ON SEMISEPARATED SETS VIA (a)-TOPOLOGIES

R. NIRMALA* AND N. RAJESH**

*4/10, North street, Oviyampalayam, Namakkal, Tamilnadu, India.
**Department of Mathematics, Rajah Serfoji Government College, Thanjavur, Tamilnadu, India.

Abstract: In this paper, we introduce and study the notion of operation-semiconnected sets in (a)topological spaces which is a set equipped with countable number of topologies. Several properties of these notions are discussed.

1. INTRODUCTION

The notion of bitopological space (X, τ_1, τ_2) (a non empty set X endowed with two topologies τ_1 and τ_2) is introduced by Kelly [3]. The authours in [1, 2, 4] studied the properties of a nonempty set equipped with more topologies. Ogata [5] defined an operation on a topological space (X, τ) as a mapping from τ into the power set of X such that such that $V \subset V^\gamma$ for each $V \in \tau$, where V^γ denotes the value of γ at V. A subset A of X is said to be γ-open if for each $x \in A$ there exists an open neighbourhood U of x such that $U^\gamma \subset A$. The concept of (a)-γ-semiopen sets in (a)topological spaces are introduced and studied in [6]. An operation γ on (a)topological space $(X, \{\tau_n\})$ is a mapping $\gamma: \cup\tau_n \to P(X)$ such that $U \subset \gamma(U)$ for each $U \in \cup\tau_n$. Throughout the paper, N denotes the set of natural numbers. The elements of N are denoted by i, m, n etc. The (τ_n)-closure (resp. (τ_n)-interior) of a set A is denoted by τ_n-cl(A) (resp. τ_n-Int(A)). By τ_m-Int(A) and τ_m-cl(A), we denote the τ_m-interior of A and τ_m -closure of A in $(X, \{\tau_n\})$, respectively. If there is no scope of confusion, we denote the (a)topological space $(X, \{\tau_n\})$ by X.

2. PRELIMINARIES

Definition 2.1. [6] If $\{\tau_n\}$ is a sequence of topologies on a set X, then the pair $(X, \{\tau_n\})$ is Called an (a) topological space.

Definition 2.2. [7] Let X be an (a)topological space. A subset S of X is said to be (m, n)-γ-semiopen if $S \subset \tau_{m\gamma}$-cl($\tau_{m\gamma}$-Int(S)).

The complements of (m, n)-γ-semiopen set isCalled a (m, n)-γ-semiclosed set.

Definition 2.3. [7] Let X be an (a)topological space. A subset S of X is said to be (a)-γ-semiopen if S is (m, n)-γ-semiopen for all $m \neq n$.

Definition 2.4. [7] For a subset A of (X, τ_n, γ), (a)-scl_γ(A) denotes the intersection of all (a)-γ-semiclosed sets containing A.

3. 0N (A)-Γ–SEMISEPARATED SETS

In this section, we introduce the notion (a)-γ-semiseparated sets and study some of their basic properties.

Definition 3.1. Two nonempty subsets A and B of an (a) topological space (X, $\tau_{n,}\gamma$) are said to be (a)-γ-semiseparated if A∩(a)sCl_γ(B)=(a) scl_γ(A) ∩B=∅. If X=A∪B such that A and B are (a)-γ-semiseparated sets, then we say that A and B form (a)-γ-semiseparation of X.

Remark 3.2. Each two (a)γ-semiseparated sets are always disjoint, since A∩B⊂A∩(a)-sCl_γ(B)= ∅.

Remark 3.3. (1) If A and B are (a)γ-semiseparated, then A and B are disjoint.
(2) If A ≠ ∅ is a subset of B and B is (a)-γsemiseparated from C, then A and C are (a)-γ- semiseparated.
(3) If A and B are (a)-γ-semiseparated and A and C are (a)-γ-semiseparated then A and B∪C (a)-γ-semiseparated.

Theorem 3.4. For the subsets A and B of a (a)-topological space (X, τ_n, γ), the following statements are equivalent:
(1). A and B are (a)γ-semiseparated.
(2). There exists (a)-γ-semiclosed sets F_1 and F_2 satisfying A⊂F_1⊂ (X\B) and B⊂F_2⊂ (X\A).
(3). Thee exist (a)-γ-semiopen sets G_1 and G_2 satisfying A⊂G_1⊂ (X\B) and B⊂G_2⊂ (X\A).

Proof: The proof is clear.

Proposition 3.5. Let A and B be the subsets of a (a)-γ-topological space (X, τ_n, γ). If A and B are (a)-γ-semiseparated, Φ≠D⊂B, then C and D are (a)-γ-semiseparated.

Proof. Since A and B are (a)-γ-semiseparated sets, A∩(a)-sCl_γ(B)=Φ and (a)-sCl_γ(A)∩B=Φ. By hypothesis C⊂A, we have (a)-sCl_γ(C)∩D=Φ. Similarly, we have C∩(a)-sCl_γ(D) = Φ. Therefore, C and D are (a)-γ-semiseparated sets.

Theorem 3.6. If for an (a)-γ-semiclosed subset S of a (a)topological space(X, τ_n, γ), A and B are (a)-γ-semiseparated sets such that S = A∪B, then A and B are (a)-γ-semiclosed sets.

Proof. S = A∪B where, A∩(a)-sCl_γ(B)=Φ and (a)-sClγ(A)∩B=Φ. Now S∩(a)-sCl_γ(A) = (A∪B) ∩sCl_γ(A) =A. As the intersection of (a)-γ-semiclosed sets is (a)-γ-semiclosed, A is (a)-γ-semiclosed. Similarly B is (a)-γ-semiclosed.

Theorem 3.7. Let A and B be nonempty subsets in a (a) topological space (X, τ_n, γ). The following statements hold.
(1). If A and B are (a)-γ-semiseparated A_1⊂A, B_1⊂B, then A_1 and B_1 are so.
(2). If A∩B = Φ such that A and B are (a)-γ-semiclosed ((a)-γ-semiopen), then A and B are (a)-γ-semiseparated.
(3). If A and B are (a)-γ-semiclosed ((a)-γ-semiopen) and if H = A∩(X\B), and G = B∩(X\A), then H and G

are (a)-γ-semiseparated.

Proof. (1). $A_1 \subset A$, (a)-sClγ(A_1) ⊂ (a)-sClγ(A). Then (a)-sClγ(A)∩B= Φ implies (a)-sClγ(A)∩B_1=Φ and (a)-sClγ(A_1)∩B_1=Φ. Hence A_1 and B_1 are (a)-γ-semiseparated.

(2). A = (a)-sCl$_\gamma$(A), B = (a)-sCl$_\gamma$(B) and A∩B = Φ (a)-sClγ(A)∩B=Φ and (a)-sClγ(B)∩A=Φ. Hence A and B are (a)-γ-semiseparated sets. If A and B are (a)-γ-semiopen, then there existsCompliments are (a)-γ-semiclosed. If A and B are (a)-γ-semiopen, then X\A and X\B are (a)-γ-semiclosed. Since H⊂X\B, (a)-sCl$_\gamma$(H)⊂(a)-sClγ(X\B) = X\B and so (a)-sCl$_\gamma$(H)∩B = Φ. Thus G∩ (a)-sCl$_\gamma$(H) = Φ. Similarly H∩ (a)-sClγ(G) = Φ. Hence H and G are (a)-γ-semiseparated sets.

Theorem 3.8. The sets A and B of a (a) topological space (X, τ_n, γ) are (a)-γ-semiseparated if and only if there exists U and V ∈(a)-so$_\gamma$(X, τ_n, γ) such that A⊂U, B⊂V, A∩V = Φ and B∩U = Φ.

Proof. Let A and B be (a)-γ-semiseparated sets. Set v = x\(a) - sClγ(A) and U = X\(a)-sClγ(B). Then U,V ∈(a)-so$_\gamma$(X, τ_n, γ) such that A⊂U, B⊂V, A∩V = Φ and B∩U = Φ. on the other hand, let U,V ∈ (a)-SO$_\gamma$(X, τ_n, γ) such that A⊂U, B⊂V, A∩V = Φ and B∩U = Φ. since X\V and X/U are (a)-γ-semiclosed sets, (a)-sClγ(A) ⊂ X\V ⊂ X\B and (a)-sClγ(B) ⊂ X\U ⊂X\A. Thus (a) - sClγ(A)∩B = Φ and (a)-sClγ(B) ∩A = Φ.

Theorem 3.9. Let A and B be nonempty disjoint subsets of a (a)-topological space (X, τ_n, γ) and E = A∪B. Then A and B are (a)-γ-semiseparated if and only if each of A and B are(a)-γ-semiclosed ((a)-semiopen) in E.

Proof. Let A and B are (a)-γ-semiseparated sets. By definition 3.1 A contains (a)-γ-semilimit points of B. Then B contains all (a)-γ-semilimit points of B which are in (a)-γ-semiclosed in A∪B. Therefore B is (a)-γ-semiclosed in E. similarly A is (a)-γ-semiclosed in E.

Theorem 3.10 Let (X, τ_n, γ) be a (a) topological space. If A and B are (a)-γ-semiseparation of X itself, then A and B are (a)-γ-semiclosed sets of (X, τ_n, γ).

Proof. Since A and B are (a)-γ-semiseparated, A∩(a)-scl$_\gamma$(B) = (a)-scl$_\gamma$(A) ∩B = Φ. Then A∩(a)-scl$_\gamma$(B) = Φ if and only if B is (a)-γ-semiclosed in A∪B = X. similarly, we can show that A is (a)-γ-semiclosed in X.

Definition 3.11. Let (X, τ_n, γ) be a (a) topological space. Points are called (a)-γ-semiseparated from (a)-γ-semiseparated from (a)-γ-semiclosed sets in X if for all (a)-γ-semiclosed setsC⊂X and for each X ∈ X\C, $\{x\}$ and C are (a)-γ-semiseparated.

Theorem 3.12. Let (X, τ_n, γ) be a (a)-γ topological space in which points are (a)-γ-semiseparated from (a)-γ-semiclosed set and let S be the pair of (a)-γ-semiseparated sets in X. then fo each subset A of X the (a)-γ-semiclosure of A is (a)-scl$_\gamma$(A) = $\{x \in X : \{\{x\}, A\} \notin S\}$.

Proof. Let x∉ $\{x \in X : \{\{x\}, A\} \notin S\}$.Then$\{\{x\}, A\} \in S$. We have $\{x\}$ ∩(a)-scl$_\gamma$(A) = Φ. Thus x∉(a)-scl$_\gamma$(A) and hence (a)-scl$_\gamma$(A) ⊂$\{x \in X : \{\{x\}, A\} \notin S\}$. Suppose that x∉ (a)-scl$_\gamma$(A). Then (a)-scl$_\gamma$(A) is a (a)-

γ-semiclosed set disjoint from $\{x\}$ and thus by hypothesis, $\{\{x\}, A\} \in S$. Hence $x \notin \{x \in X : \{\{x\}, A\} \notin S\}$.

In general, if x∈(a)-scl$_\gamma$($\{y\}$) in a (a)topological space, then it need not to be the case that y∈(a)-scl$_\gamma$($\{x\}$) However when points are (a)-γ-semiseparated from (a)-γ-semiclosed sets, this is the case, in fact this provides us with alternate characterizations of the axiom.

Definition 3.13. (a)-γ topological space (X, τ_n, γ) isCalled (a)-γ-semisymmetric if distinct points in X are (a)-γ-semiseparated.

Theorem 3.14. Let(X, τ_n, γ) be a (a)-γ topological space. Then the following are equivalent:

(1). Points are (a)-γ-semiseparated from (a)-γ-semiclosed sets in X.

(2). For all x, y ∈ X, x ∈ (a)-scl$_\gamma$($\{y\}$) if and only if y ∈ (a)-scl$_\gamma$($\{x\}$).

(3). X is (a)-γ-semiseparated.

Proof. Suppose that (1) holds. If x ∈ (a)-scl$_\gamma$($\{y\}$), Then $\{x\}$ and $\{y\}$ are not (a)-γ-semiseparated and hence y ∈ (a)-scl$_\gamma$($\{x\}$). If $\{x\}$ and $\{y\}$ are topologically distinct, then one of them, say x, hasa (a)-γ-semiopen set U which does not contain y. We have (a)-scl$_\gamma$($\{y\}$) ⊂ X\U. This implies that $\{x\}$and (a)-scl$_\gamma$($\{y\}$) are (a)-γ-semiseparated and $\{x\}$ and $\{y\}$ are (a)-γ-semiseparated. Hence (1) implies (2) and (1) implies (3). Suppose that (2) is true. Let C ⊂ X be (a)-γ-semiclosed and let x ∈X\C. For each y ∈ C, x ∉(a)-scl$_\gamma$($\{y\}$) and hence y ∉ (a)-scl$_\gamma$($\{x\}$). Thus (a)-scl$_\gamma$($\{x\}$) ∩ C = Φ. Hence (2) implies (1). Finally suppose that (3) is true and suppose that x ∉ (a)-scl$_\gamma$($\{y\}$) . Then $\{x\}$ and $\{y\}$ are topologically distinct and hence (a)-γ-semiseparated. Thus (a)-sClγ($\{x\}$∩ $\{y\}$) = Φ, that is y ∉ (a)-scl$_\gamma$($\{x\}$). Hence (3) implies (2).

Theorem 3.14 tells us that when X is (a)-γ-semisymmetric, the collection of (a)-γ-semiseparated sets uniquely determines the family of (a)-γ-semiopen sets of the topology of X. Note that (a)-γ-semisymmetry really is necessary.

Theorem 3.15. Let S be the sets of unordered pairs of subsets of a set X such that

(1). If $\{x, y\} \in$ S, Then A and B are disjoint.

(2). If A ⊂ B and $\{B, C\} \in$ S, then $\{A, C\} \in$ S,

(3). If $\{A, B\} \in$ S and $\{A, C\} \in$ S then $\{A, B \cup C\} \in$ S,

(4). If $\{\{x\}, B\} \in$ S for each x ∈ A and $\{\{y\}, A\} \in$ S for each y ∈B, then $\{A, B\} \in$ S and

(5). If $\{\{x\}, B\} \notin$ S and for each y ∈ B, $\{\{y\}, A\} \notin$ S, then $\{\{x\}, A\} \notin$ S.

Then there exists a unique (a)-γ-semisymmetric family of the (a)-γ-semiopen sets of the topology of X for which S is the collection of (a)-γ-semiseparated sets.

Proof: Let (a)-scl$_\gamma$(A) = $\{x \in X : \{\{x\}, A\} \notin S\}$for every subset A of X. if x ∉ (a)-scl$_\gamma$(A), then $\{\{x\}, A\}$∈S and hence x ∉ A. thus A ⊂ (a)-scl$_\gamma$(A) for each subset A. if x ∈(a)-scl$_\gamma$(A), then $\{\{x\}, A\} \notin S$and hence$\{\{x\}, B\} \notin S$, that is x∈ (a)-scl$_\gamma$(B). Thus (a)-scl$_\gamma$(A) ⊂ (a)-scl$_\gamma$(B) whenever A ⊂ B. In particular, since A ⊂(a)-scl$_\gamma$(A), (a)-scl$_\gamma$(A) ⊂(a)-scl$_\gamma$((a)-scl$_\gamma$(A)). If x ∈(a)-scl$_\gamma$((a)-scl$_\gamma$(A)), then $\{\{x\}, (a) - scl\gamma(A)\}$

$\notin$ S and hence, $\{\{x\}, A\} \notin S$. Furthermore, by the final condition, (a)-scl$_\gamma$((a)-scl$_\gamma$(A)) ⊂(a)-scl$_\gamma$(A) and thus, (a)-scl$_\gamma$((a)-scl$_\gamma$(A)) = (a)-scl$_\gamma$(A) for each A ⊂ X. (1) since X ⊂(a)-scl$_\gamma$(X) ⊂ X, then (a)-scl$_\gamma$(X) = X. (2) if (a)-scl$_\gamma$(A_α) = A_α for all α ∈ Δ, then (a)-scl$_\gamma$($\cap_{\alpha\in\Delta}A_\alpha$) ⊂(a)-scl$_\gamma$($A_\alpha$) = A_α for each α ∈ Δ, since $\cap_{\alpha\in\Delta}A_\alpha \subset A_\alpha$ for each α ∈ Δ. Hence, (a)-scl$_\gamma$($\cap_{\alpha\in\Delta}A_\alpha$) ⊂ $\cap A_\alpha$. Also since $\cap_{\alpha\in\Delta}A_\alpha$ ⊂ (a)-scl$_\gamma$($\cap_{\alpha\in\Delta}A_\alpha$), then(a)-scl$_\gamma$($\cap_{\alpha\in\Delta}A_\alpha$) = $\cap_{\alpha\in\Delta}A_\alpha$. (3) if (a)-scl$_\gamma$(A) = A and (a)-scl$_\gamma$(B) = B, then (a)-scl$_\gamma$(A ∪ B) = $\{x \in X : \{\{x\}, A \cup B\} \notin s\} = \{x \in X : \{\{x\}, A\} \notin s\}$ or $\{\{\{x\}, B\} \notin s\}$ = $\{x \in X : \{\{x\}, A\} \notin s\} \cup \{x \in X : \{\{x\}, B\} \notin s\}$ = (a)-scl$_\gamma$(A) ∪ (a)-scl$_\gamma$(B) = A ∪ B. hence, (a)-sClγ is the (a)-γ-semicloser operator of a topology τ on X. if y ∉ c = $\{x \in X : \{\{x\}, C\} \notin S\}$, then $\{\{y\}, C\} \in S$. Thus points are (a)-γ-semiseparated from (a)-γ-semiclosed sets in X. suppose that $\{A, B\}$ ∈S, then A ∩ (a)-scl$_\gamma$(B) = A ∩ $\{x \in X : \{\{x\}, B\} \notin s\}$ = $\{x \in A : \{\{x\}, B\} \notin s\}$ = Φ. Similarly (a)-scl$_\gamma$(A) ∩ B = Φ. Hence, if $\{A, B\}$ ∈S , then A and B are (a)-γ-semiseparated in τ. Now suppose that A and B are (a)-γ-semiseparated. Then $\{x \in A : \{\{x\}, B\} \notin s\}$ = A ∩ (a)-scl$_\gamma$(B) = Φ and $\{x \in A : \{\{x\}, B\} \notin s\}$ = (a)-scl$_\gamma$(A) ∩ B= Φ. Hence, $\{\{x\}, B\} \in s$ for each x∈ A and $\{\{y\}, A\} \in s$ for each y∈B. Thus$\{A, B\} \in s$. Hence S is the collection of pairs of sets (a)-semiseparated by (a)-SO$_\gamma$(X) and by theorem 3.14, S determines (a)-γ SO$_\gamma$(X) uniquely.

4. PROPERTIES OF (A)-Γ-SEMICONNECTED SPACES

In this section we introduce and study (a)-γ-semiconnected space and also investigate some of their basic properties.

Definition 4.1.A subset A of (a)topological space (X, τ_n, γ) is said to be (a)-γ-semiconnected if it cannot be expressed as union of two (a)-γ-semiseparated sets. Otherwise, the set A is called (a)-γ-semidisconnected.

Lemma 4.2. Let A ⊂ B∪ C such that A be a nonempty (a)-γ-semiconnected set in a (a)-γ- topological space (X, τ_n, γ) and B, C be (a)-γ-semiseparated sets. Then only one of the following conditions holds:

(1). A ⊂ B and A ∩ C = Φ.

(2). A ⊂ C and A ∩ B = Φ.

Proof: Since A ∩ C = Φ, A ⊂ B. Also if A∩B = Φ, then A ⊂ C. since A ⊂ B∩C, then both A ∩ C = Φ and A ∩ B = Φ cannot hold simultaneously. Similarly suppose that A∩B≠Φ and A∩C≠Φ, then by theorem 3.7 (1), A∩B and A∩C are (a)-γ-semiseparated sets such that A = (A∩B) ∪ (A∩C) which contradicts with the (a)-γ-semiconnectedness of A. Hence one of the conditions (1) and (2) must be hold.

Theorem 4.3. If an (a)-γ-semiconnected set S of a (a)-topological space (X, τ_n, γ) is connected in A∪B, where A and B are (a)-semiseparated sets, then either S⊂A or S⊂B.

Proof: we have S = (S∩A) ∪ (S∩B) where S∩A and S∩B are(a)-γ-semiseparated sets. So either S∩A = Φ or S∩B = Φ and hence either S⊂B or S⊂A.

Theorem4.4. A subset M of (a)-topological space (X, τ_n, γ) is an (a)-semiconnected set if their exists an (a)-γ-semiconnected set satisfying C⊂M⊂(a)-scl$_\gamma$(C).

Proof. Let M = A∪B, where A and B are (a)-γ-semiseparated sets. Then either C⊂A or C⊂B and hence either M⊂(a)-$scl_γ$(C)⊂(a)-$scl_γ$(A)⊂(X\B) or M⊂(X\A). Therefore either B = Φ or A = Φ.

Corollary 4. If C is an (a)-γ-semiconnected set of a (a)-topological space (X, $τ_n$, γ), then (a)-$scl_γ$(C) also.
Proof: Follows theorem 4.4.

Theorem 4.6.if$\{M_α : α ∈ Δ\}$ is a family of (a)-γ-semiconnected set of a (a)-topological space (X, $τ_n$, γ) satisfying the property that any two of which are not (a)-γ-semiseparated, then M = $\bigcup_{α∈Δ} M_α$ is (a)-γ-semiconnected.

Proof: Let M = A∪B, where A and B are (a)-γ-semiseparated sets. Then for each α∈Δ either $M_α$ ⊂A or $M_α$⊂B. since any two members of the family $\{M_{αα} : αα ∈ Δ\}$ are not (a)-γ-semiseparated. Either $M_α$⊂A for each α ∈Δ or $M_α$⊂B for each α ∈Δ. So either B = Φ or A = Φ.

Corollary 4.7. If M = $\bigcup_{αα∈Δ} M_α$ where each $M_α$ is (a)-γ-semiconnected set of a (a)-topological space (X, $τ_n$, γ) and also $M_α ∩ M_{α'} ≠ Φ$ for α ∈Δ, then M is (a)-γ-semiconnected.

Proof: Follows from theorem 4.

Corollary 4.8. If M = $\bigcup_{αα∈Δ} M_α$ where each $M_α$ is (a)-γ-semiconnected set of a (a)-topological space (X, $τ_n$, γ) and also $\bigcap_{αα∈Δ} M_α ≠ Φ$ for each α ∈Δ, then M is (a)-γ-semiconnected.

Proof: suppose that M = $\bigcup_{αα∈Δ} M_α$ is not (a)-γ-semiconnected. Then we have $\bigcup_{α∈Δ} M_α$ = H∪G where H and G are (a)-γ-semiseparated sets in X. since $\bigcap_{αα∈Δ} M_α ≠ Φ$, we have a point x in $\bigcap_{αα∈Δ} M_α$. since x∈$\bigcup_{αα∈Δ} M_α$, either x ∈ G or x ∈ H. since x∈$M_α$ for each α ∈Δ. By theorem 4.3, $M_α$ ⊂ H $M_α$ ⊂ G. since H and G are disjoint, $M_α$ ⊂ H for all α ∈Δ and hence $\bigcup_{αα∈Δ} M_α$ ⊂ H. this implies that G is empty. This is a contradiction. Suppose that x∈G. By similar way, we have that H is empty. This is a contradiction. Thus $\bigcup_{αα∈Δ} M_α$ is (a)-γ-semiconnected.

Theorem 4.9. For an (a)-topological space (X, $τ_n$, γ) the following statements are equivalent:

(1). XC is (a)-γ-semiconnected.

(2). X cannot be expressed as the union of two nonempty disjoint (a)-γ-semiopen sets.

(3). X contains no nonempty proper subset which is both (a)-γ-semiopen and (a)-γ-semiclosed.

Proof: (1) ⇒ (2): Suppose that X is (a)-γ-semiconnected and if X can be expressed as the union of two nonempty disjoint sets A and B such that A and B are (a)-γ-semiopen sets. Consequently A ⊂ X\B. then (a)-$scl_γ$(A) ⊂ (a)-$scl_γ$(X\B) = X\B. therefore, (a)-$scl_γ$(A) ∩ B = Φ. Similarly we can poveA∩ (a)-$scl_γ$(B) = Φ. This is a contradiction to the fact that X is (a)-γ-semiconnected. Therefore, X cannot be expressed as the union of two non-empty disjoint (a)-γ-semiopen sets.
(2)⇒(3): if X cannot be expressed as the union of two nonempty disjoint sets A and B such that A and B are (a)-γ-semiopen sets. If X contains the nonempty proper subset A which is both (a)-γ-semiopen and (a)-γ-

semiclosed. Then X = A ∪ (X\A). Hence A and X\A are disjoint (a)-γ-semiopen sets whose union is X. this is the contradiction to our assumption. Hence, X contains no nonempty proper subset which is both (a)-γ-semiopen and (a)-γ-semiclosed.

(3) ⇒ (1): suppose that X contains no nonempty proper subset which is both (a)-γ-semiopen and (a)-γ-semiclosed and X is not (a)-γ-semiconnected. Then X can be expressed as the union of two nonempty disjoint sets A and B such that((a)-scl_γ(A) ∩ B) ∪ (A∩ (a)-scl_γ(B)) = Φ. Since A∩B = Φ, A = X\B and B = X\A. since (a)-scl_γ(A) ∩ B = Φ, (a)-scl_γ(A) ⊂ X\B. hence (a)-scl_γ(A) ⊂ A. therefore A is (a)-γ-semiclosed. Similarly B is (a)-γ-semiclosed. Since A = X\B, A is (a)-γ-semiopen. Therefore there exists a nonempty proper set A which is both (a)-γ-semiopen and (a)-γ-semiclosed. This is a contradiction to our assumption. Therefore X is (a)-γ-semiconnected.

Theorem 4.10.A topological space (X, τ_n, γ) is (a)-γ-semiconnected if, and only if X is not the union of any two any two (a)-γ-semiseparated sets.

Proof: Let A and B be two (a)-γ-semiseparated sets such that X = A∪B. therefore (a)-scl_γ(A) ∩ B = A∩ (a)-scl_γ(B) = Φ. Since A⊂(a)-scl_γ(A) and B⊂ (a)-scl_γ(B), then A ∩ B = Φ. Now (a)-scl_γ(A) ⊂ X\B = A. Hence (a)-scl_γ(A) = A. then A is (a)-γ-semiclosed. By the same way we can show that B is (a)-γ-semiclosed which contradicts with theorem 4.9 (2). Conversely, let A and B be any two disjoint nonempty and (a)-γ-semiclosed sets of X such that X = A∪B. Then (a)-scl_γ(A) ∩ B = A∩ (a)-scl_γ(B) = Φ, which contradicts with the hypothesis.

Theorem 4.11.A topological space (X, τ_n, γ) is (a)-γ-semiconnected if, and only if for every pair of points x, y in X, there is an (a)-γ-semiconnected subset of X which contains both x and y.

Proof: the necessity is immediate since the (a)-γ-semiconnected space itself contains these two points. For the sufficiency, suppose that for any two points x and y, there is an (a)-γ-semiconnected subset $C_{x,y}$ of X such that x, y $\in C_{x,y}$. Let a ∈ X be a fixed point and {$C_{x,\,y}$: x ∈ X be a class of all (a)- γ- semiconnected subsets of X which contain the points a and x, then X = $\cup_{x\in X} C_{a,x}$ *and* $\cap_{x \in X} C_{a,x} \neq \emptyset$. Therefore, X is (a)-γ-semiconnected.

REFERENCES

[1] Bose, M. K. and Mukharjee, A. On countable families of topologies on a set, Novi Sad J. Math. 40 (2), 7-16, 2010.
[2] Bose, M. K. and Tiwari, R. On increasing sequences of topologies on a set, Riv. Mat. Univ. Parma. (7) 7, 173-183, 2007.
[3] Kelly, J. C., Bitopological spaces, J. Proc. London Math. Soc. (1963), 13, 71-89.
[4] Kov´ar, M. M. A note on the comparison of topologies, Int. J. Math. Math. Sci. 26, 233-237, 2001.
[5] Ogata, H., Operation on topological Spaces and associated topology, Math. Japonica (1991), 36, 175-184.
[6] Roy Choudhuri, A., Mukharjee, A. and Bose, M. K., Hyperconnectedness and extremal disconnectedness in (a)topological spaces, Hacettepe Journal of mathematics and Statistics 44(2), 289-294, 2015.
[7] Tyagi B.K, Sheetal Luthra and Harsh V. S. Chauhan, On New Types of Sets Via γ-open Sets in (a)Topological Spaces, CUBO A Mathematical Journal, 20 (2), 41-52, 2018.