OUTCOME OF A UNIQUE FIXED POINT THEOREM BY USING RATIOAL INEQUALITY

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Abstract: The motive of this paper is outcome of a unique fixed point theorem by using rational inequality and finally, investigated a unique fixed point theorem which is a generalization of many authors of fixed point theory.

Key words and phrases : Orbitally continuous, Self maps, Cluster point, Fixed point, Metric Space.

Ams(2010) subject classifications: Primary 47H10 Secondary 54H25,34B15

1. Introduction And Preliminary

Obtained fixed point for self maps On metric space by altering distances between the points with use of certain continuous function was Khan, Swaleh and Sessa [6], then Sastry and Babu [10] discussed and established the existence of fixed points for the orbit of single self maps and pairs of self maps by using control function which is not monotonic with several examples to emphasize the importance of control function. Recently Babu and Ismail [1],Dosenvi et.al [4],Mihet [7] Sayyed et.al. [11,12], Pourmoslemi et.al.[9] ,Gomicki [5] , Dolhare and Khanpate [3], Suhas and Dolhare [14], Mishra et. al. [8] and many authors proved a fixed point theorem for self maps by altering distances between the points under more general conditions.

Delbosco [2] and Skof *[13]* have established fixed point theorems for selfmaps of complete metric spaces by altering the distances between the points with the use of a function $\phi : R^+ \rightarrow R^+$ satisfying the following properties :

1. ϕ is continuous and strictly increasing in R^+ ;
2. $\phi(t) = 0$ if and only if $t = 0$;
3. $\phi(t) \geq M\, t^{\mu}$ for every $t > 0$, where $M > 0$, $\mu > 0$ are constant

For achieving result we shall refer the following theorems,

THEOREM1.1 Let T be continuous selfmap of a metric space (X,d). Such that for some x_0 in X, the sequence $\{T^n x_0\}$has a cluster point in z in X and there exist $\phi \in \Phi$ such that

$$\phi(d(Tx , Ty)) < c\phi(d(x, y)) + \frac{(1-c)}{2}[\phi(d(x, Tx)) + \phi(d(y, Ty))] \text{ for all distinct } x, y \text{ in } X,$$

where $0 \leq c < 1$. Then z is the unique fixed point of T.

THEOREM 1.2 Let T be a selfmap on a metric space (X, d). Suppose there exists a point x_0 in X such that the orbit $O(x_0) = \{T^n x_0; n =0, 1, 2,.....\}$ has a cluster point z in X. If T is orbitally continuous at z and Tz there exists a $\phi \in \Phi$ such that $\phi(d(Tx, Ty)) < (d(x, y))$

for each x, $y = Tx \in \overline{O(x_0)}$, $x \neq y$, then z is a fixed point of T.

THEOREM1.3. Let T be a continuous self map of a metric space (X, d) such that for some x_0 in X, the sequence $\{T^n x_0\}$has a cluster point in z in X and there exist $\phi \in \Phi$ such that

$$\phi(d(Tx , Ty)) < \max \{ \phi(d(x, y), \phi(d(x, Tx), \phi(d(y, Ty)\}$$

for all x,y in X, then z is a unique fixed point of T.

THEOREM 1.4. Sayyed et.al [7]. Let T be a selfmap on a metric space (X, d). Suppose there exists a point x_0 in X such that the orbit $O(x_0) = \{T^n x_0; n =0, 1, 2,.....\}$ has a cluster point z in X. If T is orbitally continuous at z and Tz there exists a $\phi \in \Phi$ such that

$$\phi(d(Tx , Ty)) \leq \frac{\alpha\, \phi(d(x,Tx))\Phi(d(x,ty)) + \phi(d(y,Ty))\phi(d(y,Tx))}{\phi(d(x,Ty)) + \phi(d(y,Tx))} + \beta\, \phi(d(x,y))$$

For all $x,y \in X$ and α and β are non negative with $0 \leq \alpha+\beta < 1$. Then z is a fixed point of T.

2.MAIN RESULT

THEOREM 2.1 Let (X,d) be a metric space and V is a self map on X, then there exists a point $x_0 \in X$ such that the orbit $O(x_0) = \{V^n x_0 ; n= 0,1,2,... \}$has a cluster point z in X. If V is orbitally continuous at z and V_z and there exists $\phi \in \Phi$ such that

$$\phi(d(Vx,Vy)) \leq a_1 \frac{\phi(d(x,Vx))[1+\phi(d(y,Vy))]}{1+\phi(d(Vx,Vy))} + a_2 \frac{\phi(d(y,Vy))[1+\phi(d(x,Vx))]}{1+\phi(d(x,y))} + a_3 \phi(d(x,y))$$

___ ___ ___ A*

For all x,y ∈ X and a_1 , a_2 and a_3 are non negative with $0 \le a_1 + a_2 + a_3 \le 1$. Then z is a fixed point of V.

PROOF: Defined the sequence $\{x_n\}$by $x_n = V^n x_0$, for n =1,2,... . If $x_n = x_{n+1}$, for some n then $x_m = x_n$ for some $m \ge n$ so that $z = \lim_{n\to\infty} x_m$ and $V_z = z$ by orbital continuity of V.

Taking $x_n \neq x_{n+1}$, if $\alpha_n = \phi(d(x_n, x_{n+1}))$ then by A* it follows that

$$\alpha_{n+1} = \phi(d(x_{n+1}, x_{n+2}))$$

$$= \phi(d(Vx_n, Vx_{n+1}))$$

$$\le a_1 \frac{\phi\big(d(x_n, Vx_n)\big)[1+\phi(d(Vx_{n+1}, x_{n+1}))]}{1+\phi(d(Vx_n, Vx_{n+1}))} + a_2 \frac{\phi\big(d(x_{n+1}, Vx_{n+1})\big)[1+\phi(d(x_n, Vx_n))]}{1+\phi(d(x_n, x_{n+1}))}$$

$$+ a_3\, \phi(d(x_n, x_{n+1})) ,$$

i.e. $\alpha_{n+1} = \frac{a_1 + a_3}{1 - a_2} \alpha_n$, so that $\{x_n\}$ is a strictly decreasing sequence of positive numbers and hence converges, say to $\alpha \ge 0$.

Let k(n) be a sequence of positive integers such that $\{x_{kn}\}$ converges to z. Then $\{\alpha_{k(n)}\}$converges to α. By the continuity of ϕ and orbital continuity of V at z.

$$\alpha = \lim_{n\to\infty} \phi(d(x_{k(n)}, x_{k(n)+1})) = \phi(d(z, Vz))$$

It is enough to show that α = 0. We observe that z and Vz belong to $\overline{0(x_0)}$.

Suppose $\alpha \neq 0$ then by A* and the orbital continuity of V at z and Vz.

$$0 \neq \alpha = \lim_{n\to\infty} \alpha_{kn+1} = \lim_{n\to\infty} \phi\,(d(x_{kn+1}, x_{kn+2})) = \phi\,(d(Vz, V(Vz))) < \phi(d(z, Vz)) = \alpha .$$

Which is a contradiction, hence α = 0. Since ϕ vanishes at zero it follows that Vz = z .

<u>COROLLARY 2.1:</u> Let V be a continuous self map of a metric space (X,d) such that for some x_0 inn X, the sequence $\{V^n x_n\}$ has a cluster point z in X and there exist $\phi \in \Phi$ such that

$$\phi(d(Vx, Vy)) \le a_1 \frac{\phi\big(d(x, Vx)\big)[1+\phi(d(y, Vy))]}{1+\phi(d(Vx, Vy))} + a_2 \frac{\phi\big(d(y, Vy)\big)[1+\phi(d(x, Vx))]}{1+\phi(d(x, y))} + a_3\, \phi\,(d(x, y))$$

For all x,y ∈ X and a_1 , a_2 and a_3 are non negative with $0 \le a_1 + a_2 + a_3 \le 1$. Then z is a unique fixed point of V.

3.CONCLUSION

In this paper, proved a unique fixed point theorem by using contractive type ration inequality for self maps. These results can be extended to any directions and can also be extended to fixed point theory of multi-valued mappings, compatible, weakly compatible, ordered and many mappings.

4.ACKNOWLEDGEMENTS

The author would like to give his sincere thanks to the editor and the anonymous referees for their valuable comments and useful suggestions in improving and advancing the article.

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