng, 2016). Porosity is defined as the proportion of void volume to total volume. Permeability is the ability of a porous material to pass a fluid and can therefore differ in different directions. Poroelasticity is involved in the study of the time-dependent bond between a solid skeleton's deformation and fluid flow in a porous medium. Porous materials are solid structures consisting of pores or voids. Examples of natural porous materials include rocks, soils and biological tissues, and artificial porous materials include foams, ceramics and paper products.

Literature Review

Terzaghi (1923) was the first who presented the one-dimensional theory of soil consolidation and suggested certain assumptions for the study of a fully saturated soil. Biot (1941, 1956) was the first to propose a three-dimensional linear poroelasticity hypothesis for a saturated soil. Saturated soils are modelled as deformable, linear, porous, elastic materials saturated with compressible fluids, according to this theory. In porous elastic medium, McNamee and Gibson (1960a) proposed that stresses and pore pressure can be represented using two displacement functions. These routines are in handy when dealing with situations involving a semi-infinite body or infinite layer. By applying uniform pressure to the surface, McNamee and Gibson (1960b) solved the issues of the semi-infinite clay layer over a circular area or over an infinite strip. Schiffman and Fungaroli (1965) explored the consolidation of a semi-infinite solid due to homogeneous tangential pressure on the surface using three displacement functions. Hankel-Laplace transform techniques are used to tackle the problem of a unit tangential load distributed uniformly over a circular area. For a pervious and an impervious surface, the excess pore pressures at a point are analysed.

In some cases, the poroelastic medium is modelled as a half-space for consolidation problems, ignoring the curvature of Earth. When the dimensions of the loaded area are much smaller as compared to the thickness of the soil stratum, the medium is modelled as a finite layer. Gibson et al. (1970) solved the axially symmetric and plane strain consolidation of a clay stratum on a smooth impermeable foundation. The formulas for the immediate and final settlement of the layer's surface have been numerically assessed. The problem of a layer adhering to a rough, inflexible foundation that is subjected to general surface loading was solved by Booker (1974). It is assumed that the pore fluid is incompressible. For different Poisson's ratios, the solution for specific scenarios such as uniformly loaded strip, square, and circle has been calculated. Lee and Sills (1981) demonstrated that a soil stratum of fill applied in a soft, moist condition can be consolidated under its own weight and can withstand enormous strains. There are no limitations on the size of the project. Using the Fourier transform method, Booker and Small (1982a) established a method for analysing the consolidation of a horizontally stratified soil. This approach reduces consolidation partial differential equations to ordinary differential equations. A finite layer technique is used to solve ordinary differential equations. Booker and Small (1982b) built on Booker and Small (1982a) findings 's to solve the problem of horizontally fixed soil consolidation under both axially symmetric and general surface loads. Integral transformations were employed by Booker and Randolph (1984) to propose a solution for cross-anisotropic deformation and flow properties in a soil medium. The ratio of the soil's horizontal to vertical permeability has a big influence on the numerical values for circular and rectangular surface loads.

A boundary integral equation approach for linear porous elasticity was developed by Cheng and Liggett (1984). When compared to the finite element method, this paper demonstrated that the boundary integral equation method is more efficient and accurate. For soil consolidation, the model's practical applications are taken into account. Booker and Small (1987) found that a layered soil subjected to strip, circular, or rectangular surface loading, as well as fluid withdrawal owing to pumping, might be consolidated. For the time-dependent field variables, they used the Laplace transform. Roeloffs (1988) calculated the stress and pore pressure variations generated by a constant periodic deviation of water level on the homogeneous poroelastic half-space using linked Biot equations of elastic deformation and pore fluid flow. Rajapakse and Senjuntichai (1993) proposed three-dimensional general solutions for the analysis of asymmetric problems involving a poroelastic solid with compressible elements. To find explicit general solutions, Fourier, Hankel and Laplace transforms are used. A set of fundamental solutions is derived from the general solutions.

The axisymmetric consolidation response of a poroelastic layer sitting on a rigid impermeable basis generated by circular foundation was studied by Selvadurai and Yue (1994). The undrained Poisson's ratio of the poroelastic medium is found to have a considerable impact on the rigid circular indentor's magnitude and consolidation rate. Yue et al. (1994) conducted an analytical study of the excess pore-fluid pressure in a limited seabed layer caused by a compressible pore fluid. The poroelastic seabed layer is supported by a rough, impermeable substrate. The layer's surface is believed to be either entirely permeable or completely impermeable. The solution is found using the Fourier transform method. Kalpna and Chander (2000) employed the Green's function approach to calculate stresses and pore pressure in a poroelastic half-space induced by finite reservoir surface load, as well as solving the inhomogeneous diffusion equation for pore pressure computation. The permeability anisotropy of oil/gas reservoirs can vary up to 100:1 in the horizontal to vertical direction, according to Ganbe and Kurashige (2001). As a result, the effect of permeability anisotropy on the consolidation of a poroelastic material due to external applied load is important to investigate in geophysical or engineering applications.

Wang and Fang (2003) developed the non-axisymmetric Biot's consolidation issue in cylindrical dimensions by introducing state variables. The state variables stresses, displacements, and pore pressure are the primary unknowns. Three cases are presented for the lower surface: pervious and impervious rough stiff bases, and porous elastic half-space. Chen (2004) studied the anisotropy of pore fluid permeability and compressibility and proposed analytical consolidation solutions for multi-layered poroelastic half space. The state vector approach is used to solve the basic governing equations.

Using the finite layer technique, Mei et al. (2004) investigated a cross-anisotropic elastic constitutive model for the consolidation of a layered soil. Both the fluid and solid elements were considered to be incompressible. It is discovered that cross anisotropy has a significant impact on instant settlement, final settlement, and consolidation behaviour. Conte (2006) proposed a computationally effective method for analysing linked consolidation in unsaturated soils under plane strain and axially symmetric circumstances. The Fourier transform is utilised to find the solution for plane strain loading, while the Hankel transform is employed for axial symmetry. Ai and Wang (2008) devised a new strategy for resolving the axisymmetric Biot's consolidation problem for a finite soil layer. The Laplace and Hankel transform domain is used to determine the relationship between displacements, stresses, excess pore water pressure, and flux at a location in a finite soil layer. They demonstrated the method's correctness and efficiency in solving the consolidation problem numerically. The cylindrical coordinate system was used to solve Biot's consolidation of a finite soil layer by Ai et al. (2008). Between the ground surface and an arbitrary depth, the displacements, stresses, excess pore water pressure, and flow are estimated using the Laplace and Hankel transform domains.

Ai and Wu (2009) proposed a more efficient analytical method for resolving planar strain consolidation in a limited soil layer with anisotropic permeability. In the Laplace-Fourier transform domain, the relationship of basic variables between the ground surface and the depth is established using the governing equations. The actual answer is obtained by numerically inverting the Laplace transform and the Fourier transform. The anisotropy of permeability has a significant impact on the consolidation behaviour of soils, according to numerical data. Using linearized Biot's theory, Singh et al. (2009) explored the consolidation of a poroelastic half-space through anisotropic permeability under axisymmetric surface stresses. The stiffness is calculated using a stiffness formula. They assumed that both the fluid and solid elements were compressible. The disc loading issue has been thoroughly examined. The analytical answers were obtained using Laplace-Hankel transform techniques.

The influence of negative Poisson's ratio on the two-dimensional consolidation of a porous elastic half-space for normal strip loading and axisymmetric normal disc loading was investigated by Rani et al. (2010). Using the eigen value technique and a Laplace-Hankel transform, Ai et al. (2011) solved the governing equations of Biot's consolidation analytically. The Laplace-Hankel transform is numerically inverted to yield the global stiffness matrix equation for multilayered soils. The accuracy of this strategy is demonstrated using numerical examples. The quasi-static deformation of a clay layer overlying on a rough-rigid impermeable base by axisymmetric surface stresses was explored analytically and quantitatively by Rani et al. (2011). On the consolidation process, the impacts of fluid compressibility, solid component compressibility, and permeability anisotropy are investigated. The axisymmetric problem for circular loading is examined, along with numerical calculations.

Biot's consolidation of multilayered soils of arbitrary depth subjected to non-axisymmetric loadings was studied by Ai and Zeng (2012) using a computationally efficient and stable technique. The Fourier expansion and Laplace-Hankel technique are used to solve the governing equations of Biot's consolidation. In the transformed domain, the analytical layer-element approach for characterising the link between generalised displacements and stresses of a soil layer is precisely derived. To establish the relationship between generalised stresses and displacements of a single soil layer, analytical layer-element solutions to Biot's consolidation through permeability anisotropy and incompressible solid and fluid components were reported by Ai et al. (2012). A global stiffness matrix is then built and solved, taking into account the continuity and the boundary conditions of the soil layers. The true solution is possible. The inversion of the Laplace-Fourier transform can be used to determine the true solution.

Using Biot's linearized theory for fluid saturated porous materials, Kumari and Miglani (2012) investigated the plane strain deformation of a poroelastic half space which is in welded contact with an elastic half-space having line-load in elastic half space. The integral equations for stresses and displacements are produced by applying boundary conditions at the interface. Graphically depicted are the undrained displacements, stresses, and pore pressure for poroelastic half space. In the cylindrical polar coordinate system, Ai et al. (2013) developed a precise global stiffness matrix for the non-axisymmetric consolidation of layered porous elastic materials through permeability anisotropy and compressible elements. The analytical layer-element is obtained by applying Hankel, Laplace and Fourier transforms to the state variables in the governing equations.

Using Biot's consolidation theory of fluid-infiltrated porous materials, Ai and Hu (2015) calculated the consolidation of two-dimensional and three-dimensional multilayered poroelastic materials among compressible fluid and solid constituents, as well as anisotropy in permeability due to external load. The Laplace-Fourier transformation techniques were utilised to reduce the partial differential equations to ordinary ones. The findings show that the compressibility of the elements has a significant impact on the consolidation process. Puri and Rani (2016) extended Gibson et al. (1970)'s solution for the plane strain deformation of a clay stratum covering a smooth rigid foundation caused by normal strip and normal line loading. The pore pressure and consolidation expressions have been explicitly obtained. The effect of fluid and solid ingredient compressibility, as well as the permeability ratio, has been numerically investigated. The axisymmetric consolidation of multi-layered poroelastic soils through compressible components was investigated by Liang et al. (2017). In the Laplace-Hankel transformed domain, formulas for displacements, stresses, and pore pressure are produced.

The deformation of a porous elastic soil stratum lying over an elastic-half space subjected to axisymmetric surface loads was studied by Rani and Rani (2017). The fluid and solid components are assumed to be compressible, and the permeability varies both horizontally and vertically. The solutions for displacement, stresses and pore pressure are attained using Laplace-Hankel transform techniques. The influence of compressibility and anisotropy on fluid and solid component permeability has been investigated. Rani and Rani (2018) explored the quasi-static axisymmetric deformation of a stratum sitting over a smooth-rigid pervious or impervious base due to surface loads. For both pervious and impervious bases, expressions for displacement, stresses and pore pressure of a soil strata have been attained using Laplace-Hankel transform techniques. The effect of permeability anisotropy and compressibility of the fluid and solid components on the consolidation procedure is examined.

Conclusion

This paper discussed both the axisymmetric and the plane strain consolidation problems of different poroelastic medium under surface loads. The study is related to normal strip, circular, or rectangular surface loading. The most extensively used methodologies in this category were discovered to include the Laplace-Hankel transform techniques, transfer matrix approach, Laplace-Fourier transform, Green's function approach, state vector approach, numerical, and analytical procedures. The poroelasticity theory is also discussed, which has been applied to a wide range of geophysical and engineering challenges.

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