Comparison of Divide Sum and Harmonic Mean Method for Solving Transportation Problems

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Abstract:- This Paper is an attempt to Give comparison between Divide Sum and Harmonic Mean Method for Solving Transportation Problems. Both Methods are studied for finding the initial basic feasible solution of transportation problems. In this paper I have given two examples to show that the divide sum method is better for finding solutions to transportation problems.
Keywords:- Harmonic mean, Divide Sum method, Initial Solution

Introduction:- Linear programming is a major branch of Operation research under which we optimize linear function of variables subject to some linear restrictions. Further transportation problem is a special case of linear programming which includes the transshipment of a homogenous product, which are originally available at different supply points, to various demand stations in such a manner that the total cost of transportation is least. A certain class of linear programming problem known as transportation problems arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the contacts of determining optimum shipping pattern. The general transportation problem can be stated as, a product may be transported from factories to retail stores. The factories are the sources and the stores are the destinations. The amount of products that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destination have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of m origin and it is required that the given quantities of the product be shipped to each of n destinations. The minimum cost of shipping a unit of the product from any origin to any destination is known. The shipping schedule which minimizes the total cost of shipment is to be determined. The problem can be formulated as:

Min Z=

subject to

= , i = 1, 2, ...m,

= , j = 1, 2, ...n.

≥ 0, for all i, j.

For each supply point i,(i = 1, 2, ..., m) and demand point j,(j = 1, 2, ..., n)

=unit transportation cost from i th source to j th destination

=amount of product transported from i th source to j th destination

=amount of supply at i th source.

=amount of demand at j th destination.

where and are given non-negative numbers and assumed that total supply is equal to total demand, i.e. Xm i=1 ai = Xn j=1 bj , then the transportation problem is called balanced otherwise it is called unbalanced. The aim is to minimize the objective function satisfying the above mentioned constraints.

Note:- In this paper I have used the Harmonic method technique proposed by M Palanivel and M Suganya A New Method to solve the Transportation Problem. And Divide Sum Method A New technique for solving transportation Problems by Deep S.M , Tuteja A, Singh Sandeep. Further 12 examples are studied showing the Divide sum method is stronger than Harmonic Mean Method.

Example 1

Table 1: Input data

Destination → source ↓	$D1$	$D2$	$D3$	supply (a_i)
$S1$	6	4	3	30
$S2$	2	1	6	40
$S3$	4	8	3	50
Demand (bj)	60	25	35	

Table 2: Input data and initial solution for example number 1

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
1	$[c_{ij}]_{3\times3}$=[6 4 3; 2 1 6; 4 8 3]; $[a_i]_{3\times1}$=[30, 40, 50]; $[b_j]_{1\times3}$=[60, 25, 35]	x_{13} = 30, x_{21} = 15, x_{22} = 25, x_{31} = 45, x_{33} =5	340	340

Example 2.

Table 3: Input data

Destination → source ↓	D_1	D_2	D_3	supply (a_i)
S_1	17	20	23	34
S_2	26	20	26	52

S_3	25	17	28	51
Demand (b_j)	51	40	46	

Solution:

Table 4:

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
2	$[c_{ij}]_{3\times3}$=[17 20 23 ;26 20 26; 25 17 28]; $[a_i]_{3\times1}$=[34, 52, 51];	x_{11} = 34, x_{21} = 6, x_{23} = 46, x_{31} = 11, x_{32} = 40	2885	2885

	$[b_j]_{1\times 3}$=[51, 40, 46]			

Example 3:

Table 5:

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
3	$[c_{ij}]_{4\times 6}$=[1 2 1 4 5 2; 3 3 2 1 4 3; 4 2 5 9 6 2; 3 1 7 3 4 6]; $[a_i]_{4\times 1}$=[30, 50, 75, 20]; $[b_j]_{1\times 6}$=[20, 40, 30, 10, 50, 25]	x_{13} = 30, x_{24} = 10, x_{25} = 40, x_{31} = 20, x_{32} = 20, x_{35}	450	430

		=10, $x36$=25, $x42$=20		

Input data and solution obtained in other examples is shown in tables

Table 6: Input data and initial solution for example number 4 to 12

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
4	$[c_{ij}]_{3\times3}$=[10 20 30 ;20 30 40; 30 40 50]; $[a_i]_{3\times1}$=[10, 15, 25]; $[b_j]_{1\times3}$=[20, 15, 15]	x_{11} = 10, x_{21} = 10, x_{23} = 5, x_{32} = 15, x_{33} = 10	1600	1600

Table 7

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
5	$[c_{ij}]_{4\times4}$=[12 4 8 16 ;18 15 12 9; 10 8 20 16; 6 12 10 14]; $[a_i]_{4\times1}$=[30, 20, 40 10]; $[b_j]_{1\times4}$=[20, 20, 20, 40]	x_{12} = 20, x_{13} = 10, x_{24} = 20, x_{31} = 20, x_{34} = 20 x_{43} = 10	960	940

Table 8

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
6	$[c_{ij}]_{4\times6}$=[1721 3442 1721 6884 8605 3442 ;5163 5163 3442 1721 6884 5163; 6884 3442 8605 15489 10326 3442; 5163 1721 12047 5163 6884 10326];	x_{13} = 31, x_{24} = 10, x_{25} = 41, x_{31} = 21, x_{32} = 20, x_{35} = 10, x_{36} =	791660	755519

	$[a_i]_{4\times1}$=[31, 51, 76 21]; $[b_j]_{1\times6}$=[21, 41, 31, 10, 51, 25]	25, x_{42} = 21		

Table 9

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
7	$[c_{ij}]_{4\times6}$=[578 1156 578 2312 2890 1156 ;1734 1734 1156 578 2312 1734; 2312 1156 2890	x_{13} = 6882, x_{24} = 2220, x_{25} =	57742200	55175880

	5202 3468 1156; 1734 578 4046 1734 2312 3468]; $[a_i]_{4\times1}$=[6882, 11322, 16872, 4662]; $[b_j]_{1\times6}$=[4662, 9102, 6882, 2220, 11322, 5550]	9102, x_{31} = 4662, x_{32} = 4440, x_{35} = 2220, x_{36} = 5550, x_{42} = 4662		

Table 10

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
8	$[c_{ij}]_{4\times3}$=[10 9 8 ;10 7 10; 11 9 7; 12 14 10]; $[a_i]_{4\times1}$=[8, 7, 9, 4]; $[b_j]_{1\times3}$=[10, 10, 8]	x_{11} = 6, x_{12} = 2, x_{22} = 7, x_{32} = 1, x_{33} = 8, x_{41} = 4	240	240

Table 11:

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
9	$[c_{ij}]_{4\times3}$=[470 423 3760 ;470 329 470; 517 423 329; 564 658 470]; $[a_i]_{4\times1}$=[184, 161, 207, 92]; $[b_j]_{1\times3}$=[230, 230, 189]	x_{11} = 138, x_{12} = 46, x_{22} = 161, x_{32} = 23, x_{33} = 184, x_{41} = 92	259440	259440

Table 12:

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
10	$[c_{ij}]_{3\times3}$=[828 552 414; 276 138 828; 552 1104 414]; $[a_i]_{3\times1}$=[420, 560, 700]; $[b_j]_{1\times3}$=[840, 350, 490]	x_{13} = 420, x_{21} = 210, x_{22} = 350, x_{31} = 630, x_{33} =70	656880	656880

Table 13

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
11	$[c_{ij}]_{3\times3}$=[2499 2940 3381;3822 2340 3822; 3675 2499 4116]; $[a_i]_{3\times1}$=[3026, 4628, 4539]; $[b_j]_{1\times3}$=[4539, 3560, 4094]	x_{13} = 2670, x_{21} = 1335, x_{22} = 2225, x_{31} = 4005, x_{33} =445	37744455	37744455

Table 14

Ex.	Input data	Obtained allocations by DSM	Obtained cost by DSM	Optimal solution
12	$[c_{ij}]_{3\times3}$=[2484 1656 1242;828 414 2484; 1656 3312 1242]; $[a_i]_{3\times1}$=[4620, 6160, 7700]; $[b_j]_{1\times3}$=[9240, 3850, 5390]	x_{13} = 330, x_{21} = 165, x_{22} = 275, x_{31} = 495, x_{33} =55	21677040	21677040

3.3 Comparative study and result analysis

The chi-square test represents a useful method of comparing experimentally obtained results with those to be expected theoretically on some hypothesis. The equation of the chi-square $(\chi)^2$ is stated as follows: $(\chi)^2 = ((f_o - f_e)^2 \div f_e)$ (Chi-square formula for testing agreement between observed and expected results) In which f_o = frequency of occurrence of observed or experimentally determined facts f_e = expected frequency of occurrence of some hypothesis. The difference between observed and expected frequencies are squared and divided by the expected number in each case, and the sum of quotients is $(\chi)^2$. The more closely the observed results approximate to the expected, the smaller the chi square and the closer the agreement between observed data and the hypothesis being tested. Contrariwise, the larger the chi-square the greater the probability of a real divergence of experimentally observed from expected results. We compare the computed value of chi-square with the standard table against the given degrees of freedom. The number of $d.f = (r - 1)(c - 1)$ in which r is the number of rows and c the number of columns in which the data are tabulated. In the present study the Divide sum method developed by the investigator was checked against the existing method Harmonic Mean method. For this twelve statements were solved with both the methods and results are presented in table 15.

Table 15: Table showing the performance of DSM against HM method

Example Number	IBFS by DS method	IBFS by HM method	Comparison
3.2.1	340	375	DSM is better
3.2.2	2885	3169	DSM is better
3.2.3	450	460	DSM is better
3.2.4	1600	1600	no difference
3.2.5	960	960	no difference
3.2.6	791660	808870	DSM is better
3.2.7	57742200	60308520	DSM is better
3.2.8	240	248	DSM is better
3.2.9	259440	268088	DSM is better

3.2.10	656880	724500	DSM is better
3.2.11	37744455	41460027	DSM is better
3.2.12	21677040	23908500	DSM is better

3.3.1 Interpretation

Σ

From the table 3.23 it is observed that out of the twelve problems, in ten problems the Divide Sum Method appears better as compared to Harmonic Mean method whereas in rest two problems there exist no difference in the solutions by either of the method. To check the significance difference between the performance of two methods in solving the problems, Chi Square (2) method was used. As in the present research the cell frequencies are small so the Yates correction was used. Therefore the formula used to find the Chi-Square, following formula was used: $(\chi)^2 = ((f_o - f_e - 0.5)^2 \div f_e)$ and Chi-square for difference between two methods is given in the table 3.24

Table 3.24: Table showing the Chi-square for difference between DSM and HM method

	DSM superior to HM method	No difference between DSM and HM method	Chi-square
observed frequency	10	2	4.08
observed frequency	6	6	

From the table 3.24 the value of Chi square showing the difference between the performance of two methods, divide sum method and harmonic mean method came out to be 4.08 which is significant at 0.05 level of significance because it is greater than the tabulated value of Chi-square (3.841) with one degree of freedom. Hence it can be concluded that the divide sum method is effective as compared to the harmonic mean method.

References:-

(1) M Palanivel, M Suganya A New method to solve Transportation Method Juniper Publishers (2018).

(2) Deep S.M Tuteja A, Singh Sandeep Divide sum method a new technique for solving Transportation problems. Trends and Challenges in Mathematics(2018).