A STUDY ON TRANSLATION OF Q-FUZZY SUBSEMIRING OF A SEMIRING

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Abstract : **In this paper, we made an attempt to study the algebraic nature on translation of Q- fuzzy subsemiring of a semiring and we introduce the some theorems in translation of Q-fuzzy subsemiring of a semiring**
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KEY WORDS: fuzzy subset, **Q**- fuzzy subset, Q-fuzzy subsemiring, Q-fuzzy translation.

INTRODUCTION: There are many concepts of universal algebras generalizing an associative ring (*R ;* + *;* .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b+c) = a. b+a. c and (b+c) .a = b. a+c. a for all a, b and c in R. A semiring R is said to be additively commutative if a+b = b+a for all a, b in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0+a = a = a+0 and a.0 = 0 = 0.a for all a in R. After the introduction of fuzzy sets by L.A.Zadeh[6], several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subnearrings and ideals was introduced by S.Abou Zaid[13]. A.Solairaju and R.Nagarajan [3] have introduced and defined a new algebraic structure called Q-fuzzy subgroups. We introduce the concept of Q-fuzzy translation of Q-fuzzy subsemiring of a semiring and established some results.

1. PRELIMINARIES:

1.1 Definition: Let X be a non–empty set. A **fuzzy subset A** of X is a function A : X → [0, 1].

1.2 Definition: Let X be a non-empty set and Q be a non-empty set. A **Q-fuzzy subset** A of X is a function A: X×Q → [0,1].

1.3 Definition: Let (R, +, ·) be a semiring and Q be a non empty set. A Q-fuzzy subset A of R is said to be a Q-**fuzzy subsemiring (QFSSR)** of R if the following conditions are satisfied:

(i) $\mu_A(x + y, q) \geq \min\{\mu_A(x , q), \mu_A(y , q)\}$,

(ii) $\mu_A(xy,q) \geq \min\{ \mu_A(x,q), \mu_A(y,q) \}$,for all x and y in R and q in Q.

1.4 Definition: Let A be a Q-fuzzy subset of X and α ε [0,1-Sup{A(x,q): x ε X , 0<A(x,q)<1}]. Then T = T_α^A is called a Q-fuzzy translation of A if T(x,q)=A(x,q)+α, for all x in X and q in Q.

1.1.1 Example: Consider the set X={0,1,2,3,4,5} and Q={p}. Let A = {((0,p),0.6), ((1,p),0.43), ((2,p),0.28), ((3,p),0.35), ((4,p),0.15), ((5,p),0.51)} be a Q-fuzzy subset of X and α = 0.15. The Q-fuzzy translation of A is T = $T_{0.15}^A$ ={((0,p),0.75), ((1,p),0.58), ((2,p),0.43), ((3,p),0.50), ((4,p),0.30), ((5,p),0.66)}.

1.5 Definition: Let (R,+,·) and (R',+,·) be any two semirings Q be a non empty set. Let f:R→R' be any function and A be a Q-fuzzy translations of Q-fuzzy subsemiring in R, V be a Q-fuzzy subsemiring in f(R) = R', defined by $\mu_V(y,q) = \sup_{x\in f^{-1}(y)} \mu_A(x,q)+\alpha$, for all x in R and y in R' and q in Q. Then A is called a pre-image of V under f and is denoted by $f^{-1}(V)$.

1.6 Definition: Let (R, +, **.**) and (R', +, **.**) be any two semirings. Then the function f : R → R' is called a **semiring homomorphism** if it satisfies the following axioms:

(i) f(x+y) = f(x) + f(y),

(ii) f(xy) = f(x) f(y), for all x and y in R.

1.7 Definition: Let (R , +, .) and (R', +, .) be any two semirings. Then the function f : R → R' is called a **semiring anti-homomorphism** if, it satisfies the following axioms:

(i) f(x + y) = f(y) + f(x),

(ii) f(xy) = f(y) f(x), for all x and y in R.

2. PROPERTIES ON Q-FUZZY TRANSLATION OF Q-FUZZY SUBSEMIRING OF A SEMIRING

2.1 Theorem: If `E and `H are two Q-fuzzy translations of Q-fuzzy subsemiring A of a semiring (R, +, ·) , then their intersection `E ∩ `H is a Q-fuzzy subsemiring of R.

Proof: Let x and y belong to R and q in Q. Let `E=$T_\alpha^A$={<(x,q),$\mu_A$(x,q)+α>/x in R and q in Q} and `H= T_β^A={<(x,q),μ_A(x,q)+β>/x in R and q in Q }be two Q-fuzzy translations of Q-fuzzy subsemiring. Let C=`E∩`H and C={<(x,q), μ_C(x,q)>/ x in R and q in Q}, where $\mu_C(x,q)=\min\{\mu_A(x,q)+\alpha,\mu_A(x,q)+\beta\}$. Case (i): α≤β. Now $\mu_C(x+y,q)=\min\{\mu_{`E}(x+y,q),\mu_{`H}(x+y,q)=\min\{\mu_A(x+y,q)+\alpha,\mu_A(x+y,q)+\beta\}=\mu_A(x+y,q) +\alpha=\mu_{`E}(x+y,q)$, for all x and y in R and q in Q. And $\mu_C(xy,q)=\min\{\mu_{`E}(xy,q),\mu_{`H}(xy, q)=\min\{\mu_A(xy,q)+\alpha,\mu_A(xy,q)+\beta\}=\mu_A(xy,q)+\alpha=\mu_{`E}(xy,q)$, for all x and y in R and q in Q. C=T_α^A={<(x,q),μ_A(x,q)+α>/x in R and q in Q} is a Q-fuzzy translation of Q-fuzzy subsemiring a of the semiring (R,+, ·). Case (ii): α≥β. Now $\mu_C(x+y,q)= \min\{\mu_{`E} (x+y,q),\mu_{`H} (x+y, q) =\min\{\mu_A (x+y,q)+\alpha,\mu_A(x+y,q)+\beta\}= \mu_A (x+y, q)+\beta = \mu_{`H}(x+y,q)$, for all x and y in R and q in Q. And μC(xy,q)= min{μE(xy,q),μ`H (xy, q) = min{μA(xy,q)+α,μA(xy,q)+β}=μA(xy,q)+β=μ`H(xy,q), for all x and y in R and q in Q. C = `H = T_β^A ={<(x, q), μA (x, q)+β >/x in R and q in Q } is a Q-fuzzy translation of Q-fuzzy subsemiring a of the semiring (R, +, ·). Hence all cases, intersection of any two Q-fuzzy translations of Q-fuzzy subsemiring A of a semiring (R, +, ·) is a Q-fuzzy translation of A.

2.2 Theorem: The intersection of a family of Q-fuzzy translations of Q-fuzzy subsemiring A of a semiring (R, +, ·) is a Q-fuzzy translation of A.

Proof: Let x and y belong to R and q in Q. Let A_i = T_α^A={<(x, q), μA (x, q)+α_i>/x in R and q in Q} be a family of Q-fuzzy translations of Q-fuzzy subsemiring A of the semiring (R, +, ·). Let C= $\bigcap_{i\in I} A_i$ and C={<(x, q), μA (x, q)>/x in R and q in Q}

Where A(x , q) = $\inf_{i\in I}$ A (x,q)+α_i=A(x,q)+ $\inf_{i\in I}$ α_i . Clearly C is also a Q-fuzzy translation of a Q-fuzzy subsemiring A of R.

2.3 Theorem: If `E and `H are two Q-fuzzy translations of Q-fuzzy subsemiring A of a semiring (R, +, ·) , then their union `E ∪ `H is a Q-fuzzy subsemiring of R.

Proof: Let x and y belong to R and q in Q. Let `E = $T_\alpha^A$={<(x, q), μA (x, q)+α>/x in R and q in Q} and `H = T_β^A ={<(x, q), μA (x, q)+β >/x in R and q in Q }be two Q-fuzzy translations of Q-fuzzy subsemiring. Let C=`E∪`H and C={< (x, q), μC (x, q)>/ x in R and q in Q}, where μC(x,q)=max{μA(x,q)+α,μA(x,q)+β}. Case (i): α≤β. Now μC(x+y,q) = max{μE(x+y,q),μ`H(x+y,q) = max{μA(x+y,q)+α,μA(x+y,q)+β} = μA(x+y,q)+β=μ`H(x+y,q), for all x and y in R and q in Q. And μC(xy,q) = max{μE(xy,q),μ`H(xy,q)=max{μA(xy,q)+α,μA(xy,q)+β}=μA(xy,q)+β= μ`H(xy,q), for all x and y in R and q in Q. Therefore C= T_β^A={<(x,q),μA(x,q)+β>/x in R and q in Q} is a Q-fuzzy translation of Q-fuzzy subsemiring a of the semiring (R,+,·). Case (ii): α≥β. Now μC(x+y,q)=max{μE(x+y,q),μH(x+y,q)}=max{μA(x+y,q)+α,μA(x+y,q)+ β}=μA(x+y,q)+α=μE(x+y, q) , for all x and y in R and q in Q. And μC(xy,q)= min{μE (xy,q),μH (xy,q)=min{μA(xy,q)+α,μA(xy,q)+β}=μA(xy,q)+α=μE(xy,q), for all x and y in R and q in Q. Therefore C=`H =T_α^A ={<(x, q), μA (x, q)+α >/x in R and q in Q } is a Q-fuzzy translation of Q-fuzzy subsemiring A of the semiring (R, +, ·). Hence all cases, union of any two Q-fuzzy translations of Q-fuzzy subsemiring A of a semiring (R, +, ·) is a Q-fuzzy translation of A.

2.4 Theorem: The union of a family of Q-fuzzy translations of Q-fuzzy subsemiring A of a semiring (R, +, ·) is a Q-fuzzy translation of A.

Proof: Let x and y belong to R and q in Q. Let A_i = T_α^A={<(x, q), μA (x, q)+α_i>/x in R and q in Q} be a family of Q-fuzzy translations of Q-fuzzy subsemiring A of the semiring (R, +, ·). Let C= $\bigcup_{i\in I} A_i$ and C={<(x, q), μA (x, q)>/x in R and q in Q}, where A(x , q) = $\sup_{i\in I}$ {A (x,q)+α_i}=A(x,q)+ $\sup_{i\in I}$ α_i . Clearly C is also a Q-fuzzy translation of a Q-fuzzy subsemiring A of R.

2.5 Theorem: If T is a Q-fuzzy translation of a Q-fuzzy subsemiring A of a semiring R, then T is a Q-fuzzy subsemiring of R.

Proof: Assume that T is a Q-fuzzy translation of a Q-fuzzy subsemiring A of a semiring R. Let x and y in R and q in Q. We have T(x+y,q)= A(x+y,q)+α ≥ min{A(x,q),A(y,q)}+α=min{A(x,q)+α,A(y,q)+α}=min{T(x,q),T(y,q)}which implies that T(x+y,q)≥min{T(x,q),T(y,q)},for all x, y in R and q in Q. Again, T(xy,q) = A(xy,q)+α≥min{A(x,q),A(y,q)}+α=min{A(x,q)+α,A(y,q)+α}=min{T(x,q), T(y,q) } which implies that T(xy,q)≥min{T(x,q),T(y,q)}, for all x, y in R and q in Q. Hence T is A Q-fuzzy Subsemiring of R.

2.6 Theorem: Let (R, +, .) and (R', +, .) be any two semirings Q be a non-empty set. The homomorphic image of a Q-fuzzy translation of a Q-fuzzy subsemiring A of R is a Q-fuzzy of subsemiring of R'.

Proof: Let (R, +, .) and (R', +, .) be any two semirings Q be a non-empty set. Let f : R → R' be a homomorphism. Then, f(x+y)=f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R. Let T_α^A be a Q-fuzzy translation of a Q-fuzzy subsemiring A of R. Let V be the homomorphic image of T_α^A under f. We have to prove that V is a Q-fuzzy of subsemiring of R'. Now, for f(x), f(y) in R' and q in Q, we have V(f(x)+f(y) ,q)= V(f(x+y),q)≥ T_α^A(x+y,q)= A(x+y,q)+α≥min{A(x,q),A(y,q)}+α=min{A(x,q)+α, A(y,q) +α } = min {T_α^A(x,q), T_α^A(y,q) } which implies that V(f(x)+f(y),q) ≥ min { V(f(x),q), V(f(y),q) }for f(x), f(y) in R' and q in Q. Again, V(f(x)f(y),q) = V(f(xy),q)≥T_α^A(xy,q)=A(xy,q)+α≥min{A(x,q),A(y,q)}+α=min{A(x,q)+α,A(y,q)+α}=min{T_α^A(x,q),T_α^A(y,q)} which implies that V(f(x)f(y),q)≥min{V(f(x),q),V(f(y),q) }.

Hence V is a Q-fuzzy of subsemiring of R'.

2.7 Theorem: Let (R, +, .) and (R', +, .) be any two semirings and Q be a non-empty set. The homomorphic pre-image of a Q-fuzzy translation of a Q-fuzzy subsemiring of R' is a Q-fuzzy of subsemiring of R.

Proof: Let (R, +, .) and (R', +, .) be any two semirings Q be a non-empty set.

Let f : R → R' be a homomorphism. Then, f(x+y) = f(x) + f(y) and f(xy) = f(x) f(y), for all x and y in R . Let T_α^v be a Q-fuzzy translation of a Q-fuzzy subsemiring V of R'. Let A be the homomorphic pre-image of T_α^v under f. We have to prove that A is a Q- fuzzy of subsemiring of R. Let x and y in R and q in Q. Then, A(x+ y,q) = T(f(x+y),q)=T(f(x)+f(y),q)= V(f(x) +f(y) ,q)+α ≥ min { V(f(x),q), V(f(y),q) }+α ≥ min{V(f(x),q)+α,V(f(y),q)+α}= min{T(f(x),q),T(f(y),q)}= min{A(x,q),A(y,q)} which implies that A(x+y,q)≥min{A(x,q),A(y,q)}for x and y in R and q in Q. Again, A(xy,q)=T(f(xy),q)=T(f(x)f(y),q)=V(f(x)f(y),q)+α≥min{V(f(x),q)+α,V(f(y),q)+α }= min { T(f(x),q), T(f(y),q) } = min{A(x,q),A(y,q)} which implies that A(xy,q) ≥ min { A(x,q),A(y,q)}for x and y in R and q in Q. Hence A is a Q-fuzzy of subsemiring of R.

2.8 Theorem: Let (R, +, .) and (R', +, .) be any two semirings and Q be a non-empty set. The anti-homomorphic image of a Q-fuzzy translation of a Q-fuzzy subsemiring A of R is a Q-fuzzy subsemiring of R'.

Proof: Let (R, +, .) and (R', +, .) be any two semirings Q be a non-empty set.

Let f : R → R' be an anti-homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x, y ∈ R. Let T_α^A be a Q-fuzzy translation of a Q-fuzzy subsemiring A of R. Let V be the anti-homomorphic image of T_α^A under f. We have to prove that V is a Q-fuzzy subsemiring of R'. Now, for f(x), f(y) in R' and q in Q, V(f(x)+f(y),q)=

V(f(y+x),q)≥T_{α}^{A}(y+x,q)=A(y+x,q)+α≥min{A(x,q),A(y,q)}+α=min{A(x,q)+α,A(y,q) +α}=min{T_{α}^{A}(x,q), T_{α}^{A}(y,q) } which implies that V(f(x)+f(y),q)≥min{V(f(x),q), V(f(y),q) }for f(x), f(y) in R' and q in Q. Again, V(f(x)f(y),q)= V(f(yx),q) ≥ T_{α}^{A}(yx,q)= A(yx,q)+α ≥ min{A(x,q),A(y,q) }+α = min {A(x,q)+α, A(y,q) +α } = min{T_{α}^{A}(x,q),T_{α}^{A}(y,q)} which implies that V(f(x)f(y),q)≥min{V(f(x),q),V(f(y),q) }.
Hence V is a Q-fuzzy of subsemiring of R'.

2.9 Theorem: Let (R, +, .) and (R', +, .) be any two semirings and Q be a non-empty set. The anti-homomorphic pre-image of a Q-fuzzy translation of a Q-fuzzy subsemiring of R' is an Q-fuzzy subsemiring of R.

Proof: Let (R, +, .) and (R', +, .) be any two semirings Q be a non-empty set.
Let f : R → R' be a homomorphism. Then, f(x+y) = f(y) + f(x) and f(xy) = f(y) f(x), for all x and y in R . Let T_{α}^{v} be a Q-fuzzy translation of a Q-fuzzy subsemiring V of R'. Let A be the homomorphic pre-image of T_{α}^{v} under f. We have to prove that A is a Q- fuzzy of subsemiring of R. Let x and y in R and q in Q. Then, A(x+ y,q) = T(f(x+y),q)=T(f(y)+f(x),q)= V(f(y) +f(x) ,q)+α ≥ min { V(f(x),q), V(f(y),q) }+α ≥min{V(f(x),q)+α,V(f(y),q)+α}=min{T(f(x),q),T(f(y),q)}=min{A(x,q),A(y,q)} which implies that A(x+y,q)≥min{A(x,q),A(y,q)}for x and y in R and q in Q. Again, A(xy,q)=T(f(xy),q)=T(f(y)f(x),q)=V(f(y)f(x),q)+α≥min{V(f(x),q)+α,V(f(y),q)+α }= min { T(f(x),q), T(f(y),q) } = min{A(x,q),A(y,q)} which implies that A(xy,q) ≥ min {A(x,q),A(y,q)}for x and y in R and q in Q. Hence A is a Q-fuzzy of subsemiring of R.

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