EFFECT OF TANGENT MODULUS ON RESIDUAL STRESSES OF A SPHERICAL VESSEL SUBJECTED TO INTERNAL PRESSURE

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Abstract : The paper explains the effect of tangent modulus on mechanical residual stresses of a spherical vessel. FE analysis of high pressure spherical vessel considering von Mises yield criterion is carried out to predict behavior of stresses within the plastic zone. Residual stress distribution in autofrettaged spherical vessel considering different tangent modulus are evaluated. The material model is currently bilinear and allows consideration of strain hardening.

Keywords: Autofrettage, Strain hardening, Tangent modulus, FE Analysis, Residual stress.

I. INTRODUCTION

There had been several work related to the distribution of residual stress and strain caused by autofrettage[1]-[5]. For boundary value problem, Jahed et. al. [6] gave the linear elastic solution and studied the elastic -plastic analysis by using axisymmetric method for residual stress distribution[7].They also sketched the stress-strain behaviour of alloy steel for the autofrettage process[8].Investigations of Lazzarin et. al. [9] explained the unloading behavior of the autofrettage process while considering the Bauschinger effect. They proposed that the model of stress-strain behavior during unloading process would be either bilinear isotropic or kinematic hardening. Parker[10]designed the ratio of numerical autofrettage pressure to the ideal autofrettage pressure in view of Tresca and plane stress condition and verified that one with available solution. He also investigated double and triple autofrettage process along with the cause of Bauschinger effect[11].It had been investigated that the fatigue life of cylinder can be improve and it is related to the geometry and the ratio of autofrettage pressure to the yield strength. Livieri et. al. [12] gave the analytical solution for residual stress distribution of cylindrical vessel considering Bauschinger effect. Huang et. al. [13] gave an autofrettage model considering actual tensile-compressive stress-strain curve , plain strain and modified yield criterion. Effect of strain hardening and Bauschinger effect have been discussed and New model were compared with numerical results and experimental data. The residual stresses were in good conformity with numerical solution and experimental data. Gibson et. al. [14] mentioned the necessitate for FE analysis for plain stress and plain strain condition with a range of end condition. Adibi-Asl et. al. [15] proposed few practical analytical expressions for bilinear material model and Ramberg- Osgood model considering Bauschinger effect. Also studied the effect of Bauschinger effect depending on different loading. Hojjati et. al. [16] used Equivalent von -Mises stress as yield criterion for the optimum autofrettage radius and optimum autofrettage pressure of strain hardened cylinder. Plain stress and plain strain condition were used for theoretical and FE simulation. Huang and Moan[17] projected and analytical model for reverse yielding related to the plasticity. Hamid et.al.[18] used different autofrettage pressure for different cycle with similar method executed by Parker[17].Parker and Huang [19] studied the thick walled spherical vessel and proposed numerical solution .

It is a well known fact that during loading process if yielding is initiate in inner surface of spherical vessel, then during removal of internal pressure re-yielding may happen near the inner surface. After removing pressure (unloading) residual stresses are set on the vessel. The purpose of present work is to observe effect of tangent modulus on residual stress of the vessel. Research in this analysis of autofrettaged spherical vessel had not been received appropriate attention by the research community.

II. THEORY

For spherical coordinate system, The primary assumption considering origin at the centre for a spherical coordinate system are:

$$\frac{d\sigma_r}{dr} - \frac{2}{r}\left(\sigma_t - \sigma_r\right) = 0 \text{ .... (1)}$$

$$\frac{d\left(\varepsilon_t^e + \varepsilon_t^p\right)}{dr} + \frac{\left(\varepsilon_t^e + \varepsilon_t^p\right) - \left(\varepsilon_r^e + \varepsilon_r^p\right)}{r} = 0 \text{......................(2)}$$

Von Mises yield criterion

$$\sigma_{eq} = \sigma_t - \sigma_r \text{ .... } (3)$$

This study include residual stress prediction on an uniaxial loading-unloading stresses of strain hardening material model. The stress-strain behavior of engineering material within the elastic region is linear, upto the initial yield stress. But post yielding behavior is state by one of the following model: bi-linear, multi-linear and non-linear. Here a bi-linear model is consider for analysis. Bi-linear model permit the structural feature of the model to developed with less complexities than material with non linearity. Bilinear isotropic and bilinear kinematic hardening models are most preferred models for this purposes. These models are defined in two stages in ANSYS: first one is elastic and then plastic behavior. To model the apparent drop in reverse yield strength of material, a kinetic hardening model was selected. Strain hardening is expressed in terms of tangent modulus (E_t) which is the slope of the stress-strain curve. The effect of changing the tangent modulus on residual stress is investigated.

For the bilinear material model as presented in the relation between equivalent stress and equivalent plastic strain can be stated as below

$$\sigma_{eq} = \sigma_y + \mathrm{E}_p \varepsilon_{eq}^{p} \text{ .... } (4)$$

Where

$$E_p = \frac{E_t E}{E - E_t} \text{ .... } (5)$$

In the case of kinematic hardening, if the tensile stress reaches the value of (σ_{eq}) during plastic deformations, then on unloading, the yielding stress comes to a constant value of $(2\sigma_y)$

A bilinear kinematic hardening quadratic axi-symmetric 8-node elements have been used for inelastic finite element analysis. In ANSYS the vessel was modeled as a hollow sphere and Axi-symmetric element 8 node 183 were used for meshing. The material properties used here, E = 206 GPa, σ_y = 850MPa, and ϑ = 0.3. Dimension chosen for modeling are as inside radius = 0.12m, outside radius = 0.24m and radius ratio, k = 2. An FEA procedure for simulating bilinear kinematic stress-strain behavior during mechanical autofrettage has been implemented and investigated. FE analysis of autofrettage process has been conducted for spherical vessel considering von Mises yield criterion to predict behavior within the plastic zone.

III.COMPARISON AND DISCUSSION

If the internal pressure of a pressure vessel is increased to a critical value yielding begins at the radius where the yield criterion is first satisfied. As $(\sigma_t - \sigma_r)$ has the greatest value at $r = r_i$, hence yielding will begin at the inner radius. It is assumed that in following figures, r = 0 and 0.12 (along x axis) correspond to internal radius (r_i = 0.12m) and external radius (r_o= 0.24m) of spherical vessel respectively.

Figure1 shows the elastic-plastic loading hoop and radial stresses for different tangent modulus with same pressure, p=1100Mpa. With lower tangent modulus yielding is developed more easily. It is seen that with increasing tangent modulus hoop stress near bore is increasing and at the out side radius is decreasing. But for radial stress there is no significant change in stress value at inside and outside radius. Figure 2 shows the completely elastic unloading hoop and radial stress for different tangent modulus with same pressure. For elastic unloading there no effect of tangent modulus because re autofrettage is not happened with the applied pressure. Figure 3 shows the residual stress for autofrettaged spherical vessel with different tangent modulus. It is very clear from the figure with increasing tangent modulus residual compressive hoop stress near bore is decreasing and at the outside radius positive hoop stress is decreasing. With decreasing compressive hoop stress near bore is loosing potential benefit of autofrettage. Figure 4 shows the residual stress distribution if re-autofrettage is happen during unloading.

For re-autofrettage pressure taken as 1300 Mpa, it clear with increasing tangent modulus residual stress near bore is less compressive and at the outside radius positive hoop stress is decreasing with increasing tangent modulus. No considerable change is found on radial residual stress for both autofrettage and re-autofrettage cases at inside and outside radius .

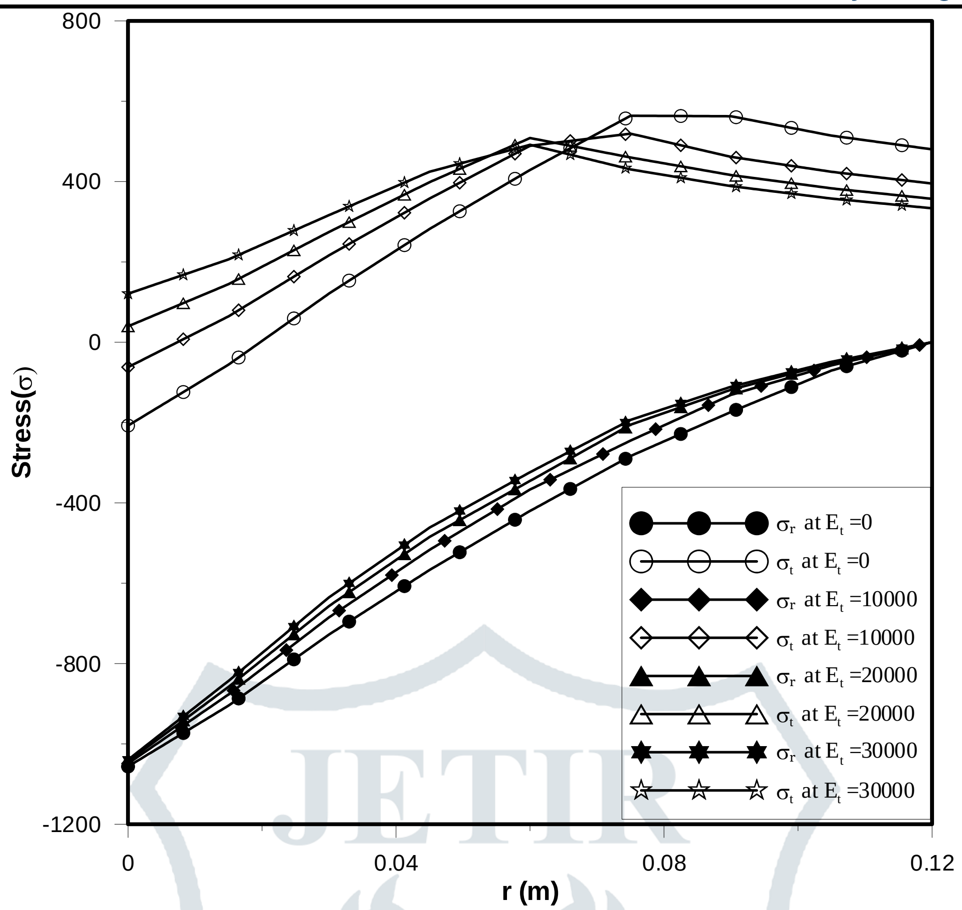

Fig.1: Loading hoop and radial stresses for different tangent modulus

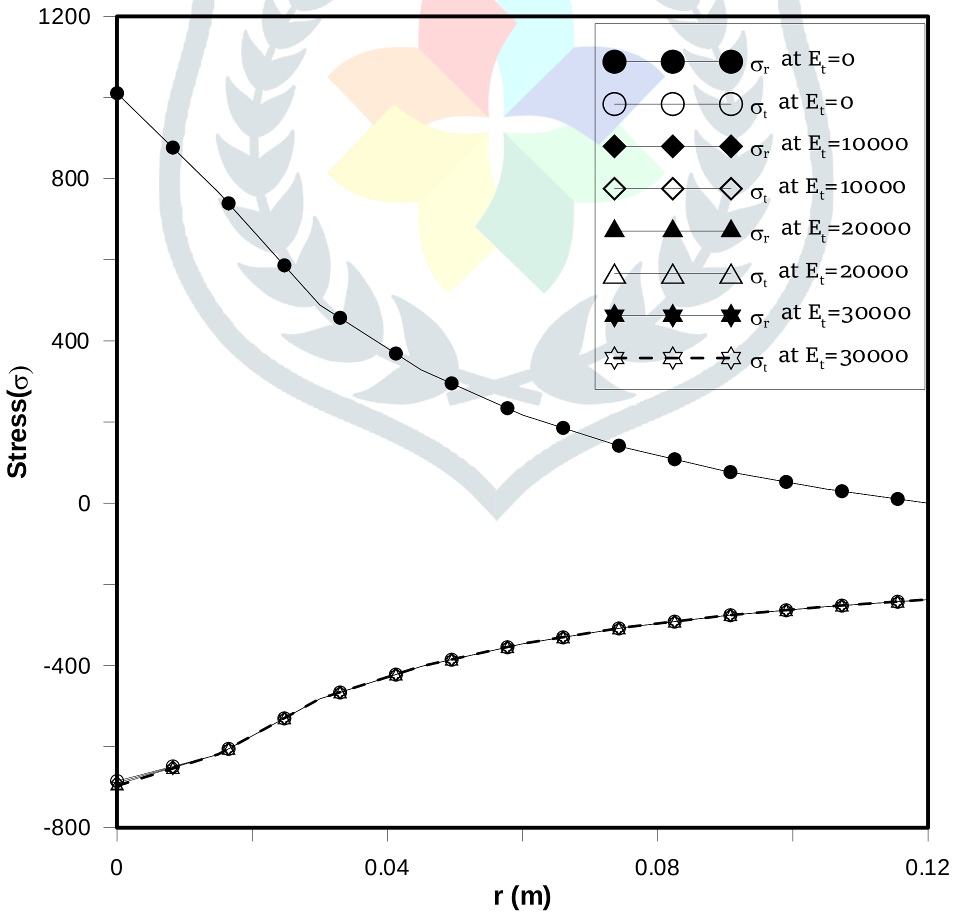

Fig.2 : Elastic unloading hoop and radial stresses with same pressure and different tangent modulus

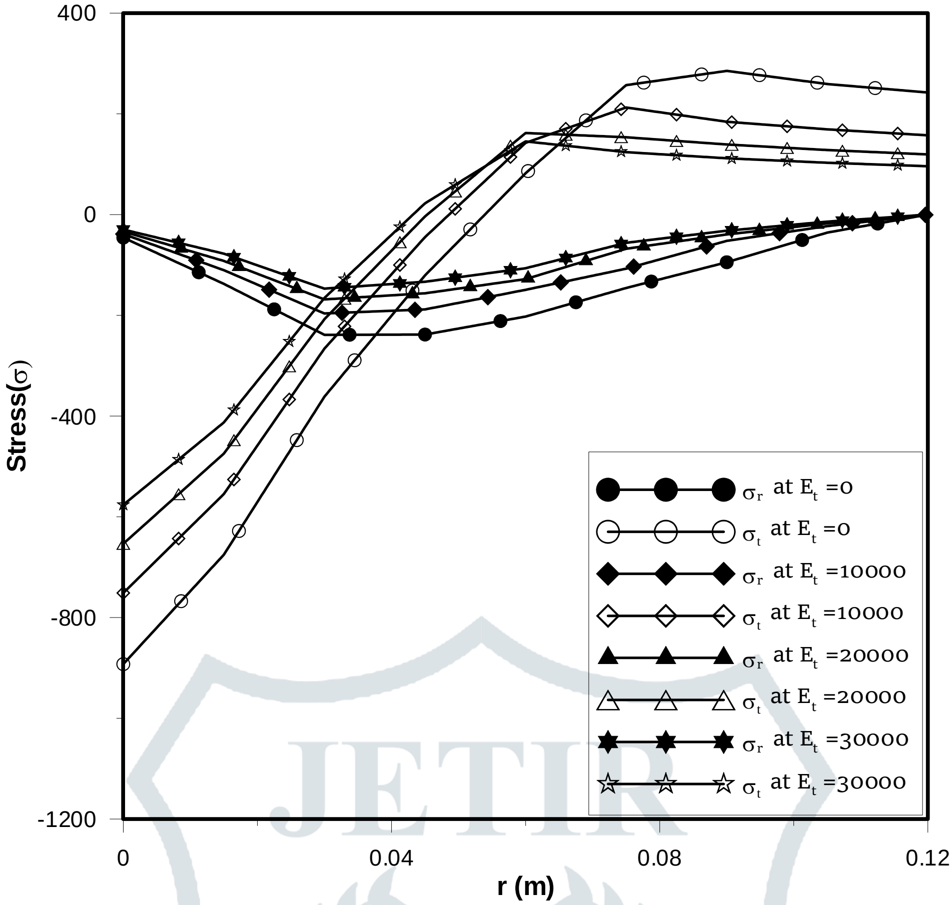

Fig.3: Residual hoop and radial stresses for different tangent modulus

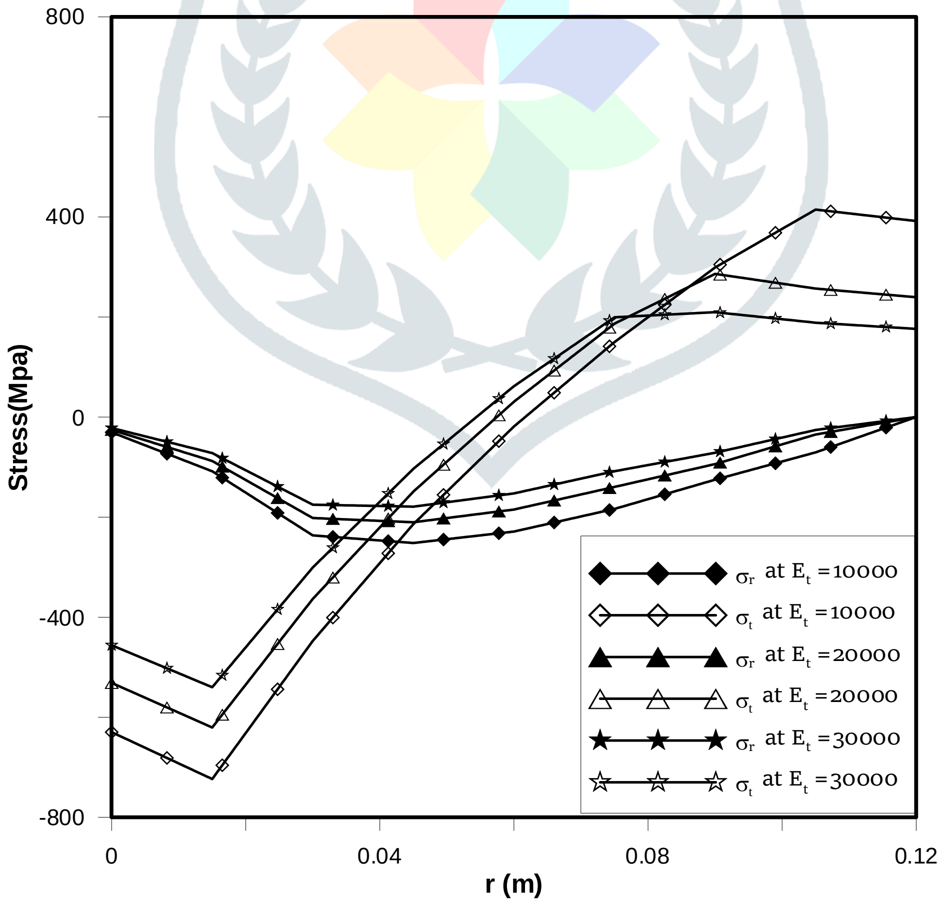

Fig.4: Residual stresses for re-autofrettage with different tangent modulus

IV.CONCLUSION

The residual stresses are highly dependent on the tangent modulus. Kinematic strain hardening material have less compressive residual stress near bore than elastic perfectly plastic material $(E_t = 0)$.With the increasing tangent modulus, residual hoop stress near bore is less compressive and thus loosing the benefit of autofrettage. At the outer radius, results are quite different, here with increasing tangent modulus positive hoop stress is decreased. Effect of tangent modulus is noticed in both elastic and plastic zone. Due to re-autofrettage hoop stress near bore is less compressive and also with increasing tangent modulus compressive hoop stress near bore is decreased.

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LIST OF NOTATIONS

ε	Deformation (strain)
u	Displacement
σ_{eq}	Equivalent stress
$\varepsilon_t, \varepsilon_\phi, \varepsilon_r$	Elastic Principal strain
ε_{eq}	Equivalent strain
σ_t	Hoop stress,
r_i	Inside Radius(m)
r_c	Loading first yield radius
σ_y	Loading Yield stress in tention
r_o	Outside Radius(m)
E_p	Plastic modulus
$\varepsilon_r^p, \varepsilon_t^p, \varepsilon_\phi^p$	Plastic strains
$\sigma_t, \sigma_\phi, \sigma_r$	Principal stresses
p	Pressure (Mpa)
ϑ	Poisson's ratio
r	Radius (Wall Thickness)
σ_r	Radial stress
k	Ratio of r_o to r_i
E_t	Tangent modulus (loading phase) in Mpa
E_{tu}	Tangent modulus (unloading phase) in Mpa
r_d	Unloading yield radius
$\sigma_y^{'}$	Unloading yield stresses, in compression
E	Young's modulus; modulus of elasticity(Gpa)

Superscript *e= Elastic and p=Plastic*