A NEW ADDITION ASSIGNED METHOD FOR ASSIGNMENT PROBLEM

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ABSTRACT:

In this paper introduced a new approach to make an Addition Assigned Method (AAM) for Assignment Problem. This method is quite simple steps and reducing time for doing sum and also comparing with optimum assignment cost for Hungarian Method.

Index Terms: Addition Assigned Method (AAM), Assignment Problem , Hungarian Method, Optimal Cost.

INTRODUCTION:

The Assignment problem is a optimization problem in the field of Operation Research. It is always a degenerate form of a transportation problem, but the transportation technique or simplex method cannot be used to solve the assignment problem because of degeneracy.

The Assignment problem assigned the total cost in available jobs to different men/machines in an organization/manufacturing units under the condition that is one job is given to one machine and one machine has to only one jobs.

The aim of the Assignment problem is minimize the cost/loss

MATHEMATICAL PRELIMINARIES OF AN ASSIGNMENT PROBLEM

Consider an assignment problem of assigning n jobs to n machines (one job to one machine). Let C_{ij} be the unit cost of assigning i^{th} machine to the j^{th} job

$$X_{ij} = \begin{cases} 1, \text{if } j^{th} \text{ job is assigned to } i^{th} \text{ machine} \\ 0, \text{ otherwise} \end{cases}$$

The technique used for solving assignment problem makes use of the following two theorems:
Theorem 1: The optimum assignment schedule remains unaltered if we add (or) subtract a constant from all the elements of row or column of the assignment cost matrix.
Theorem 2: If for an assignment problem all $c_{ij}>0$, then an assignment schedule (x_{ij}) which satisfies $\Sigma c_{ij}x_{ij}=0$, must be optimal.

ADDITION ASSIGNED METHOD (AAM) ALGORITHM :

(*Balanced problem*)

Let A,B,C,D,…Z denotes the resources/jobs and I,II,III,IV,… denotes the destination/machine.

Step 1: First check it is balanced one (if it is equal no. of rows and equal no. of columns) if not it is unbalanced.

Step 2: Find the smallest value on the each row wise and mark it * symbols

Step 3: Write the suitable smallest value obtained the corresponding jobs to machine in the below mode

That is	jobs	→	machines
	A	→	I
	B	→	II
	C	→	III … so on

Step 4: Check that all resources have unique small value mark it eliminating corresponding small value rows and columns destination , it is obtained resource have unique optimal value. Further if it is not unique go to next step.

Step 5: First select any one resource have unique destination and eliminate the corresponding small value rows and columns and check out any two or more resources have same column destination (or) repeating small column destination go to sub steps follows:

(i) If any two or more resources have same small value destination columns (batch wise unique resources) Suppose that have small value obtained that resources and destination as

JOBS		MACHINES
A	→	I
B	→	I
C	→	II,III
D	→	III

We added the small and next small value in the A,B → resources row wise we get any some additional value . (A,B as batch wise follows of I column destination).

A→I → Greater value (added small and next small value)

B→I→ Small value (added small and next small value)

(ii) First choose addition greater value obtained resources and select the remaining row small value of that A→ resource and eliminate the corresponding small value row and column.

(iii) Next go to B→ resources and select the remaining row small value of that B →resource and follows the process that next batch of C, D→ resources (it is different and same destination)

(iv) The above procedure to select all resources have unique destination.

Further any resources have same small value arrived in row wise (or) same addition greater value obtained go to next step.

(v) Any resources have same small value arrived in row wise or same addition greater value obtained that we skip the resources go to next greater addition value and do the above procedure.

(vi) Finally marking that all the destination and added the assigning optimal cost.

The above iteration all jobs have assigned exactly unique small value destination .

NUMERICAL ILLUSTRATIONS:

In the section, we provide numerical examples to illustrate the ADDITION ASSIGNED METHOD (AAM) ALGORITHM FOR ASSIGNMENT PROBLEM.

EXAMPLE 1:

Four different jobs can be done on four different machines. The set up and take down time costs are assumed to be prohibitively high for change over. The matrix below gives the cost in rupees of processing job “i” on machine “j”.

Jobs / Machines	M1	M2	M3	M4
J1	5	7	11	6
J2	8	5	9	6
J3	4	7	10	7
J4	10	4	8	3

How should the jobs be assigned to the various machines so that the total cost is minimized?[question from the book resource management techniques]

SOLUTION:

Step 1: It is a balanced one (if it is equal no. of rows and equal no. of column)
Rows =4=Columns

Step 2: find the small value and mark it *

Jobs / Machines	M1	M2	M3	M4
J1	5 *	7	11	6
J2	8	5 *	9	6
J3	4 *	7	10	7
J4	10	4	8	3*

Step 3: write the suitable smallest value resources rows and destination columns

J1 → M1
J2 → M2
J3 → M1
J4 → M4

Step 4: follows that choose resources have unique destination

J2 → M2
J4 → M4 (mark it)

Jobs / Machines	M1	M2	M3	M4
J1	5	7	11	6
J2	8	5	9	6
J3	4	7	10	7
J4	10	4	8	3

Eliminating the corresponding rows and columns

Jobs / Machines	M1	M2	M3	M4
J1	5	7	11	6
J2	8	5	9	6
J3	4	7	10	7
J4	10	4	8	3

Step 5: two resources have same small value column destination so that apply sub step of step 5.

(i) J1 → M1
J3 → M1

So that adding the small and next small value in the J1,J2 → resources we get as

J1→ 5+11=16 (greatest added value)
J2→ 4+10=14 (smallest added value)

(ii) First select the greatest added value resources have corresponding remaining small value destination.
J1→M1 and marked the small value

Jobs / Machines	M1	M3
J1	5	11
J3	4	10

Eliminating the corresponding small value rows and columns.

Jobs / Machines	M1	M3
~~J1~~	~~5~~	~~11~~
J3	4	10

(iii) Finally J3→M3 is obtained
J3→M3 → 10

Jobs / machines	M3
J3	**10**

This problem obtained optimal value for all resources to unique destination

Go to step (vi)

J1 → M1 → 5
J2 → M2 → 5
J3 → M3 →10
J4 → M4 → 3
Total = 23 is the optimal cost value.

EXAMPLE 2:

The assignment cost of assigning any one operator to any one machine is given in the following table

machine / operators	I	II	III	IV
A	10	5	13	15
B	3	9	18	3
C	10	7	3	2
D	5	11	9	7

Find the optimal assignment by ADDITION ASSIGNED METHOD ?

SOLUTION:

Step 1: it is balanced
Step 2:

machine / operators	I	II	III	IV
A	10	5*	13	15
B	3 *	9	18	3 *
C	10	7	3	2 *
D	5 *	11	9	7

Step 3:

A→ II (UNIQUE)
B → I (or) IV
C → IV
D → I

Step 4: First select that A→ II

machine / operators	I	II	III	IV
~~A~~	~~10~~	~~5~~	~~13~~	~~15~~
B	3	9	18	3
C	10	7	3	2
D	5	11	9	7

Step 5: Next doing the addition process for remaining resources

B → 3+3=6 (next greatest value)
C → 3+2=5 (smallest value)
D → 5+7=12 (greatest value)
First choose D → I

machine / operators	I	III	IV
B	3	18	3
C	10	3	2
~~D~~	~~5~~	~~9~~	~~7~~

Next to choose B → IV

machine / operators	III	IV
~~B~~	~~18~~	~~3~~
C	3	2

Finally get C → IV

machine / operators	III
C	**3**

The optimal cost values are
A→ II → 5
B → IV → 3
C → III → 3
D → I → 5
Total = 16

COMPARING WITH HUNGARIAN OPTIMAL SOLUTION

EXAMPLE NO.	HUNGARIAN METHOD	ADDITION ASSIGNED METHOD
1	**23**	**23**
2	**16**	**16**

UNBALANCED ASSIGNMENT PROBLEM FOR ADDITION ASSIGNED METHOD (AAM)

First check it is balanced or unbalanced (if it is no. of rows is not equal to no. of columns)
Further , if the assignment problem is unbalanced is converted to balance one by adding dummy columns with zero cost elements in the cost matrix.

Next to follows all the steps in the previous balanced Addition Assigned Method Algorithm but, In the case the dummy rows (or) dummy columns are enclosed that any resources (or) destination is that small value (zero).

In the process , we consider the entire rows small value (zero) is consider the small value destination.

Now we clearly understand the Numerical illustration follows:

EXAMPLE 3:
A batch of 4 jobs can be assigned to 5 different machines. The set up time (in hours) for each job on various machines is given below:

Job / Machine	M1	M2	M3	M4	M5
J1	10	11	4	2	8
J2	7	11	10	14	12
J3	5	6	9	12	14
J4	13	15	11	10	7

First an optimal assignment of jobs to machines which will minimize the total set up time

SOLUTION:

First change to balance so we added dummy cell for job 5 (row) with zero cost elements

Job / Machine	M1	M2	M3	M4	M5
J1	10	11	4	2	8
J2	7	11	10	14	12
J3	5	6	9	12	14
J4	13	15	11	10	7
J5	0	0	0	0	0

So that it is balanced

Further follows previous algorithm

Step 1: mark small value in row wise

Job / Machine	M1	M2	M3	M4	M5
J1	10	11	4	2 *	8
J2	7 *	11	10	14	12
J3	5 *	6	9	12	14
J4	13	15	11	10	7 *
J5	0 *	0 *	0 *	0 *	0*

Step 2:

J1→ M4
J2 →M1
J3 → M1
J4 → M5
J5 → M1,M2,M3,M4,M5

Step 3: select unique destination

J1→ M4
J4 → M5

Step 4: select the small value for unique destination

Job / Machine	M1	M2	M3	M4	M5
J1	10	11	4	**2**	8
J2	7	11	10	14	12
J3	5	6	9	12	14
J4	13	15	11	10	**7**
J5	0	0	0	0	0

Eliminate the corresponding rows and columns

Job / machine	M1	M2	M3	M4	M5
J1	10	11	4	2	8
J2	7	11	10	14	12
J3	5	6	9	12	14
J4	13	15	11	10	7
J5	0	0	0	0	0

Next

Job / machine	M1	M2	M3
J2	7	11	10
J3	5	6	9
J5	0	0	0

Step 5:
Addition process

J2 → 10+7=17 (greater added value)
J3 → 5+6=11 (next greater added value)
J5 → 0+0=0 (small added value)

Sub division step 5 follows

First choose J2 → m1
We have

Job / machine	M1	M2	M3
J2	7	11	10
J3	5	6	9
J5	0	0	0

So we eliminate the corresponding rows and columns

Job / machine	M2	M3
J3	6	9
J5	0	0

Next eliminate J3 → M2

Job / machine	M2	M3
J3	**6**	9
J5	0	0

Automatically J5 assign to M3

The optimal values are

J1→ M4 = 2
J2→ M1 = 7
J3 → M2 = 6
J4 → M5 = 7
J5 → M3 = 0
Total = 22

Comparison of optimal value

Example	Hungarian method for optimal value	Addition Assigned method for optimal value
3	**22**	**22**

CONCLUSION:

Most of time we use Hungarian method to find optimal solution for a assignment problem. In this paper introduced a new Addition Assigned Method (AAM) algorithm is easy way to find optimal solution for a assignment problem . More over the optimal value is same as the Hungarian method optimal value.

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