A Theoretical model of a Newtonian fluid Model for Steady Flow of blood through Catheterized tapered artery with stenosis and velocity slip at the interface

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Abstract: An annulus is limited between an arterial stenosis formed along a tapering wall and a uniform catheter that runs parallel to it, allowing for a continuous constant laminar flow of blood. In this study, the biological fluid blood is supposed to behave as a Newtonian fluid. A velocity slip condition is applied at the catheterized wall with varying amounts of stenosis, and zero-slip at the catheter border is taken. Analytical expressions are obtained for axial velocity, flow rate, wall shear stress and their variations with different flow parameters are illustrated in figures. The behavior of these flow variables has been investigated in this constrained circular region. It's worth noting that the insertion of an axial slip will increase velocity and flow rate on the one hand, while decreasing wall shear stress and apparent viscosity on the other. There includes a discussion of the effects of stenosis and tapering. A brief explanation of the physiological amplifications of this theoretical modeling under blood flow conditions is also included.

Keywords: Laminar flow, annulus, stenosis, catheterized tapered artery, viscosity.

1. Introduction:

Blood flow through tapered tubes is crucial not only for understanding the flow behavior of the wonderful body fluid in arteries, but also for the development of prosthetic blood vessels (How and black,1987). An unnatural and unexpected growth is reported to occur at the lumen of an artery under certain pathological situations. This is known as stenosis, which is a type of CVS disease known as atherosclerosis.. Due to a reduction in blood supply to different organs and tissues or obstruction of an artery, this undesired growth and its presence at one or more spots in blood vessels causes significant circularatory illnesses (Young 1970, Caro 1973). As stenosis progresses through mild, moderate, and severe levels, difficulties in the body arise as a result of arterial blockages that partially or entirely cut off blood supply or completely scaled off an artery. This causes CVS disorders such as myocardial infarction, heart block, stroke, and coronary thrombosis, among others (Boyd 1963, Guyton 1970, Puniyani and Nimi 1998, Dintenfass 1981, Biswas 2000). Aside from that, artificial catheters are sometimes implanted in uniform and stenosed arteries for medical reasons and for clinical and subclinical objectives. For blood flow through a stenosed artery, the pressure-flow relationship changes, and this is exacerbated by the insertion of a catheter in the stenotic region, which increases the impedance or resistance to flow and changes the pressure distribution (Biswas and Bhattacherjee 2003, Biswas and Chakraborty 2010) quickly and efficiently. Blood flow through stenosed and catheterized restricted arteries appears to be crucial. Hydrodynamic variables may play a role in the creation, development, and progression of artery stenosis, according to research (Young and Tsai, 1979; caro,1971). Artificial catheters are sometimes implanted in stenosed arteries for a variety of medical and therapeutic reasons.

The presence of a flexible catheter in a restricted section of an artery changes the pressure-flow relationship significantly, since it increases the impedance or frictional resistance to flow and changes the pressure distribution (Young and Tsai, 1979; Biswas and Nath, 1999). The study of blood flow in the annular zone of a mammalian artery has already piqued the curiosity and attention of scientists Because of its potential physiological importance and numerous clinical uses. It's worth noting that physiological fluid blood can behave like a Newtonian fluid in certain flow circumstances (Schlichting, 1968). A Newtonian fluid's laminar flow across an annulus has been studied (Yuan, 1969). In a catheterized stenosed uniform artery, the annular blood flow was examined (Young and Tsai, 1979; Biswas and Nath,1999; Jayaraman and Tewari, 1995; Biswas and Bhattacherjee,2003). Blood flow in tapered arteries with stenosis has recently been modeled (Liu et al., 2004).

Theoretical, computational, and experimental studies of blood flow in arteries have been conducted in recent years (Biswas,2000; Fung,1981; Forrester and Young,1970; Liu et al.,2004; How and Black,1987). The complicated shape of arteries (bending, bifurcating, branching, tapering, discontinuous, etc.) is also a significant component that impacts local hemodynamic (Guyton, 1970; Puniyani and Niimi, 1998). Much of the study on arterial flow in recent decades has been inspired by the association between artery flow, particularly wall shear stress, and the sites where atherosclerosis occurs.

It is now well understood that the most vulnerable sites are those where shear stresses are modest or change rapidly in time or distance. These situations are more likely to occur when the channel is curved, bifurcated, has a junction, a side branch, or any other abrupt change in flow shape 9Berger and Jou, 2000). There is little doubt that artery tapering is an important feature of the mammalian arterial system, and the occurrence of stenosis along the tapered wall can significantly alter the flow status. We're interested in studying the steady annular flow of blood via a catheterized tapered channel with stenosis because of the foregoing concerns. Blood is used as a Newtonian fluid in this scenario, and an axial slip velocity is applied to the stenotic wall.

2. Mathematical Formulation and Flow geometry

The flow geometry for the steady flow of an incompressible Newtonian fluid in an annular region between a constricted tapered tube of undisturbed radius R_0 and a co-axial uniform circular pipe of radius kR_0 (k<<1). is shown in Figure 1.

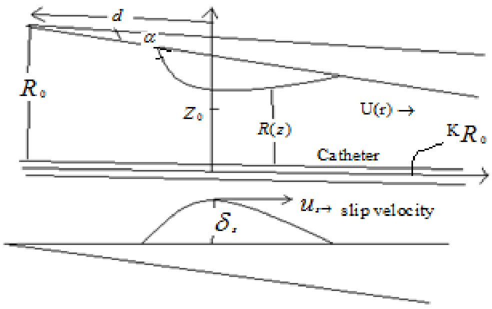

Fig1.Schematic diagram of a catheterised tapered artery with stenosis

A catheter is co-axial to the tapering vascular segment with an axially symmetric stenosis, which is theoretically described as a rigid tube with a circular section. The constricted tapering artery's shape (fig.1) is theoretically described as (Liu et.al, 2004)

$$R(z) = R_0 - m(z+d) - \frac{\delta_s \cos\alpha}{2}\left(1+\cos\frac{\pi z}{z_0}\right), |z| \le z_0$$

$$= R_0 - m(z+d), |z| \ge z_0 \qquad (2.1)$$

Where R(z) signifies the radius of the tapered arterial segment in the stenotic region, R_0 is the constant radius of the straight artery in the non-stenotic region, α is the tapering angle, δ_s is the maximum height of stenosis, d the stenosis location, z_0 is the half length of the stenosis and $m = \tan\alpha$ represents the slope of the tapered vessel. Let (r, θ, z) be the system of co-ordinates, used to analyze the flow in the geometry as stated above, where r and θ are along the radial and circumferential directions and z-axis is taken along the axis of the artery.

3. Governing Equations:

Let us consider a steady, Laminar flow of blood through an axially non symmetric but radially symmetric stenosed artery - a circular tube with a catheter of radius kR_0 (k<<1) co-axial to it and one-dimensional flow obeying the constitutive equation for a Newtonian fluid. Fluid velocity vector has the form $\vec{V} = (0,0,u(r))$ in cylindrical polar system (r,θ,z) representing the radial, circumferential and axial coordinate respectively. The equations of motion governing the fluid flow in (r,θ,z) coordinate system (schlichting, 1968) are written as follows

$$\frac{\partial p}{\partial r} = 0, \qquad (3.1)$$

$$\frac{\partial p}{\partial \theta} = 0, \qquad (3.2)$$

$$\frac{\mu}{r}.\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\partial p}{\partial z} \qquad (3.3)$$

Where $u = u(r)$ denotes the axial velocity, μ is the viscosity of blood and p the pressure.

As a result of equation (3-5), the governing equation of fluid flow is given by

$$C + \frac{\mu}{r}.\frac{\partial}{\partial r}\left(r\frac{\partial \mu}{\partial r}\right) = 0 , \quad kR_0 \le r \le R(z) \qquad (3.4)$$

where $C=-\frac{dp}{dz}$, is the pressure gradient.

In order to study the annular flow of blood between the stenotic wall and a catheterized tapered artery, we shall consider the following cases:

4. Boundary Conditions:

The boundary conditions for the present problem are given by

$$u(r)=u_s \text{ at } r=R(z) \tag{4.1}$$

$$u(r)=0 \text{ at } r=kR_0 \tag{4.2}$$

Where u_s is the axial velocity slip at the stenotic wall (Biswas,2000)

5. Solutions of the problem:

The general integral of equation (3.4) is obtained as

$$u(r)+\frac{cr^2}{4\mu}=B+A\ln r, \qquad kR_0\le r\le R(z) \tag{5.1}$$

Where A and B are constants of integration. As a result of applying condition (3.1) in equation (5.1) the expression for velocity function becomes

$$u(r)=\frac{C}{4\mu}\left[(kR_0)^2-r^2\right]+\frac{\ln\left(\frac{r}{kR_0}\right)}{\ln\left(\frac{R(z)}{kR_0}\right)}\left[u_s+\frac{c}{4\mu}\left\{R(z)^2-(kR_0)^2\right\}\right], kR_0\le r\le R(z) \tag{5.2}$$

The **rate of flow** for the steady, incompressible and laminar flow is obtained by integrating the quantity

$Q=2\pi\int_{r=kR_0}^{R(z)} ru(r)dr$ which becomes with the help of the equation (4.2)

$$Q=\pi u_s\left[(R(z))^2-\frac{(R(z))^2-(kR_0)^2}{2\ln\left(\frac{R(z)}{kR_0}\right)}\right]+\frac{\pi c}{8\mu}\left[(R(z))^4-(kR_0)^2-\frac{\left\{(R(z))^2-(kR_0)^2\right\}}{\ln\left(\frac{R(z)}{kR_0}\right)}\right] \tag{5.3}$$

Expression **for wall stress** is defined by

$$\tau_{R(z)}=-\mu\frac{\partial u(r)}{\partial r}]_{r=R(z)} \tag{5.4}$$

Which becomes with the help of equation (4.2)

$$\tau_{R(z)}=\frac{cR(z)}{2}-\frac{\frac{\mu}{R(z)}}{\ln\left(\frac{R(z)}{kR_0}\right)}\left[u_s+\frac{c}{4\mu}\left\{(R(z))^2-(kR_0)^2\right\}\right] \tag{5.5}$$

Apparent viscosity can be computed with the help of formula $\mu_a=\frac{\pi c(R(z))^4}{8Q}$

Where Q has the representation in equation (5.3)

Thus with the help of equation (5.3), we get

$$\mu_a=\left[\frac{8u_s}{c(R(z))^2}\left[1-\frac{1-\left(\frac{kR_0}{R(z)}\right)}{2\ln\left(\frac{R(z)}{kR_0}\right)}\right]+\frac{1}{\mu}\left\{-\left(\frac{kR_0}{R(z)}\right)^4-\frac{\left\{1-\left(\frac{kR_0}{R(z)}\right)^2\right\}^2}{\ln\left(\frac{R(z)}{kR_0}\right)}\right\}\right] \tag{5.6}$$

The non dimensional form of the flow variables and flow geometry can be expressed by using the following non-dimensional variables:

$$\bar{R} = \frac{R(z)}{R_0}, \quad \bar{\delta_s} = \frac{\delta_s}{R_0}, \mathrm{u}_0 = \frac{cR_0^{\ 2}}{4\mu}, \quad \mathrm{Q}_0 = \frac{\pi c R_0^{\ 4}}{8\mu}$$

$$\bar{Q} = \frac{Q}{Q_0}, \bar{\mathrm{u}}_\mathrm{s} = \frac{u_s}{u_0}, \bar{r} = \frac{r}{R_0}, \overline{kR_0} = \frac{kR}{R_0}, \bar{\tau}_{R(z)} = \frac{\tau_{R(z)}}{\tau_0}, \bar{\mu}_\mathrm{a} = \frac{\mu_a}{\mu}$$

Velocity function:

$$\bar{u}(r) = \left[\left(\overline{kR_0}\right)^2 - \bar{r}^2\right] + \frac{\ln\left(\frac{\bar{r}}{\overline{kR_0}}\right)}{\ln\left(\frac{\bar{R}(z)}{\overline{kR_0}}\right)}\left[\bar{u}_s + \left\{\left(\bar{R}(z)\right)^2 - \left(\overline{kR_0}\right)^2\right\}\right], \overline{kR_0} \le \bar{r} \le \bar{R}(z), (5.7)$$

Flow Rate:

$$\bar{Q} = \bar{u}_s\left[2\left(\bar{R}(z)\right)^2 - \frac{\bar{R}(z)^2 - \left(\overline{kR_0}\right)^2}{2\ln\left(\frac{\bar{R}(z)}{\overline{kR_0}}\right)}\right] + \left[\left(\bar{R}(z)\right)^4 - \left(\overline{kR_0}\right)^4 - \frac{\left\{\left(\bar{R}(z)\right)^2 - \left(\overline{kR_0}\right)^2\right\}}{\ln\left(\frac{\bar{R}(z)}{\overline{kR_0}}\right)}\right] \quad (5.8)$$

Wall shear stress:

$$\bar{\tau}_{R(z)} = \bar{R}(z) - \frac{1}{2\bar{R}(z)\ln\left(\frac{\bar{R}(z)}{\overline{kR_0}}\right)}\left[\bar{u}_s - \bar{v}_s + \left\{\left(\bar{R}(z)\right)^2 - \left(\overline{kR_0}\right)^2\right\}\right] \quad (5.9)$$

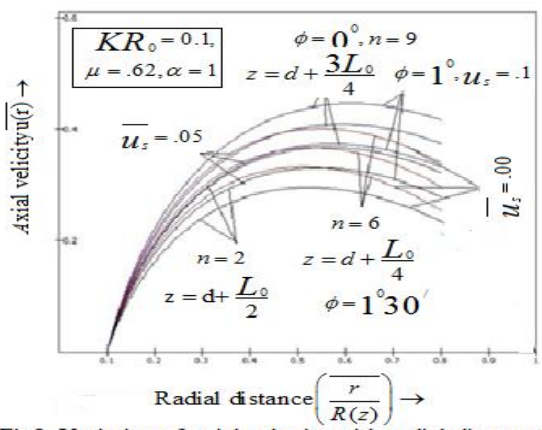

Fig2. Variation of axial velocity with radial distance for different slip velocities

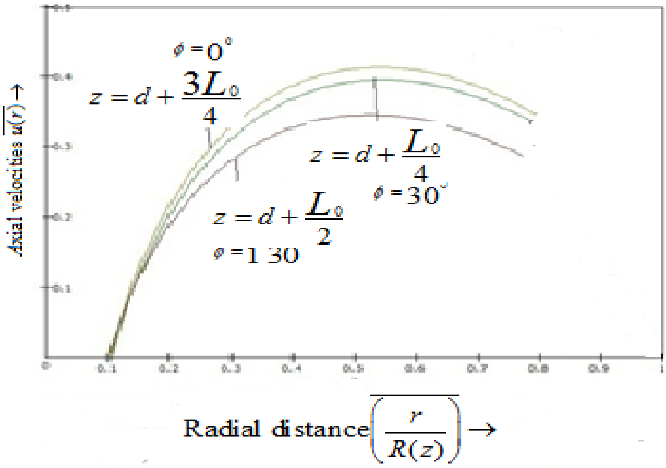

Fig.3 Variation of axial velocity against radial distance

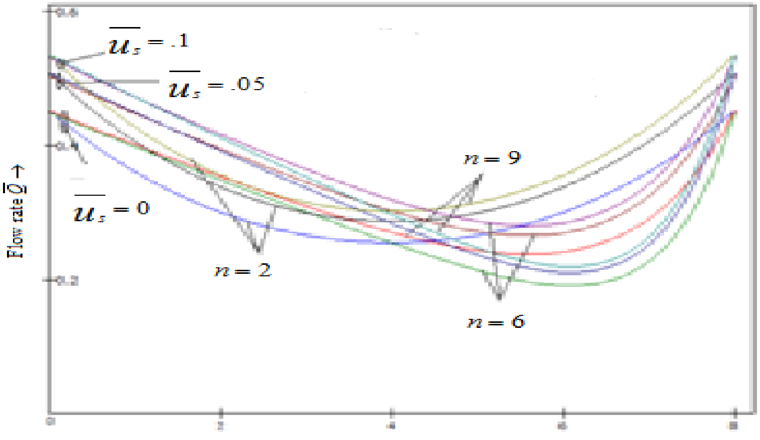

Fig.4 Variation of flow rate against axial distance for n=2,6,9 and $\overline{u_s} = 0, .05, .1, \phi = 0^{\circ}, 30^{\circ}, 1^{\circ}30^{\prime}$

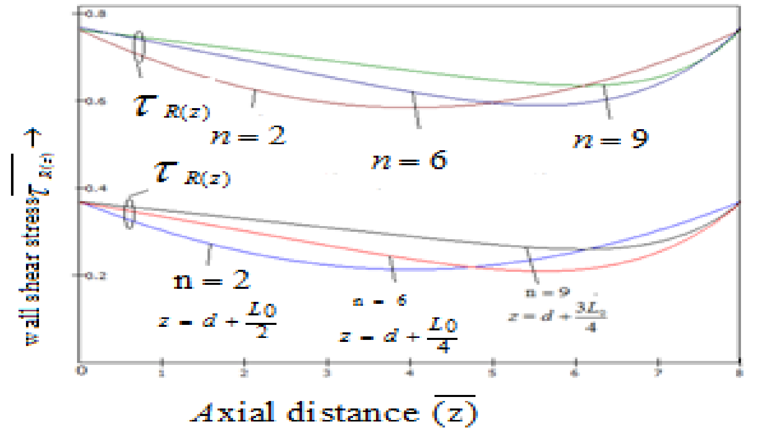

Fig5. Variation of shear stress against axial distances for different values of n and $\overline{\delta_s} = .12,.15, \phi = 0^{\circ}, 30^{\circ}, 1^{\circ}30'$

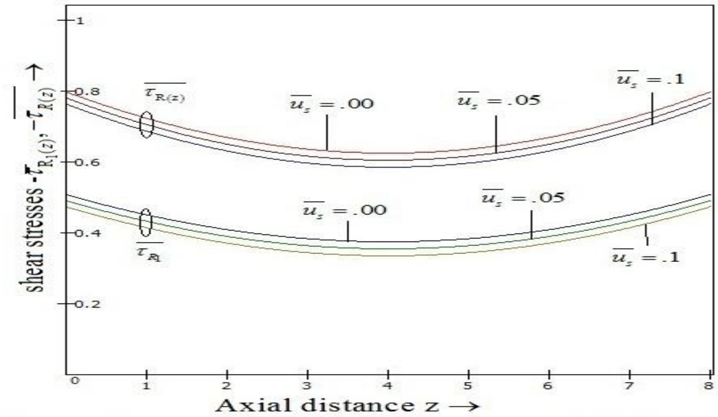

Fig.6 Variation of shear stress against axial distanc for $kR_0 = .1, \overline{u}_s = 0, .05, .1, n = 2$

6. **Results and Discussions**:

In carrying out the present work for blood flow through an annular region in (Fig 1) $kR_o \leq r \leq R(z)$ between a stenotic wall R(z) and a catheterized artery (k << 1, catheter radius kR_0 << R_0, artery radius), the following estimates for the constricted region, such as stenosis length L_0 = 8, and d its location in the region $d \leq z \leq d+L_0$ (Biswas, 2000; Chaturani and Biswas,1983), stenosis development in asymmetric manner and maximum heights of stenosis $\overline{\delta_s}$ in dimensionless form) equals to $1-\sqrt{3}/2,\ 1-1/\sqrt{2}$, and $1/2$ corresponding to an abnormal growth of 25, 50 and 75 percents in three respective and gradual cases of mild, moderate and severe formations at the lumen of an artery, have been used in developing the current mathematical analysis. In view of this, analytical expressions for axial velocity, flow rate, pressure gradient, resistance to flow, wall shear stress have been obtained in this study and their graphical representations are shown in Figs. (2-6).

The present model includes the following cases:

When $kR_0 = 0$ and $R(z) = R_1(z)$, it results in Newtonian flow model of an arterial stenosis with no-slip (u_s = 0) and slip $u_s \neq 0$ respectively. In case R(z) = R_1(z) = R_0 and $u_s \geq 0$, it reduces to annular flow between co-axial cylindrical tube models of Newtonian fluid with slip and no-slip conditions.

When R(z) = R_1(z) = R_0, kR_0 = 0 and u_s = 0, it expresses a Poiseuille flow of blood inside a uniform tube with slip or zero-slip at the boundary. If R(z) $\neq R_1$(z) $\neq$ R_0, $kR_0 \neq 0$ and $u_s \geq 0$, then it represents a two-layered annular flow of blood through a uniform artery with slip or no-slip at interface.

In the analysis, the combined influences of several parameters have been developed, in cases of uniform region. To analyze the quantitative effect of uniform artery, maximum height of stenosis δ_i , δ_s, slip velocities ($u_s \geq 0$) at the interface, Newtonian behaviour of blood, two-layered flow etc., computer codes have been developed for the numerical evaluations of the analytic results obtained for velocity, flow rate, wall shear stress and pressure gradient for parameter values δ_s =0.15, δ_i =0.12, u_s =0.00,.05,.1, Q=0.5, 1.0, 1.5 (Verma and Parihar2009,2010) and viscosities μ_1 =1.2 cp, μ_2 = 2 cp, $\mu_2' = .62$,α = 0.82 (< 1) for a full scale location from z = d to d+L_0 ,and $0 \le r \le R_1(z)$, $R_1(z) \le r \le R(z)$ for PPL and core regions have been used. In the forgoing analysis, an attempt is taken up to address the variations of velocity, flow rate characteristics etc., due to such parameters.

6.1. Axial Velocity profiles

A comparison of velocity profiles that have been obtained from eqs. (5.7), for slip and no-slip cases, maximum heights of stenosis and different axial locations for z = d+L_0/2, d+L_0/4, d+3L_0/4 for shape parameter n=2, 6, 9 and for other parameter values, is shown in Figs (2-3). As tube radius r/R ranges from 0 (at tube axis) to 1 (at wall) on either side of axis, velocity decreases from a greater value at axis to a smaller one slip velocity at interface and, then to a minimum magnitude zero-slip velocity at boundary. As expected, velocity increases with slip at interface. Its values are higher for flows with slip ($u_s > 0$) than those with no-slip ($u_s = 0$). Also it is observed from Figs. (2-3) that $\overline{u_1}\,|_{n=9} < \overline{u_1}\,|_{n=2} < \overline{u_1}\,|_{n=6}$. Although, it shows a little deviation from parabolic profile, in to the core region, its behaviour is parabolic in the peripheral region. For symmetric and asymmetric stenoses, it behaves differently. As slip velocity increases, velocity increases in all three forms of stenosis formation at an artery wall.

6.2. Variation of flow rate

Figure 4 depicts the flow rate fluctuations as a function of several factors. From the start point to the stenotic throat, it reduces in amplitude before increasing to the termination position. As the shape parameter n decreases, it is shown that $\overline{Q}$ decreases. The greatest magnitude is attained at the stenotic throat for n = 2 (symmetric case) at z = d+L_0/2 and away from the throat for n > 2 (asymmetric case) at z = d+3L_0/4. In all cases of stenosis, flow rate increases as slip velocity increases.

6.3 Variation of wall shear stress:

The variation of wall shear stress in the annular region is shown in Figures 5 and 6. It diminishes as the slip velocity increases. It displays a higher magnitude in the middle of the stenosis and a lower magnitude at the beginning and end of the stenosis. It climbs to a larger value at the other end of stenosis due to the minimal constriction.

7. Conclusion:

This work investigated the steady flow of blood (a Newtonian fluid) via a catheterized stenosed artery under the conditions of slip at the fluid layer interface with minor asymmetric stenosis, velocity slip, and zero-slip at the catheter and tube wall. Three scenarios involving a slide and no-slip at the interface, as well as a catheter border, are explored. The following are some of the study's most important findings: (a) Poiseuille flow of blood (Newtonian Fluid) with slip or zero-slip at vessel wall and non-Newtonian fluid with slip or zero-slip at stenotic wall are included in the current model, as are annular flow models between co-axial cylindrical tubes and a Newtonian fluid with slip and no-slip, and Newtonian fluid models of blood flow in a catheterized stenosed uniform artery with slip or zero-slip conditions.

(b) As expected, velocity increases with axial slip and grows to greater magnitudes with increasing levels of slip, but velocity decreases with increasing stenosis height.

(c) The flow rate increases as the stenosis progresses, reaching its highest magnitudes at each end of the restricted annular region and its lowest at the stenosis's throat.

(d) Wall shear stress falls with velocity slip and increases with the amount of slip velocity, but it is reduced with velocity slip. It decreases in magnitude from the end of stenosis to the place of lowest constriction, and hence increases in value at the other end of stenosis.

It's worth noting that by using a velocity slip at the interface; wall shear stress can be lowered significantly, which helps to prevent damage or rupture of the artery endothelium. Also, by injecting axial velocity at the slip at interface, the diseased arterial system and pressure flow-relationship in a uniform stenosed artery can be improved. As a result, one would hope to find a device (drugs or equipment) that can cause slip and be utilised to treat and cure PPL and arterial illnesses, as well as rupture or damage to the artery endothelium. Consider simply pressure decrease in addition to the previous experimental studies on blood flow through stenosed arteries. Determine the wall shear stress slip at interface in two-layered flow, velocity and flow rate, and other parameters in stenosed and annular flow in a catheterized stenosed artery could be of interest and importance. Such research could aid in determining the growth, development, and progression of an arterial stenosis, as well as investigating the pressure-flow relationships and behaviour of flow variables in the two-layered annular region caused by the invention of a catheter in an arterial stenosis, which could aid in a better understanding of stenotic and arterial diseases such as angina, pectoris, myocardial infarction, stroke, and thrombosis.

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