SEMI*δ- LOCALLY CLOSED AND CONTINUOUS FUNCTIONS

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Abstract: *The purpose of this paper to introduce a new class of set namely Semi*δ- locally closed and new class of function namely Semi*δ- locally closed continuous and study their properties.*

Keywords: semi*δ-locally closed set and semi*δ-locally closed continuous function.

1.INTRODUCTION :

Bourbaki [3] defined a subset of a topological space as locally closed if it is the intersection of an open set and a closed set. Stone [10] used the term FG for locally closed subset. Ganster and Reilly [6] used locally closed sets to define LC-continuity and LC-irresoluteness.
The concept of semi*δ-open sets [4], have been initiated by Pious Missier .S and Reena .C. The purpose this paper is introduced to new class of set called semi*δ- locally closed set and new class of function called semi*δ- locally closed continuous functions in topological spaces.

2. PRELIMINARIES

Definition 2.1: A subset A of a topological space (X, τ) is called a **semi*δ-open [7]** if there exists a δ-open set U in X such that U⊆A⊆ Cl*(U).

Definition 2.2: The complement of semi*δ-open set is **semi*δ-closed [6]** It is denoted by S*δC(X,τ).

Definition 2.3: The δ-interior of a subset A of X is union of all regular open set contained in A. The subset A is called **δ-open[4]** if A=δint(A).The complement of δ-open is **δ-closed**.

Definition 2.4: If A is a subset of a topological space X, the **semi*δ-closure[6]** of A is defined as the intersection of all semi*δ-closed sets in X containing A. It is denoted by s*δ Cl(A).

Definition 2.5: A subset A of topological space (X, τ) is called **locally closed[1]** if A=U∩F where U is open and F is closed in (X, τ).

Definition 2.6: A subset of A of topological space (X, τ) is called **δ-locally closed[4]** if A=U∩F where U is δ-open and F is δ-closed in (X, τ).

Definition 2.7: A function f :(X, τ) ⟶(Y, σ) is said to be **semi*δ-irresolute** [5] if f^{-1}(V) is semi*δ-open in (X, τ) for every semi*δ-open set V in (Y, σ).

Definition 2.8: A function f: (X, τ) → (Y, σ) is said to be **semi*δ-continuous** [5] if f^{-1}(V)is semi*δ-open in (X, τ) for every open set V in (Y, σ).

Definition 2.9: A function f: (X, τ) → (Y, σ) is said to be **δ-continuous** [4] if (V)is δ-open in (X, τ) for every open set V in (Y, σ).

Definition 2.10: A subset of A (X, τ) is called **δ-dense[4]** if δ cl(A)=X.

Theorem 2.11: [7] Every δ-open set is semi*δ-open.

Theorem 2.12: [7] Every δ-closed set is semi*δ-closed.

3.SEMI*δ- LOCALLY CLOSED

Definition 3.1 A subset A of a topological space (X, τ) is called a **semi*δ-locally closed** if A= S∩F where S is semi*δ-open and F is semi*δ-closed. The class of all semi*δ-locally closed is denoted by S*δ- LC(X, τ).

Example 3.2: Let X={a, b, c}, τ={φ, {a}, {b}, {a, b},{a, c}, X}, S*δ-(X, τ)={φ,{b}, {a, c}, X} and S*δ C(X, τ)= {φ, {b}, {a, c}, X}. Then S*δ-LC(X, τ)={φ, {b}, {a, c}, X}.

Definition3.3: A subset A of a topological space (X, τ) is called a **semi*δ-locally closed*** if A=S∩F where s is semi*δ-open and F is δ-closed. The class of all semi*δ-locally closed* is denoted by S*δ-LC*(X, τ).

Example 3.4: Let X= {a, b, c}, τ= {φ, {a}, {b}, {a, b}, {b, c}, X}, S*δ-O(X, τ)={φ, {a}, {b, c}, X} and δ C(X, τ)={φ, {a}, {b, c}}. Then S*δ-LC*(X, τ) = {φ, {a}, {b, c}, X}.

Definition 3.5: A subset A of a topological space (X, τ) is called **semi*δ-locally closed****, if A=S∩F where S is δ-open and F is semi* δ-closed. The class of all semi* δ-locally closed** is denoted by S*δ-LC**(X, τ).

Example 3.6: Let X = {a, b, c}, τ ={φ, {a}, {b}, {a, b}, {a, c}, X}, S* δC(X, τ)={φ, {a, c}, {b}, X} and δ O(X, τ)={φ, {b}, {a, c}, X}. Then S*δ-LC**(X, τ) ={φ, {b}, {a, c}, X}

Theorem 3.7: If a subset A of (X, τ) is δ-locally closed then it is semi*δ-locally closed, semi*δ-locally closed* and semi*δ-locally closed**.
Proof: Let A=P∩Q where P is δ-open and Q is δ-closed in (X, τ). Since every δ-open set semi* δ-open and every δ-closed set is semi* δ-closed, A is semi*δ-locally closed, semi*δ-locally closed* and semi*δ-locally closed**.

Remark 3.8: The converse of the above theorem need not be true as shown in the following example.

Example 3.9: Let X={a, b, c}, τ={φ, {a}, {c}, {a, c}, X}, S*δO(X, τ)={φ, {a}, {c}, {a, b}, {b, c}, {c, a}, X}, S*δC(X, τ)={φ, {a}, {b}, {c}, {a, b}, {b, c}, X} and δ O(X, τ)={φ, {a}, {c}, {a, c}, X}, δ C(X, τ)={φ, {b}, {a, b}, {b, c}, X}. Then S*δ-LC(X, τ) = {φ, {a}, {c}, {a, b}, {b, c}, X} and δ LC(X, τ) = {φ, X}. Here {a}, {c}, {a, b}, {b, c} is semi*δ-locally closed set, but not δ-locally closed .

Example 3.10: Let X={a, b, c}, τ={φ, {a}, {b}, {a, b}, X} S*δ-O(X, τ)={φ, {a}, {b}, {a, b}, {b, c}, {c, a}, X}, S*δ-C(X, τ)={φ, {a}, {b}, {c}, {b, c}, {a, c}, X} and δ O(X, τ)={φ,{a}, {b}, {a, b}, X} δ C(X, τ)={φ, {c}, {b, c}, {c, a}, X}. Then S*δ-LC*(X, τ) = {φ, {b, c}, {a, c}, X} and δ-LC(X, τ) = {φ, X}. Here {a, c}, {b, c} is semi*δ-locally closed*, but not δ- locally closed .

Example 3.11: Let X={a, b, c, d}, τ={φ, {a}, {b}, {a, b},{b, c}, {a, b, c}, X}, S*δ O(X, τ)={φ, {a}, {b, c}, {a, d}, {a, b, c}, {b, c, d}, X}, S*δ C(X, τ)={φ,{a}, {d}, {b, c}, {a, d}, {b, c, d}, X) and δ O(X, τ)={φ, {a}, {b, c}, {a, b, c}, X), δ C(X, τ)={φ, {b, c, d}, {a, d}, {d}, X}. Then S*δ LC**(X, τ) = {φ, {b, c}, {a}, X} and δ LC(X, τ) = {φ, X}. Here {a}, {b, c} is semi*δ- locally closed**, but not δ-locally closed.

Theorem 3.12: If a subset A of (X, τ) is semi*δ-locally closed* then it is semi*δ-locally closed.
Proof: Let A be a semi*δ-locally closed*set. Let U be a semi*δ-open set in (X, τ) and V be a δ-closed set in (X, τ). By definition of semi*δ-locally closed*, A=U∩V. Since every δ-closed is semi*δ-closed. Then A is Semi*δ-locally closed set.

Remark 3.13: The converse of the theorem need not be true as shown in the following example.

Example 3.14: Let X={a, b, c}, τ= {φ, {a}, {b}, {a, b}, X}, S*δ LC(X, τ)={φ, {a}, {b}, {b, c}, {a, c}, X} and S*δ LC*(X, τ)={ φ, {b, c}, {a, c}, X}. Here {a} and {b} is semi*δ-locally closed, but not semi*δ- locally closed*.

Theorem 3.15: If a subset A of (X, τ) is semi*δ- locally closed**, then it is semi*δ-locally closed.
Proof: Let A be semi*δ-locally closed** set. Let U be a δ-open set in (X, τ) and V be a semi*δ-closed set in (X, τ). By definition of semi*δ- locally closed**, A=U∩V. Since every δ-open set is semi*δ-open. Then A is semi*δ-locally closed set

Remark 3.16: The converse of the theorem need not be true as shown in the following example.

Example 3.17: Let X={a, b, c, d}, τ={ φ, {a}, {b},{a, b}, {b, c}, {a, b, c}}, S*δ LC(X, τ)={ φ, {a}, {b, c}, {a, d}, {b, c, d}, X}, S*δ LC**(X, τ)={ φ, {a}, {b, c}, X}. Here {b, c}, {a, d} is semi*δ locally closed, but not semi*δ locally closed**.

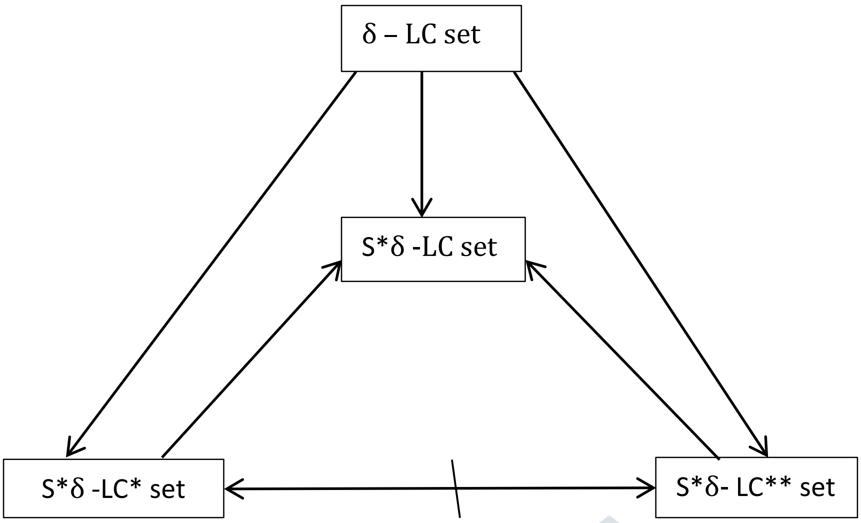
Remark 3.18: Every semi*δ locally closed* and semi* δ locally closed** are independent as shown in the following example.

Example 3.19: Let X={ a, b, c, d}, τ= {φ, {a}, {c}, {a, d}, {a, c}, {a, c, d}, X}, S*δ O(X, τ)={φ, {c}, {b, c}, {a, d}, {a, b, d}, { a, c, d}, X}, δ O(X, τ)={φ, {c}, {a, d},{a, c, d}, X} and S*δ C(X, τ)={φ, {b}, {c}, {b, c}, {a, d}, {a, b, d}, X} , δ C(X, τ)={φ, {b}, {b, c}, {a, b, d}, X}. Then S*δ LC*(X, τ) ={ φ, {b, c}, {a, b, d}, X} and S*δ LC**(X, τ)= {φ, {c}, {a, d}, X}. Therefore both are independent**.**

Remark 3.20: Every locally closed and semi*δ- locally closed are independent**.**

Example 3.21: Let X= {a, b, c, d}, τ= {φ, {a}, {a, b}, X} and τ^c ={φ, {b, c, d}, {c, d}, X}, S*δ O(X, τ)={φ, {d}, {a, c}, {b, d},{a, b, c}, {a, c, d}, X} and S* δ C(X, τ)={ φ, {b}, {d}, {a, c},{b, d},{ a, b, c}, X}. Here S*δ LC(X, τ) = {φ, {d}, {a, c}, {b, d}, {a, b, c}, X} and LC(X, τ) ={φ, X}. Therefore both are independent.

Remark 3.22: We have the following diagram.



Remark 3.23: The Union of two semi*δ- locally closed (respectively semi*δ-locally closed*, semi*δ-locally closed**) sets need not be semi*δ- locally closed(respectively semi* δ-locally closed*, semi*δ- locally closed**) as seen from the following example.

Example 3.24: Let X= {a, b, c, d}, τ={ φ, {a}, {a, b}, X}, S* δ LC(X, τ)= {φ, {d}, {a, c}, {b, d}, {a, b, c}, X}. Here {d} and {a, c} are semi*δ- locally closed, But their union {a, c, d} is not semi*δ- locally closed.

Definition 3.25: A topological space (X, τ) is said to be a **semi* δ-door space** if each subset of (X, τ) is either semi* δ-open or semi*δ-closed in (X, τ).

Example 3.26: Let X= {a, b, c, d}, τ = {φ, {a}, {b, c, d}, X} and S*δO(X, τ)={ φ, {a}, {b, c, d}, X}. Then (X, τ) is a semi* δ-door space.

Proposition 3.27: For a subset A of a space (X, τ) the following statement are equivalent

(i) A ϵ S* δ LC(X, τ)

(ii) A= U∩S*δ Cl(X, τ) for some semi* δ- open set U in (X, τ)

Proof:

$(i) \Rightarrow (ii)$

Let A ϵ S*δ LC(X, τ). Then there exist a semi* δ open set U and semi* δ closed V of (X, τ), such that A= U∩V. Since we have A ⊆ U and A ⊆ S*δ Cl(A) which implies A ⊆ U∩S*δ Cl(A). On other hand definition of semi* δ closure, S*δ Cl(A) ⊆ V. Hence

U∩ S*δ Cl(A) ⊆ U∩V = A. Therefore we have A = U∩S*δ Cl(A).

$(ii) \Rightarrow (i)$

Assume A=U ∩ S*δ Cl(A) for some semi*δ open set in (X, τ). Since semi*δ Cl(A) is semi* δ-closed, A=U ∩ S*δ Cl(A) ∈ S*δ LC(X, τ).

Theorem 3.28: For a subset of (X, τ) the following statement are equivalent.

(i) A ϵ S*δ LC(X, τ)

(ii) A = U ∩ S*δ Cl(A) for some S*δ-open set U in (X, τ).

(iii) S*δ Cl(A) – A is S*δ-closed.

(iv) A ∪ (X – S*δ Cl(A)) is semi*δ-open.

Proof: (i) ⟺ **(ii)**: It follows from the above proposition.

(iii) ⇒ **(ii):** Let U = X – (S*δ Cl(A)−A), By (iii) U is semi*-open set in (X, τ). Then we have, A = U ∩ S*δ Cl(A).

(ii) ⇒ **(iii):** Let A= U∩S*δ Cl(A) for some semi*δ –open set U in (X, τ).

S*δ Cl(A) – A = S*δ Cl(A) ∩ A^c

= S*δ Cl(A) ∩(U∩ S*δ Cl(A))^c

= S*δ Cl(A) ∩ (U^c ∪(S*δ Cl(A)^c)

= (S* δ Cl(A) ∩ U^c) ∪ (S* δ Cl(A) ∩ (S*δ Cl(A))^c)

= S* δ Cl(A) ∩ U^c

Since U is semi*δ-open, Then U^c is Semi* δ-closed and S*δ Cl(A) is semi*δ closed. Therefore, intersection of semi* δ-closed set is semi* δ-closed. Hence S* δ Cl(A)−A is semi* δ-closed.

(iii) ⇒ **(iv):** Let V=S*δCl(A)−A. Then X−V=A∪(X−S*δ Cl(A)) holds. Also X−V is semi* δ-open, since V is semi* δ-closed by (iii). Hence A∪(X−S*δ Cl(A)) is semi* δ-open.

(iv) ⇒ **(iii):** Let U= A∪(X−S*δ Cl(A)). Then X−U= S*δ Cl(A)−A, which is semi* δ-closed, since U is semi* δ-open by (iv). Hence S*δ Cl(A)−A is semi* δ-closed.

Proposition 3.29: For a subset A of (X, τ), A ϵ S* δ-LC**(X, τ), if and only if A=U∩S* δ Cl(A) for some δ-open set U in (X, τ).
Proof:
Necessity:
Let A ϵ S*δ LC**(X, τ). Then by definition, A=U∩V, where U is δ-open set and V is a semi* δ-closed set containing A. Since V is semi* δ-closed set, we have S* δ Cl(A)⊆ V, which implies that U∩S*δ Cl(A) ⊆ U∩V=A. Since A⊆U and A⊆ S* δCl(A). We have A⊆U∩ S* δ Cl(A). Therefore, A= U∩S* δ Cl(A), where U is δ-open.
Sufficiency:
Assume that A= U∩S*δ Cl(A) for some δ-open set U in (X, τ). Since S* δ Cl(A) is semi* δ-closed, we have A ∈ S*δ LC**(X, τ).

Definition 3.30: A subset A of (X, τ) is called **semi*δ-dense** if S*δ Cl(A)=X.

Example 3.31: Let X = {a, b, c} and τ ={φ, {a}, {b}, {a, b}, X}.Here S*δ Cl({a, b})=X Hence {a, b} is the semi* δ-dense set.

Proposition 3.32: In a topological space (X, τ), every semi*δ-dense set is δ-dense but not conversely.
Proof:
Let A be a semi* δ-dense set in (X, τ). Then S*δ Cl(A)=X. Since S*δ Cl(A) ⊆ δ Cl(A), and δ cl(A) ⊆ X. Therefore, we have δ Cl(A)=X. Hence A is δ-dense set.

Example 3.33: Let X={a, b, c} and τ={φ, {a}, {b}, {a, b}, X}. S*δC(X, τ) ={φ, {a}, {b}, {c}, {b, c}, {a, c}, X} and δ C(X, τ)={ φ, {a}, {b}, {a, b}, X}. Here S* δ Cl({a, b})=X. But δ Cl({a, b}) = {a, b}≠X. Therefore, semi* δ-dense is not δ-dense.

Definition 3.34: A topological space (X, τ) is called **semi* δ-submaximal** if every semi* δ-dense subset is semi*δ-open in (X, τ).

Example 3.35: Let X= {a, b, c} and τ={φ, {a}, {c}, {a, c}, X}. S*δ O(X, τ) ={φ, {a}, {c}, {a, b},{b, c}, {c, a}, X}, S*δ C(X, τ) = { φ, {a}, {b}, {c}, {b, c}, {a, b}, X}. Here S*δ Cl({a, c}) =X. Therefore, {a, c} is semi*δ-open. But it is also semi*δ-dense subset. Hence it is semi*δ-submaximal.

SEMI*δ LOCALLY CLOSED CONTINUOUS FUNCTION

Definition 4.1: A function f: (X, τ) → (Y, σ) is said to be **semi*δ-Locally closed continuous** if f^{-1}(V) is semi*δ-locally closed in (X, τ) for every open set V in (Y, σ).

Example 4.2: Let X = {a, b , c}, τ = {φ, {a}, {b}, {a, b}, X} and σ = {φ, {a}, {c}, {a, c}, Y}, S*δ LC (X, τ) = {φ, {a}, {b}, {b, c}, {a, c}, X}, S*δ LC(Y, σ) = {φ, {a}, {c}, {b, c}, {a, b}, Y}. Let f: (X, τ) →(Y, σ) be function defined by f(a)=a, f(b)=f(c)= c. Then the function is semi*δ- locally closed is continuous.

Definition 4.3: A function f: (X, τ) → (Y, σ) is said to be **semi*δ-Locally closed* continuous** if f^{-1}(V) is semi*δ-locally closed* in (X, τ) for every open set V in (Y, σ).

Example 4.4: Let X =Y= {a, b , c, d}, τ = {φ, {b}, {c,}, {b, c}, {a, b, c}, X} and σ= {φ, {b}, {c}, {b, c}, Y}, S*δ LC*(X, τ)= {φ, {b}, {c,}, {b, c}, X}, S*δ LC*(Y, σ) ={φ, {b}, {c}, {b, c}, Y}. Let f:(X, τ)→(Y, σ) be function defined by f(a)=a, f(b)=b, f(c)= c, f(d)= d. Then f is semi*δ-locally closed* continuous.

Definition 4.5: A function f: (X, τ) → (Y, σ) is said to be **semi*δ-Locally closed** continuous** if f^{-1}(V) is semi*δ-locally closed** in (X, τ) for every open set V in (Y, σ).

Example 4.6: Let X =Y= {a, b , c, d}, τ = {φ, {a}, {b}, {c, d},{a, d}, {b, c, d}, {a, c, d}, X} and σ= {φ, {b}, {c}, {b, c}, Y}, S*δ LC**(X, τ)= {φ, {a}, {b}, {c, d}, {a, b},{b, c, d}, {a, c, d}, X}, S*δ LC**(Y, σ) ={φ, {b}, {c}, {b, c}, Y}. Let f:(X, τ)→(Y, σ) be function defined by f(a)=c, f(b)=b, f(c)= a, f(d)= d. Then f is semi*δ-locally closed** continuous.

Theorem 4.7:
Let f: (X, τ)→ (Y, σ) be function. Then
i) If f is δ- locally closed continuous, then f is semi*δ- locally closed continuous,semi* δ - locally closed* continuous, semi* δ- locally closed** continuous.
ii) If f is semi*δ-locally closed* continuous (or) semi*δ-locally closed** continuous, then f is semi* δ- locally closed continuous.
Proof:
i) Suppose f is δ-locally closed continuous. Let V be an δ-open set of (Y, σ). Then f^{-1}(V) is δ- locally closed in (X, τ). Since by theorem 3.7, every δ- locally closed set is semi*δ- locally closed set, semi* δ-locally closed* set, semi*δ-locally closed** set, it follows that f is semi*δ- locally closed continuous, semi*δ- locally closed*continuous, semi*δ -locally closed** continuous.
ii) Let f be semi* δ- locally closed*continuous (or) semi*δ- locally closed** continuous function. Since every semi*δ-locally closed*and semi*δ-locally closed** set is semi*δ- locally closed set the proof follows.

Remark 4.8: Every Semi*δ- locally closed continuous is not δ- locally closed continuous as seen in the example.

Example 4.9: Let X=Y= {a, b, c, d}, τ = {φ, {a}, {b} ,{c}, {a, b}, {b, c}, {a, c}, {a, b, c}, X} and σ={φ, {a}, {b}, {a, b}, {a, b, c}, Y}, S*δ LC(X, τ)={φ, {a}, {b}, {c }, {a, b}, {b, c}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X}, , δ LC(X, τ) = {X, φ}. Let f: (X, τ) →(Y, σ) be the function defined by f(a)= a, f(b)= b, f(c)=d, f(d)=c. Therefore it is clearly semi*δ- locally closed continuous. But Here {a} in (Y, σ) does not have pre image in δ- locally closed (X, τ). Hence f is not δ-locally closed continuous.

Remark 4.10: Every Semi*δ-locally closed* continuous is not δ-locally closed continuous as seen in the example.

Example 4.11: Let X = Y= {a, b, c, d}, τ = {φ, {b} ,{c}, {b, c}, {a, b, c}, X} and σ = {φ, {a}, {b}, {a, b}, Y}, S*δ LC(X, τ) = {φ, {b}, {c }, {b, c}, X}, δ LC(X, τ) = {X, φ}. Let f: (X, τ) → (Y, σ) be the function defined by f(a)= c, f(b)= b, f(c)=a, f(d)=d. Therefore it is clearly semi*δ-locally closed*continuous. But Here {a} in (Y, σ) does not have pre image in δ- locally closed(X, τ). Hence f is not δ-locally closed continuous.

Remark 4.12: Every Semi*δ-locally closed** continuous is not δ-locally closed continuous as seen in the example.

Example 4.13: Let X=Y= {a, b, c, d}, τ = {φ, {a}, {b} ,{c}, {a, b}, {b, c}, {a, c}, {a, b, c}, X} and σ={φ, {b}, {c}, {b, c}, Y}, S*δ LC**(X, τ)={φ, {a}, {b}, {c }, {a, b}, {b, c}, {a, c}, X}, δ LC(X, τ) = {X, φ}. Let f: (X, τ) →(Y, σ) be the function defined by f(a)= a, f(b)= b, f(c)=d, f(d)=c. Therefore it is clearly semi*δ-locally closed** continuous. But Here {b} in (Y, σ) does not have pre image in δ-locally closed (X, τ). Hence f is not δ-locally closed continuous.

Remark 4.14: Every Semi*δ-locally closed continuous is not semi*δ-locally closed* continuous as seen in the example.

Example 4.15: Let X=Y= {a, b, c, d}, τ = {φ, {a}, {b} ,{c}, {a, b}, {b, c}, {a, c}, {a, b, c}, X} and σ={φ, {a}, {b}, {a, b}, {a, b, c}, Y}, S*δ LC(X, τ)={φ, {a}, {b}, {c }, {a, b}, {b, c}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X}, S*δ LC*(X, τ) = {φ, {b, d}, {a, d}, {c, d}, {b, c, d}, {a, c, d}, {a, b, d}, X}. Let f: (X, τ) →(Y, σ) be the function defined by f(a)= a, f(b)= b, f(c)=d, f(d)=c. Therefore it is clearly semi*δ-locally closed continuous. But Here {a} in (Y, σ) does not have pre image in semi*δ-locally closed* (X, τ). Hence f is not semi*δ- locally closed* continuous.

Remark 4.16: Every Semi*δ-locally closed continuous is not semi*δ-locally closed** continuous as seen in the example.

Example 4.17: Let X=Y= {a, b, c, d}, τ = {φ, {a}, {b} ,{c}, {a, b}, {b, c}, {a, c}, {a, b, c}, X} and σ={φ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, Y}, S*δ LC(X, τ)={φ, {a}, {b}, {c }, {a, b}, {b, c}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X}, S*δ LC**(X, τ) = {φ, {a}, {b}, {c},{a, b}, {b, c}, {a, c}, X}. Let f: (X, τ) →(Y, σ) be the function defined by f(a)= a, f(b)= b, f(c)=d, f(d)=c. Therefore it is clearly semi*δ- locally closed continuous. But Here {b, c} in (Y, σ) does not have pre image in semi*δ-locally closed**(X, τ). Hence f is not semi*δ- locally closed**continuous.

Definition 4.18: A function f : (X, τ) ⟶(Y, σ) is said to be **semi*δ-locally closed irresolute** if f^{-1}(V) is semi*δ-locally closed in (X, τ) for every semi*δ-locally closed set V in (Y, σ).

Example 4.19: Let X = Y = {a, b, c}, τ={ φ, {a}, {c}, {a, c}, X} and σ ={φ, {b}, {b, c}, Y}, S*δ LC(X, τ) = {φ, {a}, {c}, {b, c}, {a, b}, X} and S*δ LC(Y, σ) = {φ, {b}, {c}, {a, c}, {a, b}, Y}. Let f: (X, τ) → (Y, σ) be a map defined by f(a) = b, f(b) = a, f(c)=c. Therefore it is semi*δ-locally closed irresolute.

Definition 4.20: A function f: (X, τ) ⟶(Y, σ) is said to be **semi*δ-locally closed* irresolute** if f^{-1}(V) is semi*δ-locally closed*in (X, τ) for every semi*δ-locally closed* set V in (Y, σ).

Example 4.21: Let X = Y = {a, b, c, d}, τ={ φ, {a}, {c}, {a, d}, {a, c}, {a, c, d}, X} and σ ={φ, {b}, {c}, {b, c}, {b, d}, {b, c, d}, Y}, S*δ LC*(X, τ) = {φ, {b, c}, {a, b, d}, X} and S*δ LC*(Y, σ) = {φ, {a, c}, {a, b, d}, Y}. Let f: (X, τ) → (Y, σ) be a map defined by f(a) = b, f(b) = a, f(c)=c, f(d)=d. Therefore it is semi*δ-locally closed* irresolute.

Definition 4.22: A function f :(X, τ) ⟶(Y, σ) is said to be **semi*δ-locally closed** irresolute** if f^{-1}(V) is semi*δ-locally closed** in (X, τ) for every semi*δ-locally closed** set V in (Y, σ).

Example 4.23: Let X = Y = {a, b, c, d}, τ={ φ, {b}, {c}, {b, d}, {b, c}, {b, c, d}, X} and σ ={φ, {a}, {b}, {a, b, }, {a, d}, {a, b, d}, Y}, S*δ LC**(X, τ) = {φ, {a, c}, {a, b, d}, X} and S*δ LC**(Y, σ) = {φ, {b, c}, {a, c, d}, Y}. Let f: (X, τ) → (Y, σ) be a map defined by f(a) =c, f(b) = a, f(c)=b, f(d)=d. Therefore it is semi*δ-locally closed**irresolute.

Proposition 4.24: Let f: (X, τ) → (Y, σ) be a semi*δ –irresolute if and only if it is semi*δ –locally closed irresolute.
Proof:

Let f: (X, τ) →(Y, σ) be a semi*δ- irresolute map. Let B∈S*δ LC(Y, σ). Then there exists a semi*δ- open set U and a semi*δ- closed set V such that B= U ∩ V which implies that f^{-1}(B) = f^{-1}(U) ∩ f^{-1}(V). Since f is irresolute map. Then f^{-1}(U) and f^{-1}(V) is semi*δ-open and semi*δ-closed. Hence f^{-1}(B)∈ S*δ LC(X, τ). Therefore, f is semi*δ- locally closed irresolute. Conversely, Let f:(X, τ) → (Y, σ) be a semi*δ- locally closed irresolute. Let B ∈ S*δO(Y, σ). Let O∈ S*δLC(Y, σ). By definition of semi*δ-locally closed, O=B∩C. Since B is semi*δ-open in (Y, σ) and C is semi*δ- closed in (Y, σ) and also f is semi*δ-locally closed irresolute. Therefore f^{-1}(O) =f^{-1}(B)∩ f^{-1}(C)∈ S*δLC (X, τ). Clearly f^{-1}(B) is semi*δ-open in (X, τ) and f^{-1}(C) is semi*δ-closed in (X, τ). Hence f^{-1}(B)∈S*δ O(X, τ). Therefore, f is semi*δ- irresolute.

Proposition 4.25: Let f: (X, τ) → (Y, σ) be a semi*δ –irresolute if and only if it is semi*δ –locally closed* irresolute.

Proof:

Let f: (X, τ) → (Y, σ) be a semi*δ-irresolute map. Let B∈ S*δ LC*(Y, σ). Then there exist a semi*δ-open set U and δ-closed set W such that B=U∩W which implies that f^{-1}(B) =$f^{-1}(U) \cap f^{-1}(W)$. Since f is irresolute map. Then f^{-1}(U) and f^{-1}(W) is semi*δ- open and δ-closed. Hence f^{-1}(B) ∈ S*δ LC*(X, τ). Therefore f is semi*δ –locally closed* irresolute. Conversely, Let f:(X, τ) → (Y, σ) be a semi*δ- locally closed* irresolute. Let W ∈ S*δO(Y, σ). Let O∈ S*δLC*(Y, σ). By definition of semi*δ-locally closed*, O=W∩U. Since W is semi*δ-open in (Y, σ) and U is δ closed in (Y, σ) and also f is semi*δ-locally closed* irresolute. Therefore f^{-1}(O) =f^{-1}(W) ∩ f^{-1}(U)∈ S*δ LC*(X, τ). Clearly f^{-1}(W) is semi*δ-open in (X, τ) and f^{-1}(U) is δ-closed in (X, τ). Hence f^{-1}(W) ∈ S*δO(X, τ) . Therefore f is semi*δ-irresolute.

Proposition 4.26: Let f: (X, τ) → (Y, σ) be a semi*δ –irresolute if and only if it is semi*δ –locally closed** irresolute.

Proof:

Let f: (X, τ) →(Y, σ) be a semi*δ- irresolute map. Let B ∈ S*δ LC**(Y, σ). Then there exist a δ-open set O and semi*δ closed set V such that B=O∩V, Which implies that f^{-1}(B)= f^{-1}(O)∩f^{-1}(V). Since f is irresolute map. Then f^{-1}(O) and f^{-1}(V) is δ-open and semi*δ- closed. Hence f^{-1}(B) ∈ S* δ LC**(X, τ). Therefore f is semi*δ- locally closed** irresolute. Conversely, Let f:(X, τ) → (Y, σ) be a semi*δ-locally closed** irresolute. Let V∈ S*δO(Y, σ). Let O∈ S*δLC**(Y, σ). By definition of semi*δ-locally closed**, O=V∩U. Since V is δ-open in (Y, σ) and U is semi*δ- closed in (Y, σ). Since every δ-open is semi*δ- open. Therefore V is semi*δ-open in (Y, σ) and also f is semi*δ-locally closed** irresolute. Therefore f^{-1}(O) = f^{-1}(V) ∩ f^{-1}(U) ∈ S*δLC**(X, τ). Clearly f^{-1}(V) is δ-open in (X, τ) and f^{-1}(U) is semi*δ-closed in (X, τ). Since every δ-open is semi*δ-open, Hence f^{-1}(V)∈S*δO(X, τ). Therefore f is semi*δ- irresolute.

Remark 4.27: Semi*δ-locally closed irresolute and semi*δ-locally closed continuous are independent as following example.

Example 4.28: Let X = Y = {a, b, c, d}. Let τ={φ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}, X} and σ={φ, {a}, {b}, {a, b}, {b, c}, {a, b, c}, Y}, S*δLC(X, τ)= {φ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {c, d}, {a, d}, {b, d}, {a, c, d}, {b, c, d}, {a, b, d}, X} and S*δ LC(Y, σ) ={φ, {a}, {a, d}, {b, c}, {b, c, d}, Y}. Let f : (X, τ)→(Y, σ)be a map defined by f(a)=a, f(b) =b, f(c) = c, f(d) = d. Clearly it is semi*δ- locally closed irresolute. Here {a, b, c} in (Y, σ), f^{-1}({a, b, c}) = {a, b, c} does not semi*δ- locally closed in (X, τ). Therefore f is not semi*δ -locally closed continuous.

Example 4.29: Let X = Y = {a, b, c}. Let τ = {φ, {a}, {b}, {a, b}, X} and σ = {φ, {a}, {c}, {a, c}, Y}, S*δLC(X, τ)= {φ, {a}, {b}, {b, c}, {a, c}, X} and S*δ LC(Y, σ) ={φ, {a}, {c}, {b, c}, {a, b}, Y}. Let f : (X, τ)→(Y, σ)be a map defined by f(a)=a, f(b) = f(c) = c. Clearly it is semi*δ- locally closed continuous. Here {a, b} in S*δ LC (Y, σ) does not pre image in semi*δ-locally closed in (X, τ). Therefore f is not semi*δ- locally closed irresolute.

Proposition 4.30: For any two maps f : (X, τ)→(Y, σ) and g: (Y, σ)→ (Z, ω) then $g \circ f$: (X, τ)→(Z, ω) is semi*δ-locally closed continuous if f is semi*δ-locally closed irresolute and g is δ-locally closed continuous(resp. semi*δ- locally closed* continuous, semi*δ- locally closed** continuous)

Proof:

Let V be any open set in (Z, ω). Since g is δ- locally closed continuous (resp. semi*δ- locally closed * continuous, semi*δ- locally closed** continuous) g^{-1}(V)∈ δLC(Y, σ). Since by theorem 3.7, g^{-1}(V)∈ S*δ LC(Y, σ) . Since f is semi*δ-locally closed irresolute. $f^{-1}\left(g^{-1}(V)\right) = (g \circ f)^{-1}(V)$ ∈ S*δ LC(X, τ). Hence $g \circ f$ is semi*δ-locally closed continuous. (resp. semi*δ -locally closed* continuous, semi*δ- locally closed** continuous)

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