

# FORWARD KINEMATICS OF HUMANOID ROBOT WITH 14 DOF

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**Abstract:** Humanoid robot with the ability to perform tasks as well as a human becomes a plug-in replacement for a human. In this paper, brief elaboration of forward kinematics for 14-DOF humanoid robot legs is presented. DH-parameters and Euler angle rotation method are used for carrying out forward kinematics of legs. The effectiveness of the proposed methods are confirmed by simulation with the help of MATLAB and MATLAB Robot toolbox by using dimension of Waseda Biped humanoid Robot-2 (WABIAN-2) which is developed by Waseda University.

**Keywords - Humanoid Robot, 14-DOF, Wabian-2, DH-Parameter, MATLAB and Forward kinematics.**

## I. INTRODUCTION

The stable walking ability without falling down under any environment is the most essential for a humanoid to accomplish given tasks. Forward kinematics is a most important step for designing a good and stable biped walking.

Humanoid robot has total 14-DOF. It has 2-DOF waist and 6-DOF of two legs. There are 2-DOF of ankle, 1-DOF of knee joint and 3-DOF of hip is used for each leg. Length of upper leg and lower leg are 300 mm and distance between hip and center of waist is 125 mm.

Forward kinematics of robotic manipulator refers to use of the kinematic equation of a robot manipulator to compute the end effector position from the given value of the joint parameter. Jacques Denavit and Richard Hardenberg (1995) introduce a convention for defining the joint parameter. DH parameter helps to obtain joint matrices and link matrices to standardize the coordinate frame for spatial linkages. There are mainly two different spaces are used for kinematics modeling of robotic manipulators namely, Cartesian space and Quaternion space. The transformation of end effector in Cartesian coordinate system can be decomposed in to a translation and a rotation matrix. DH parameter used to obtain this homogeneous transformation matrix.

J. Denavit et al. [1] developed a kinematic notation method for lower pair mechanism based on matrices. This method is stand method for generating DH parameter for making final homogeneous transformation matrix of end effector of any robot. J. Znnatha et al. [2] studied about small sized and low-cost commercial humanoid robot's forward and inverse kinematics which provide close form solution rot both kinematic problems. The robot used for forward and inverse kinematics is ROBONOVA-I developed by Hi-Tech (16-DOF) which has 5-DOF legs. N. Kofinas et al. [3] studied about forward and inverse kinematics analytically for the NAO humanoid robot including software library implementation for real time.

## II. HUMANOID THREE-DIMENSIONAL MOTION

Positioning a humanoid robot in three-dimensional space can be done with base-frame origin and three planes which are perpendicular to each other. Anatomical position is shown in **Figure 1**. This position is standing position with face turned forward and arms are hanging along the side of body with the palm turned forward and legs are stretched with feet closer together. We can also set the position of palm pointing inward, occasionally.

Motion of humanoid robot can be described relative to three perpendicular planes and base-frame origin which is also shown in **Figure 1**. The base-frame origin is located at the floor. The direction of walking is pointing as the X-axis, the Y-axis is pointing to the left-hand side of walking direction and the Z-axis pointing toward upward. The definition of these planes are as follows:

**Frontal Plane:** The plane which is parallel to YZ-plane is known as the Frontal plane. Frontal plane is move along with the motion of robot.

**Sagittal Plane:** The plane which is parallel to XZ-plane is known as the Sagittal plane. Sagittal plane is stationary plane for straight walking. The plane parallel to sagittal plane which containing the Center of Mass (CoM) is known as Median Plane.

**Transverse Plane:** The plane which is parallel to the XY-plane is known as Transverse Plane. Transverse plane is stationary plane for any type of walking.

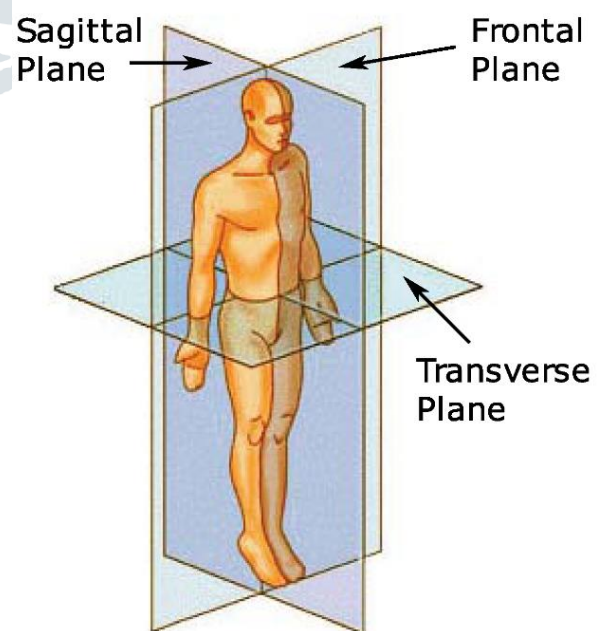


Figure 1 Positioning of humanoid robot

### III. CONFIGURATION OF DOF

Humanoid robot leg for which forward kinematics is carried out having total 12 DOF in leg. All joints are revolute joints. These revolute joints are classified according to the axis of rotation with respect to global frame. There are three types of revolute joints in humanoid robot legs:

1. Pitch Joint: The joint which rotates about Y-axis is called Pitch joint.
2. Roll Joint: The joint which rotates about X-axis is called Roll joint.
3. Yaw Joint: The joint which rotates about Z-axis is called Yaw joint.

Table 1 shows different joints which are in legs of humanoid robots.

*Table 1 Joints of humanoid robots*

Joint No.	Leg	Leg Part	Axis of Rotation	Joint Type	
1	Right Leg	Ankle joint	X	Roll joint	
2			Y	Pitch joint	
3		Knee Joint	Y	Pitch joint	
4		Hip Joint		X	Roll joint
5				Y	Pitch joint
6				Z	Yaw joint
7	Waist			Waist	X
8		Z	Yaw joint		
9	Left Leg	Ankle joint	X	Roll joint	
10			Y	Pitch joint	
11		Knee Joint	Y	Pitch joint	
12		Hip Joint		X	Roll joint
13				Y	Pitch joint
14				Z	Yaw joint

All the joints of a human legs can rotate up to some maximum angle. This rotational limits of each joint of humanoid leg is shown in **Table 2.**

Table 2 Joints limitations

Parts of Leg	Joints	Movable Range for	
		humans (°)	Wabian (°)
Ankle	Roll joint	-20 to +30	-25 to +40
	Pitch Joint	-45 to +25	-33 to +118
Knee	Pitch Joint	0 to 135	-50 to +160
Hip	Roll Joint	-20 to +45	-22 to +22
	Pitch Joint	-15 to +125	-98 to +100
	Yaw Joint	-45 to +45	-25 to +97

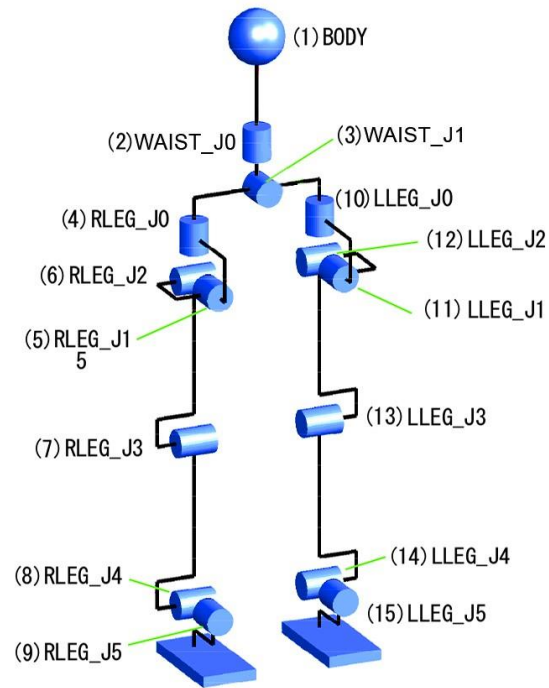


Figure 2 joints of humanoid leg

#### IV. FORWARD KINEMATICS

There are mainly two types of approach available for the solution of the forward kinematics problem.

##### 1. Geometric Approach

This approach gives easiest solution for the simple situation. However, in this approach the angles are measured relative to the direction of the previous link. The first link is an ideal link. The angle is measured relative to its base frame position. For robots with more DOF and whose links extends into 3 dimensions, the geometry gets much more tedious and process get more complicated

##### 2. Algebraic Approach

This approach involves coordinate transformations matrix. Firstly, frame assignment is carried out with respect to base frame. The joint parameter of the robot manipulator is obtained with the help of DH-parameters. For each of the link DH-parameter is obtain. There are four types of the joint parameter which we have to obtain from the frame assignment. Joint parameter required for forward kinematics are listed below:

1. Link length ( $a_i$ ): It is shortest distance between Z-axes of two consequent joint.
2. Link twist ( $\alpha_i$ ): It is angle between Z-axes of two consequent joint.
3. Joint variable:
  - a) Joint angle ( $\theta_i$ ): It is the angle of rotation of joint. Used only for the revolute joint.
  - b) Joint offset ( $d_i$ ): It is the distance of offset of joint. Used only for the prismatic joint.

Here, we are using algebraic approach because we have large number of DOF to handle. Humanoid robot leg has large number of DOF. So, Forward kinematics is carried out in two parts.

1. Forward kinematics when right leg as support leg
2. Forward kinematics when left leg as support leg

Here, in this both cases configuration of all joints remain same but only base frame is moving according to stance leg. Following are the frame assignment of humanoid leg when left leg a stance leg.

1. Forward kinematics when right leg as support leg

Forward kinematics when right leg as support leg, right leg is the leg which is fully supported on the floor while left leg perform step. Here, right leg is considered stance leg. For forward kinematics frame assignment is carried out. Following frame assignment helps to get DH-parameter. DH-parameter used to get configuration of Transformation matrix of stance leg. Frame assignment is done with respect to the base frame. Base frame is nothing but global coordinate with respect to which robot manipulator perform tasks.

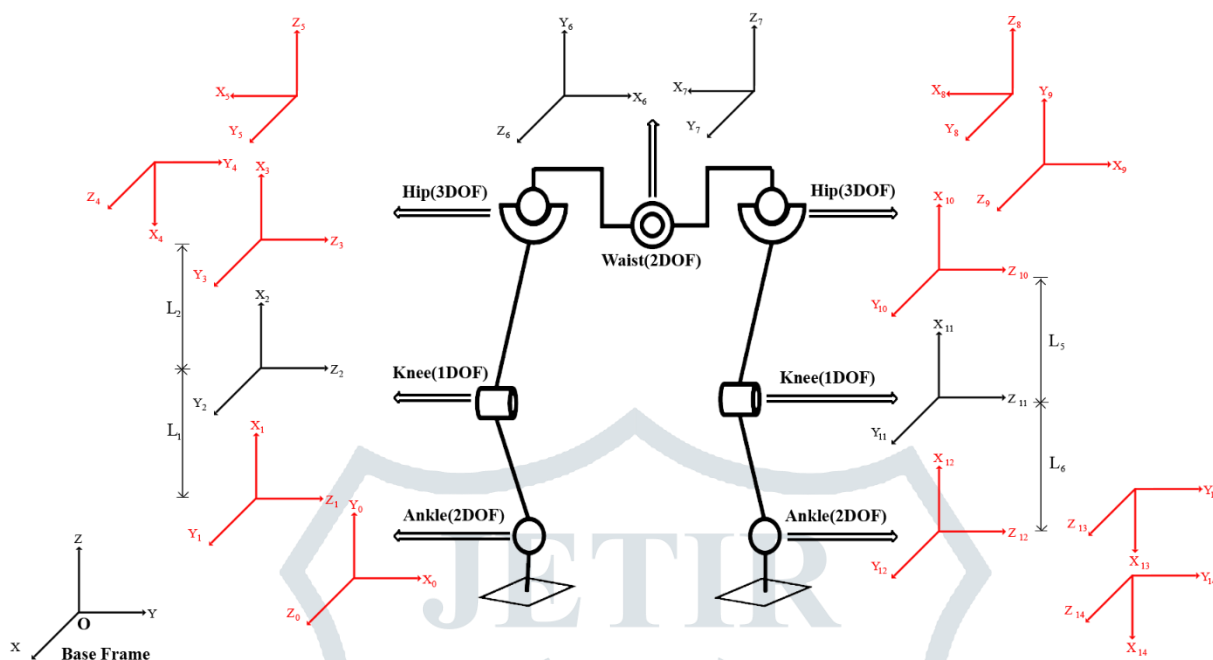


Figure 3 Joints configuration when right leg as support leg

DH-parameters for Right leg as stance leg are as followed.

Table 3 DH-parameter when right leg as support leg

Link (i)	Link length (a <sub>i</sub> )	Link twist (α <sub>i</sub> )	Joint angle (θ <sub>i</sub> )	Joint offset (d <sub>i</sub> )
0	0	π/2	π/2	0
1	0	π/2	θ <sub>1</sub> + π/2	0
2	L1	0	θ <sub>2</sub>	0
3	L2	0	θ <sub>3</sub>	0
4	0	π/2	θ <sub>4</sub> + π	0
5	0	π/2	θ <sub>5</sub> - π/2	0
6	L3	π/2	θ <sub>6</sub> - π	0
7	0	π/2	θ <sub>7</sub> + π	0
8	L4	0	θ <sub>8</sub>	0
9	0	π/2	θ <sub>9</sub> - π	0
10	0	π/2	θ <sub>10</sub> + π/2	0
11	L5	0	θ <sub>11</sub>	0
12	L6	0	θ <sub>12</sub>	0
13	0	π/2	θ <sub>13</sub> + π	0
14	0	π/2	θ <sub>14</sub>	0

Where, L1 = L2 = L5 = L6 = length of upper leg or lower leg = 300 mm and

L3 = L4 = Distance between hip and center of waist = 125 mm

Transformation matrix gives the overall transformation matrix of robotic manipulator. It is obtained by using DH-parameter and some analytical equation. With the help of transformation and current position, we can find the position of new position.

**2. Forward kinematics when left leg as support leg**

Left leg is the leg which is fully supported on the floor while right leg perform step. Here, consider left leg is stance leg and right leg is swing leg. For forward kinematics frame assignment is carried out. This frame assignment help to get DH-parameter of left leg. DH-parameter used to get configuration of Transformation matrix of stance leg. Frame assignment is done with respect to the base frame. Base frame is nothing but Global coordinate with respect to which robot manipulator perform tasks. Frame assignment of Stance leg or Right leg is shown in following figure.

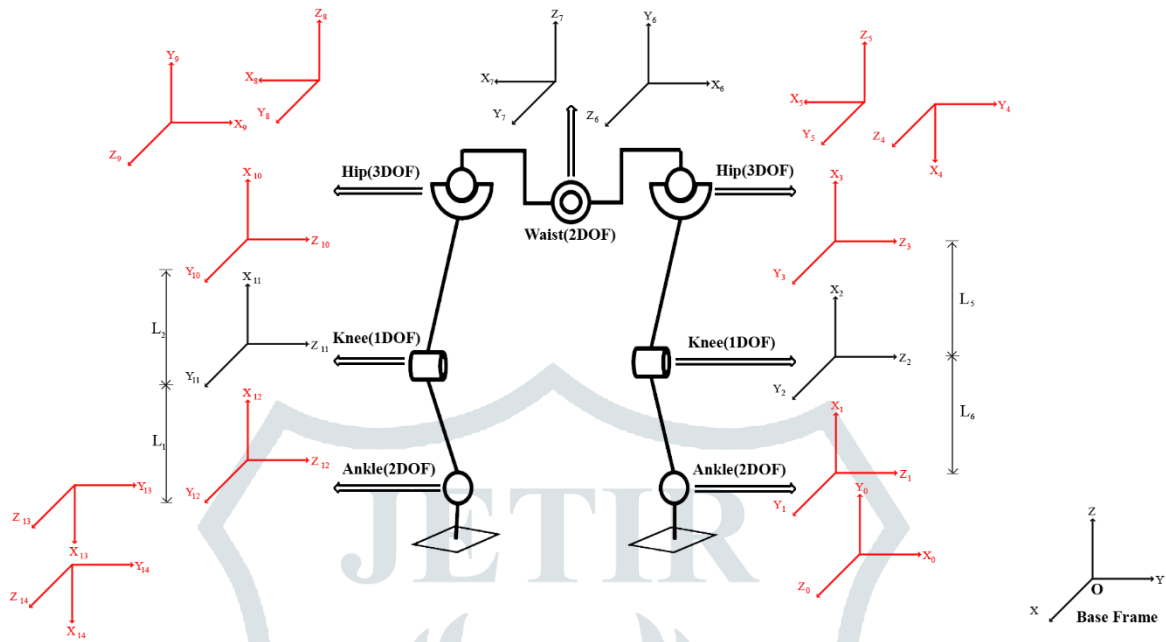


Figure 4 Joints configuration when left leg as support leg

DH-parameters for Left leg as stance leg are as followed.

Table 4 DH-parameter when left leg as support leg

Link (i)	Link length (a <sub>i</sub> )	Link twist (α <sub>i</sub> )	Joint angle (θ <sub>i</sub> )	Joint offset (d <sub>i</sub> )
0	L3*L4	π/2	π/2	0
1	0	π/2	θ <sub>1</sub> + π/2	0
2	L6	0	θ <sub>2</sub>	0
3	L5	0	θ <sub>3</sub>	0
4	0	π/2	θ <sub>4</sub> + π	0
5	0	π/2	θ <sub>5</sub> - π/2	0
6	-L4	π/2	θ <sub>6</sub> + π	0
7	0	π/2	θ <sub>7</sub> + π	0
8	-L3	0	θ <sub>8</sub> + π	0
9	0	π/2	θ <sub>9</sub> + π	0
10	0	π/2	θ <sub>10</sub> + π	0
11	-L2	0	θ <sub>11</sub> + π/2	0
12	-L1	0	θ <sub>12</sub>	0
13	0	π/2	θ <sub>13</sub>	0
14	0	π/2	θ <sub>14</sub> + π	0

Where, L1 = L2 = L5 = L6 = length of upper leg or lower leg = 300 mm and

L3 = L4 = Distance between hip and center of waist = 125 mm

Following equation is used to find transformation matrices of any joint. With the help of above equation and DH-parameter we can calculate transformation matrix of each links. Following are the calculation for the homogeneous transformation matrix of the right leg when right leg have support.

Overall transformation matrix can be calculate as follows:

$$T = {}^0T_0 {}^0T_1 {}^1T_2 {}^2T_3 {}^3T_4 {}^4T_5 {}^5T_6 {}^6T_7 {}^7T_8 {}^8T_9 {}^9T_{10} {}^{10}T_{11} {}^{11}T_{12} {}^{12}T_{13} {}^{13}T_{14} {}^{14}T_{15}$$

So, we can get the position of end effector by following equation:  $P_2 = TP_1$

Where,  $P_2$  = New Position of end effector,

$P_1$  = Current position of end effector.

This matrix multiplication can be solved with the help of MATLAB. Appendix-I shows program of translation of all revolute joints and Appendix-II shows program of forward kinematics with the help of general MATLAB codes. Appendix-III shows program of forward kinematics with the help of MATLAB Robot Toolbox. Here, we get a final homogeneous transformation matrix for both legs when right leg have support. This transformation matrix is compared with the transformation matrix generated by the robot toolbox and they are perfectly matched.

## V. RESULTS

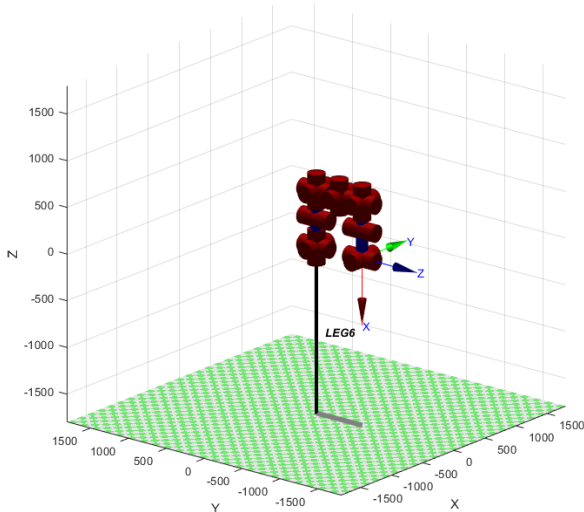


Figure 5 Joint configuration when right leg as support leg by MATLAB Robot toolbox

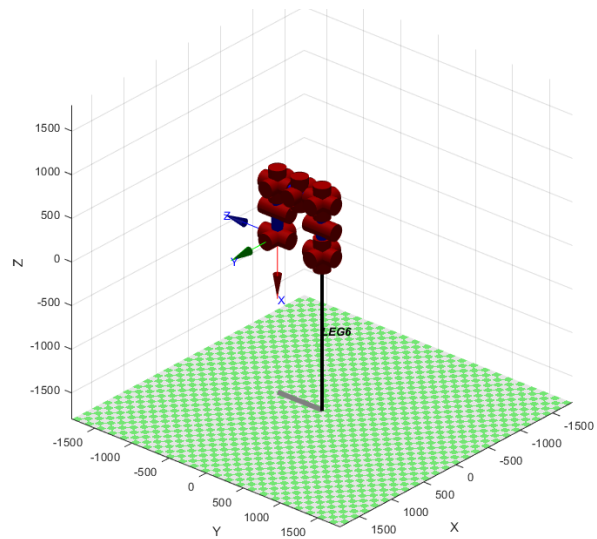


Figure 6 Joint configuration when left leg as support leg by MATLAB Robot toolbox

## VI. CONCLUSION

Forward kinematics of humanoid robot legs required perfect frame assignments. Homogeneous transformation matrices are calculated on basis of frame assignment and DH-parameters. This will help to get final homogeneous transformation matrix. With the help of MATLAB and MATLAB Robot Toolbox we can easily assign the frames to all revolute joints. Also, for such more DOF require complex matrix multiplication which is also can be easily done with help of programming in MATLAB. Programming for forward kinematics of humanoid leg can be done in both MATLAB and MATLAB Robot Toolbox. This will give location of swing foot for the given joint parameters. Final transformation matrix getting by MATLAB is compared with the transformation matrix generated by the robot toolbox and they are perfectly matched.

## REFERENCES

- [1] J. Denavit and R.S. Hartenberg, "A kinematic notation for lower pair mechanism based matrices," *Journal of applied mechanics*, pp. 215-221, 1994.
- [2] J.M. Ibarra Zannatha and R. Cisneros Limón, "Forward and Inverse Kinematics for a Small-Sized Humanoid Robot," in *International Conference on Electrical, Communications, and Computers*, Nacional, 2009.
- [3] Nikolaos Kofinas, Emmanouil Orfanoudakis and Michail G. Lagoudakis, "Complete Analytical Forward and Inverse Kinematics for the NAO Humanoid Robot," in *Journal of Intelligent & Robotic Systems*, Netherlands, 2015.
- [4] Peter I. Cork, *Robotic Toolbox*, Pinjarra Hills, AUSTRALIA: MATLAB, 2001.

APPENDIX-IAPPENDIX-III

Program of transformation of revolute joint frames with the help of MATLAB:

```
function trans_ind = transformCalculate(parameter)
d = parameter(1);
theta = parameter(2);
r = parameter(3);
alpha = parameter(4);
```

$$trans\_ind = \begin{bmatrix} \cos \theta & -\sin \theta \cos \alpha & \sin \theta \sin \alpha & r \cos \theta \\ \sin \theta & \cos \theta \cos \alpha & \cos \theta \sin \alpha & r \sin \theta \\ 0 & \sin \alpha & \cos \alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

end

APPENDIX-II

Program of forward kinematics with the help of MATLAB:

```
function [e,Transform] = Forward_kinematics(parameters)
joints = length(parameters(:,1));
dimension = 3;
Transform = eye(dimension + 1,dimension+1);
for i = 1:joints
    Transform =
    Transform*transformCalculate3(parameters(i,:));
end
Transform = [0 0 1 0.175; 1 0 0 0.250; 0 1 0 0; 0 0 0 1]*
Transform;
e_homogenous = Transform*[0; 0; 0; 1];
e = e_homogenous(1:3,1);
end
```

Program of forward kinematics with the help of MATLAB Robot toolbox:

```
L1 = 300; L2 = 300; L3 = -300; L4 = 300; L5 = -300;
L6 = -300;
```

```
L(1) = Link([0 0 0 pi/2]);
L(2) = Link([0 0 0 pi/2]);
L(3) = Link([0 0 L1 0]);
L(4) = Link([0 0 L2 0]);
L(5) = Link([0 0 0 pi/2]);
L(6) = Link([0 0 0 pi/2]);
L(7) = Link([0 0 L3 pi/2]);
L(8) = Link([0 0 0 pi/2]);
L(9) = Link([0 0 L4 0]);
L(10) = Link([0 0 0 pi/2]);
L(11) = Link([0 0 0 pi/2]);
L(12) = Link([0 0 L5 0]);
L(13) = Link([0 0 L6 0]);
L(14) = Link([0 0 0 pi/2]);
L(15) = Link([0 0 0 pi/2]);
```

```
Rob = SerialLink (L);
```

```
q1=input('q1= ');q2=input('q2= ');q3=input('q3= ');
q4=input('q4= ');q5=input('q5= ');q6=input('q6= ');
q7=input('q7= ');q8=input('q8= ');q9=input('q9= ');
q10=input('q10= ');q11=input('q11= ');
q12=input('q12= ');q13=input('q13= ');q14=input('q14= ');
th1 = 1.5708;th2=q1+pi/2;th3=q2;th4=q3;th5=q4-
pi;th6=q5-
pi/2;th7=q6+pi;th8=q7+pi;th9=q8;th10=q9+pi;th11=q10+
pi/2;th12=q11;th13=q12;
th14=q13+pi;th15=q14;
q = [th1 th2 th3 th4 th5 th6 th7 th8 th9 th10 th11 th12
th13 th14 th15];
T = Rob.fkine(q);
Rob.plot(q)
```