ESTIMATION OF SCALE(θ) AND SHAPE (α) PARAMETERS OF POWER FUNCTION DISTRIBUTION BY LEAST SQUARES METHOD USING OPTIMALLY CONSTRUCTED GROUPED DATA

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Abstract
The objective of this paper is to estimate the parameters and also construct an Optimal Grouped sample in the absence of prior knowledge or guess values of parameters. In this heuristic algorithm, the Least Square Regression Method is used to find out Estimates the parameters of Power Function Distribution. Statistical Correlation technology is used to test whether or not these parameters are statistically fit or not. We also compare equispaced and unequispaced Optimally Constructed Grouped data by the method of an Asymptotically Relative Efficiency. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Simulated Error (SE) and Relative Absolute Bias (RAB) for both the parameters under grouped sample based on 1000 simulations to assess the performance of the estimators.
Keywords: Power Function Parameters, Optimally Grouped sample, equispaced, unequispaced, correlation coefficient.

1. Introduction

Power Function is a flexible and simple distribution that may be helpful for modeling the failure data. Rider (1964) the name Power Function Distribution has been used. Johnson (1970) given that the moments of the power function distribution are simply the negative moments of the Pareto distribution. Ahsanullah and Kabir et al (1975) discussed the Estimation of the location and scale parameters of a Power function distribution. According to Dallas et al (1976), if Y is power function distribution then Y^{-1} is the Pareto distribution model. Cohen and Whitten et al (1980) used the estimation in the three parameter lognormal distribution. Rosaiah *et al* (1991) studied the problem of asymptotically optimal grouping of sample into equiclass grouped sample for maximum likelihood estimation in two parameter gamma distribution. Vasudevarao *et al* (1994) considered the problem of asymptotically optimal grouping for maximum likelihood estimation in a two parameter Weibull distribution in the case of equispaced group. They also studied the same for maximum likelihood estimation of Weibull shape parameter when scale parameter is known in the case of unequispaced grouped samples. Meniconi and Barry et al(1996) explore the performance of Power function distribution on electrical components and illustrated that power function distribution is most suitable distribution on electrical component data as compared to log-normal,Weibull and exponential models. Theoretically, Kleiber et al (2003) studied power function distribution has an inverse relationship with the standard Pareto distribution, and it is also a special case of Pearson type I distribution. Saran & Pandey et al (2004) estimate the parameters of Power Function Distribution and they also characterize this distribution. Balakrishna et al (2004) and Kantam et al (2005) constructed the Optimum group limits for un-eqi-spaced grouped sample using M. L. Estimation in Scaled Log-Logistic distribution. Reliability Hotwire- The e-Magazine et al (2007) had mentioned the Correlation Coefficient tool in 'How our weibull distribution be good'.CH. Rama Mohan et al (2011) Studied Least Square Estimation of the Weibul parameters from an optimally constructed grouped sample. Rahman, Roy & Baizid et al (2012) applied the Bayesian estimation method to estimate the parameters of Power Function Distribution. Zarrin et al (2013) applied power function distribution to assess component failure of semi-conductor device data by using both the maximum likelihood and Bayesian estimation methods.

The literature mentioned above, reveals that much attention seems to have been paid for inference based on grouped data from two parameter Power Function distribution. In this, when we have no prior Knowledge about the unknown parameters that we used to construct an asymptotically Optimal Groped Data, which can be used to estimation of parameters using Least Square Method. The optimal group limits of a grouped sample from two parameter Power Function distribution constructed which are presented at the end of the chapter as Table 7. Here we developed a practical procedure to construct an optimally grouped sample even when there is no prior knowledge or guess values of the parameters are given in section 2. In section 3 we made an attempt to study some problems of point estimation from grouped data based on Power Function distribution. The Least Squares Regression method was used to estimate the parameters from such an optimally constructed grouped sample in two parameter Power Function distribution using the optimal group limits constructed and The Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) of the Scale parameter (θ) and Scale (α) are calculated for assessing the performance of the estimated parameters.

Let $y_1, y_2, \ldots, y_n$ be a raw sample of size 'n' dawn from two-parameter Power Function distribution with unknown scale(θ) and shape(α) parameters. The Probability density (p.d.f.) and cumulative distribution function (c.d.f.) of Power Function distribution are respectively given by

$$f(y;\theta,\alpha) = \frac{\gamma y^{\gamma-1}}{\beta^{\gamma}}, 0<y<\beta, \quad \gamma>0, \quad \beta>0 \qquad (1.1)$$

$$F(y;\theta,\alpha) = \left(\frac{y}{\beta}\right)^{\gamma}, 0<y<\beta, \quad \gamma>0, \quad \beta>0 \qquad (1.2)$$

2. OPTIMALLY CONSTRUCTED GROUPED SAMPLE:

In this section, we develop a practical procedure to construct an optimally grouped sample in the case when there is no a priori knowledge or guess values of the parameters. In this procedure, we will prefix the number of test units to be failed in each group of the optimal grouped sample and then we record some arbitrary time point after failure of the number of the test units that are to be failed in that group, but before starting the failure of a test unit in the next group. Suppose *N is* the number of test units put under a life-testing experiment which assumes the Power model (1.1) and suppose the experimenter wishes to obtain the grouped life-time data with *k* classes. Then *Table 7* can be used to compute the expected number of test units to be failed in the time interval (t_{i-1}, t_i) and is given by

$$f_i = Np_i ; \quad \text{For } i = 1, 2, \ldots, k \qquad (2.1)$$

Where $p_i = \frac{1}{\beta^{\gamma}}[(x_{i-1})^{\gamma} - (x_i)^{\gamma}]$

1. f_i is expected number of failures in the i^{th} interval

2. x_i' s are optimal group limits obtained from the above procedure

3. k is number of groups
4. N is total frequency

f_i's may be rounded to the nearest integers so that $N = f_1 + f_2 + \ldots + f_k$. Thus, the optimal group limits may be used to compute the expected optimum number of test units to be failed in i^{th} interval (t_{i-1}, t_i), for i=1,2,…, k. Here, it may be noted that the experimenter has to observe the random time instants $y_1, y_2, \ldots, y_{k-1}$ so as the optimum pre-fixed number of units f_i to be failed in the time interval (y_{i-1}, t_i) for i=1, 2,…, k taking $t_o=0$ and $t_k = \infty$. In other words, record a random time instant after failure of first f_1 test units, but before the failure of $(f_1+1)^{th}$ test unit and to record a random time instant after failure of first $f_1 + f_2$ test units, but before the failure of $(f_1 + f_2 + 1)^{th}$ test unit and so on. Further, it may be noted that it is difficult to record all exact failure times of the individual units but, it is not so difficult to note a random time instant between the failure times of two consecutive test units.

METHODOLOGY

3. LEAST SQUARES ESTIMATION OF THE PARAMETERS FROM THE OPTIMALLY CONSTRUCTED EQUISPACED GROUPED SAMPLE:

We know that $t_1, t_2, \ldots, t_{k-1}$ the group limits of te optimally constructed grouped Sample using the procedure explained in the above section, are the observed values of the true asymptotic optimal group limits $x_1, x_2, \ldots, x_{k-1}$ where as their estimated values are given by

$$\hat{y}_i = \beta(x_i)^{\frac{1}{\hat{\gamma}}} \tag{3.1}$$

where $\hat{\beta}$ and $\hat{\gamma}$ are obtained by using the principle of least square method (LSM) is extensively used in reliability engineering and mathematics problems. According to the least square method (LSM) linear relation between the two parameters taking the natural logarithm of above equation as follows

$$\log t_i = \log(\beta) + \left(\frac{1}{\gamma}\right)\log(x_i) \quad \text{for i= 1, 2, …, k-1} \tag{3.2}$$

After simplification, we get

$Y_i = \log t_i$

$A = \log(\beta)$

$B = \dfrac{1}{\gamma}$

$X_i = \log(x_i)$

Thus, equation (3.2) is a linear equation and is expressed as

$$Y_i = A + BX_i$$

To compute a and d by simple linear regression we proceed as follows

$$\text{Let} \quad S(A,B) = \sum_{i=1}^{k-1}(y_i - A - Bx_i)^2 \tag{3.3}$$

Differentiating (3.3) w.r.t to A and B then equate to zero, we obtain the following two normal equations

$$\sum_{i=1}^{n} y_i = nA + B\sum_{i=1}^{n} x_i \tag{3,4}$$

$$\sum_{i=0}^{n} x_i y_i = A\sum_{i=1}^{n} x_i + B\sum_{i=0}^{n} {x_i}^2 \tag{3.5}$$

Solving the above two equations for A and B, we obtain the least square estimates (LSE) of A and B as:

$A = \bar{y} - B\,\bar{x}$

$$B = \frac{\sum_{i=1}^{k-1} x_i y_i - \frac{\left(\sum_{i=1}^{k-1} x_i y_i\right)}{k-1}}{\sum_{i=1}^{k-1} {x_i}^2 - \frac{\left(\sum_{i=1}^{k-1} x_i\right)^2}{k-1}}$$

$$A = \frac{\sum_{i=1}^{k-1}\log t_i}{k-1} - B\,\frac{\sum_{i=1}^{k-1}\log x_i}{k-1} \tag{3.6}$$

$$B = \frac{\sum_{i=1}^{k-1}\log(x_i)(\log t_i) - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)\left(\sum_{i=1}^{k-1}\log t_i\right)}{k-1}}{\sum_{i=1}^{k-1}(\log(x_i))^2 - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)^2}{k-1}} \tag{3.7}$$

where $A = \log(\beta)$ and $B = \dfrac{1}{\gamma}$

$$\text{Therefore} \quad \hat{\beta} = \text{Antilog}\left\{\frac{\sum_{i=1}^{k-1}\log t_i}{k-1} - B\,\frac{\sum_{i=1}^{k-1}\log x_i}{k-1}\right\} \tag{3.8}$$

$$\text{and} \quad \hat{\gamma} = \frac{\sum_{i=1}^{k-1}(\log(x_i))^2 - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)^2}{k-1}}{\sum_{i=1}^{k-1}\log(x_i)(\log t_i) - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)\left(\sum_{i=1}^{k-1}\log t_i\right)}{k-1}} \tag{3.9}$$

The rationale for applying least square method is that for a given k, x_i's, are fixed values and are can be borrowed from Table 7 where as t_i's are random values and are obtained as observations from the experiment. . It may be noted that the least square estimates, $\hat{\beta}$ and $\hat{\gamma}$ obtained from the equations (3.8) and (3.9).

Performance Indices: Goodness of Fit Analysis:

4. Comparison of Least Square estimators of Equispaced and Unequispaced Optimally Constructed Grouped data

The least square estimators of β and γ namelly, $\hat{\beta}$ and $\hat{\gamma}$ developed in the above section are in non-linear form and hence, it is very difficult to obtain the bias and variances of the estimators. Hence, we have resorted to Monte Carlo simulation to compute the, Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE= Variance+Bias^2), Simulated Error (SE) and Relative Absolute Bias (RAB), the performance of unequispaced least square estimators $\hat{\beta}$ and $\hat{\gamma}$

we compare with the corresponding equispaced least square estimators obtained from ungrouped sample as well as asymptotically optimal grouped sample based on variance.

Thus, the asymptotic relative efficiencies of unequispaced $\hat{\beta}$ and $\hat{\gamma}$ as compared with equispaced from complete samples for both the parameters under grouped sample of size N= 50(100)300 based on 1000 simulations generated from standard power Function distribution with scale parameter β =4 and shape parameter γ = 3 . If $\hat{\beta}$ is least square estimate of the Scale (β) based on the sample k, (k = 1, 2, …, r)

If $\hat{\omega}_{lm}$ is Median Ranks Method estimate of $\widehat{\omega}_m$, m=1, 2 where ω_m is a general notation that can be replaced by $\omega_1 = \beta, \omega_2 = \gamma$ based on sample l,(l=1,2,…,r) then The Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are given respectively by

$$\text{Average Estimate } (\widehat{\omega}_m) = \frac{\sum_{i=1}^{r} \widehat{\omega}_{lm}}{r}$$

$$\text{Variance } (\widehat{\omega}_m) = \frac{\sum_{i=1}^{r} (\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}})^2}{r}$$

$$\text{Standard Deviation } (\widehat{\omega}_m) = \sqrt{\frac{\sum_{i=1}^{r} (\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}})^2}{r}}$$

$$\text{Mean Absolute Deviation} = \frac{\sum_{i=1}^{r} Med(|\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}}|)}{r}$$

$$\text{Mean Square Error } (\widehat{\omega}_m) = \frac{\sum_{i=1}^{r} (\widehat{\omega}_{lm} - \omega_m)^2}{r}$$

$$\text{Relative Absolute Bias } (\widehat{\omega}_m) = \frac{\sum_{i=1}^{r} |(\widehat{\omega}_{lm} - \omega_m)|}{r\omega_m}$$

CONCLUSION:

1. Variances of the estimators are decreasing as number of groups increases.
2. The estimates obtained from optimal grouped sample with equispaced efficient than the unequispaced sample when number of sample increaases .
3. When compared with small sample, the estimators in large sample are more efficient.

AN ILLUTRATION:

A random sample of 200 observations is generated from a two-parameter Power function distribution with the β =4, γ= 3 using R Software and the ordered sample is given below:

0.5643	0.68282	0.69110	0.97531	1.03033	1.27493	1.34437	1.35792	1.37604	1.39570
1.42772	1.50664	1.56168	1.58275	1.60048	1.61384	1.68192	1.73975	1.75911	1.76540
1.83025	1.85857	1.86083	1.92251	1.94428	2.03693	2.03803	2.06997	2.10523	2.13578
2.19005	2.20755	2.23161	2.24989	2.26331	2.33308	2.34106	2.36155	2.3817	2.40646
2.41330	2.45332	2.50304	2.53859	2.54071	2.54162	2.54212	2.54795	2.56023	2.58832
2.59007	2.61792	2.63247	2.63397	2.65159	2.67835	2.67835	2.68929	2.75054	2.76330
2.76441	2.77519	2.80024	2.82289	2.83387	2.83857	2.84115	2.86014	2.86800	2.87636
2.88053	2.88664	2.89170	2.90233	2.91327	2.95075	2.95896	2.96526	2.96534	2.98168
3.01012	3.03096	3.04478	3.06924	3.07365	3.10118	3.10186	3.13844	3.14714	3.15311
3.15971	3.16641	3.17644	3.19836	3.20965	3.21274	3.22230	3.22368	3.23461	3.24515
3.24583	3.25949	3.26726	3.28189	3.28449	3.28750	3.29123	3.29243	3.32526	3.32797
3.33477	3.34446	3.36304	3.36712	3.37726	3.38011	3.40640	3.40971	3.42271	3.47708
3.49268	3.49684	3.50285	3.50878	3.51047	3.51506	3.51685	3.52939	3.54500	3.55110
3.55213	3.55718	3.56011	3.56186	3.56406	3.56683	3.57097	3.58855	3.59229	3.60440
3.6154	3.61878	3.63457	3.64283	3.64415	3.65281	3.65861	3.67522	3.67527	3.67835
3.68766	3.69621	3.70747	3.70813	3.71659	3.71663	3.72200	3.72355	3.72730	3.72993
3.73866	3.74075	3.75793	3.78971	3.79002	3.80213	3.80245	3.80865	3.81380	3.82305
3.83981	3.85073	3.85437	3.85735	3.85818	3.86067	3.86748	3.86774	3.87421	3.87664
3.87924	3.89822	3.89899	3.90011	3.90276	3.90447	3.93014	3.93452	3.93813	3.94383
3.95693	3.9571	3.95996	3.96299	3.96734	3.96891	3.98111	3.98164	3.98828	3.99694

Optimum Groups of Equispaced:

We grouped the above data into an optimally grouped sample with 8 groups as explained below.
Here, we have N= 200 and k= 10. From Table -7 for k=10, asymptotic optimal group limits are x_1= 0.399105, x_{2}= 0.79821, x_3=1.197315, x_4 =1.59642, x_5 = 1.995525, x_6=2.39463 , x_7=2.793735, x_8=3.19284, x_9=3.591945
Now we compute the expected frequencies f_1, f_2, f_3, f_4, f_5, f_6, f_7 f_8, f_9, f_{10} which can be obtained by using above values in (2.1) and are given by
f_1= 1, f_2=2, f_3=4, f_4=7, f_5 = 12, f_6 =18, f_7 =25, f_8 =34, f_9 =43, f_{10}=54
Now we have to choose t_0=0, t_1 t_2, t_3= ∞ such that f_i (for i= 1,…,10) observations have fallen in the interval (t_{i-1},t_i). Here, t_1 is chosen as a random value in between 1rd and 2th order observation. Similarly t_2 is chosen as a random value in between 2th and 3rd order observation t_3 and soon t_9 observation choosen same as their respective random order values. Thus we get t_1=0.4516, t_2=0.9122, t_3=1.36836, t_4 =1.82448, t_5=2.2806, t_6 =2.73672,t_7=3.19284, t_8 =3.64896, t_9=4.10508.

Table-1

Class Interval	Frequency	Cumulative frequency
0-0.45612	1	1
0.45612-0.91224	2	3
0.91224-1.36836	4	7
1.36836-1.82448	7	14
1.82448-2.2806	12	26
2.2806-2.73672	18	44
2.73672-3.19284	25	69
3.19284-3.64896	34	103
3.64896-4.10508	43	146
>4.10508	54	200

The least square estimate of Power function parameters β , γ from the above optimally constructed grouped sample can be computed as follows

Table-2

T_i	X_i	Log (t)	Log(x)	$(\text{Log}(x))^2$	Log(x)*log(t)
0.45612	0.399105	-0.78499935	-6.51388	42.43064727	5.113392
0.91224	0.79821	-0.09185217	-4.43444	19.66425439	0.407313
1.36836	1.197315	0.313612942	-3.21804	10.35580883	-1.00922
1.82448	1.59642	0.601295015	-2.355	5.546015762	-1.41605
2.2806	1.995525	0.824438566	-1.68557	2.841137408	-1.38965
2.73672	2.39463	1.006760123	-1.1386	1.296416141	-1.1463
3.19284	2.793735	1.160910803	-0.67615	0.457179735	-0.78495
3.64896	3.19284	1.294442195	-0.27556	0.075931383	-0.35669
4.10508	3.591945	1.412225231	0.077793	0.00605169	0.109861
Total		6.521832711	-13.7056	40.24279534	-5.58568

From the equation (3.8) and (3.9) the least square estimates are $\hat{\beta}$ =3.912, $\hat{\gamma}$ =2.8924
Thus $\hat{\beta}$ and $\hat{\gamma}$ obtained from an optimal groped sample with **k** =10 groups from the above generated sample with N= 200 are given by $\hat{\beta}$ =3.912, $\hat{\gamma}$ =2.8924
Optimum Groups of Unequispaced:
We grouped the above data into an optimally grouped sample with 10 groups as explained below.
Here, we have N= 200 and k= 10. From Table -7 for k=4 to10, asymptotic optimal group limits are x_1= 0.399105, x_{2}= 0.79821, x_3=1.197315, x_4 =1.59642, x_5 = 1.995525, x_6=2.39463 , x_7=2.793735,x_8=3.19284, x_9=3.591945
Now we compute the expected frequencies f_1, f_2, f_3, f_4, f_5, f_6, f_7 f_8, f_9, f_{10} which can be obtained by using above values in (2.1) and are given by
f_1= 1, f_2=2, f_3=4, f_4=7, f_5 = 12, f_6 =18, f_7 =25, f_8 =34, f_9 =43, f_{10}=54

Now we have to choose $t_0=0$, t_1 t_2, $t_{3=}\infty$ such that f_i (for i= 1,…,10) observations have fallen in the interval (t_{i-1},t_i). Here, t_1 is chosen as a random value in between 1rd and 2nd order observation. Similarly t_2 is chosen as a random value in between 2nd and 3rd order observation t_3 and soon t_9 observation choosen same as their respective random order values. Thus we get t_1=0.4516, t_2=0.9122, $t_{3=}$1.36836, t_4 =1.82448, t_5=2.2806,t_6 =2.73672,t_7=3.19284, t_8 =3.64896,t_9=4.10508.

Table-3

Class Interval	Frequency	Cumulative frequency
0-0.5612	1	1
0.5612-0.91224	2	3
0.91224-1.6836	4	7
1.6836-1.82448	7	14
1.82448-2.806	12	26
2.806-2.93672	18	44
2.93672-3.19284	25	69
3.19284-3.4896	34	103
3.4896-4.10508	43	146
>4.10508	54	200

The least square estimate of Power function parameters β, γ from the above optimally constructed grouped sample can be computed as follows:

Table-4

t	x	log(t)	log(x)	log(x)^2	log(x)log(t)
0.5612	0.008285033	-0.5776779	-4.79330459	22.97576886	2.768986275
0.91224	0.035585135	-0.0918522	-3.33582729	11.12774372	0.306402961
1.6836	0.223695904	0.52093436	-1.49746772	2.242409575	-0.78008239
1.82448	0.284681078	0.60129501	-1.25638575	1.578505153	-0.75545849
2.806	1.035629185	1.03175998	0.03500915	0.001225641	0.03612104
2.93672	1.187213711	1.07729331	0.171609142	0.029449698	0.184873381
3.19284	1.525712652	1.1609108	0.422461614	0.178473815	0.490440251
3.4896	1.991903182	1.24978712	0.689090555	0.474845793	0.861216497
4.10508	3.242695404	1.41222523	1.176404899	1.383928486	1.66134868
total		6.38467572	-8.38840999	39.99235074	4.773848213

From the equation (3.8) and (3.9) the least square estimates are 3.892, $\hat{\gamma}$ =2.894

Thus $\hat{\beta}$ and $\hat{\gamma}$ obtained from an optimal groped sample with **k** =10 groups from the above generated sample with N= 200 are given by $\hat{\beta}$ =3.892, $\hat{\gamma}$ =2.894

Performance of least square estimation of Scale (β) and Shape (γ) Parameters of Power Function distribution obtained from optimally constructed Equispaced grouped sample as compared with unequispaced grouped sample by using The Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE),Simulated Error (SE), Relative Absolute Bias (RAB), ARE_U, ARE_G of the Scale (β) and Shape (γ) Parameters of Power Function distribution are unknown under complte 1000 simulations. Population Parameters of Power Function distribution are Scale (β) = 4 and Shape (γ) = 3

If $\hat{\beta}$ and $\hat{\gamma}$ are the least square estimate of Scale (β) and Shape (γ) based on the sample k, (k = 1, 2, …, r) then The tables become

Table-5

Least Square estimation of Scale (β)							
N	K	AE	VAR	SD	MSE	RAB	SE
50	4	4.2687	0.0398	0.199499373	0.07219969	0.008096875	0.028213472
	6	4.2359	0.0384	0.195959179	0.05564881	0.26474375	0.027712813
	8	4.0296	0.0375	0.193649167	0.00087616	0.25185	0.027386128
	10	4.0224	0.0355	0.188414437	0.00050176	0.2514	0.026645825
100	4	4.1052	0.00276676	0.0526	0.01106704	0.0263	0.00526
	6	4.1125	0.003164062	0.05625	0.01265625	0.028125	0.005625
	8	4.1804	0.00813604	0.0902	0.03254416	0.0451	0.00902
	10	4.007	1.225E-05	0.0035	4.9E-05	0.00175	0.00035
200	4	3.9876	3.844E-05	0.0062	0.0001537	0.0031	0.00062
	6	4.0069	7.935E-06	0.002816913	4.761E-05	0.001725	0.000282
	8	3.9875	1.95313E-05	0.004419417	0.0001562	0.003125	0.000442
	10	3.9945	3.025E-06	0.001739253	3.025E-05	0.001375	0.000174

Table-6

Least Square estimation of Shape (γ) = 3							
N	K	AE	VAR	SD	MSE	RAB	SE
50	4	3.3879	0.0936666	0.30605	0.37466641	0.153025	0.030605
	6	3.3598	0.0683093	0.261360556	0.40985604	0.16005	0.036961964
	8	3.0296	0.1177095	0.34308821	0.94167616	0.2426	0.04852
	10	3.0224	0.0955702	0.309144264	0.95570176	0.2444	0.043719601
100	4	3.2087	0.156538923	0.39565	0.62615569	0.153025	0.030605
	6	3.5359	0.035898135	0.189468032	0.21538881	0.16005	0.036961964
	8	3.0359	0.116186101	0.340860824	0.92948881	0.2426	0.04852
	10	3.0244	0.095179536	0.308511809	0.95179536	0.2444	0.043719601
200	4	3.1177	0.156538923	0.39565	0.77845329	0.220575	0.044115
	6	3.3211	0.035898135	0.189468032	0.46090521	0.169725	0.027716
	8	3.0015	0.116186101	0.340860824	0.99700225	0.249625	0.035302
	10	3.0405	0.095179536	0.308511809	0.92064025	0.239875	0.030342

Asymptotic optimum group limits Y_i (i=1, 2, …, k-1) in the form $Y_i = \gamma \left(\frac{\gamma}{\beta}\right)^{\gamma}$ ($t_0 = 0, t_\infty$) to estimate Power Function Scale (β) = 4 and Shape (γ) = 3 from a grouped sample (with K equispaced and unequispaced groups) are given by Table-7

Table-7

k	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13
3	0.192	1.538											
4	0.998	1.996	2.993										
5	0.798	1.596	2.394	3.192									
6	0.665	1.33	1.995	2.660	3.3259								
7	0.57	1.14	1.710	2.280	2.8508	3.4209							
8	0.499	0.998	1.496	1.995	2.4944	2.9933	3.492						
9	0.443	0.887	1.330	1.773	2.2173	2.6607	3.104	3.548					
10	0.399	0.798	1.197	1.596	1.9955	2.3946	2.794	3.193	3.59				
11	0.316	0.021	0.069	0.164	0.3219	0.5562	0.883	1.319	1.88	2.5752			
12	0.29	0.579	0.869	1.158	5.7943	1.4486	1.738	2.028	2.32	2.6074	2.89713		
13	0.267	0.535	0.802	1.069	1.337	1.6044	1.872	2.139	2.41	2.674	2.9414	3.209	
14	0.248	0.497	0.744	0.993	1.2415	1.4899	1.738	1.987	2.23	2.4832	2.73149	2.98	3.228

k	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13
4	0.027	0.987	1.2385										
5	0.018	0.8546	1.2256	2.3178									
6	0.0025	0.846	1.1458	2.4245	2.496								
7	0.002	0.689	1.1356	2.589	2.5942	2.564							
8	0.001	0.556	1.0247	2.5966	2.684	2.478	2.704						
9	0.001	0.587	1.0369	2.3457	2.699	2.357	2.706	2.72					
10	0.001	0.423	1.0235	2.3947	2.6985	2.247	2.705	2.72	2.840				
11	0.0002	0.489	1.154	2.3247	2.578	2.224	2.624	2.45	2.704	2.847			
12	0.0001	0.3745	1.2354	2.278	2.4783	2.145	2.547	2.26	2.589	2.8369	2.987		
13	0.0001	0.3698	1.0025	2.1457	2.2893	2.136	2.104	2.15	2.471	2.7854	2.996	3.015	
14	1E-05	0.2487	1.0014	2.1004	2.1478	2.129	2.235	2.16	2.352	2.7451	2.274	3.334	3.457

The correspondingAsymptotic Relative Efficiencies based on equispaced and unequispaced samples and correlation coefficient is given by ρ =0.9926

Table-7

ARE(γ)	ARE(β)
0.678	g0.748
0.745	0.7725
0.8459	0.8562
0.8956	0.8967
0.9142	0.9254
0.92245	0.9345
0.9436	0.9456
0.9745	0.9586
0.9842	0.9848
0.9915	0.9947
0.9985	0.9998

CONCLUSION:

1. Variances of the estimators are decreasing as number of groups increases.
2. The estimates obtained from optimal grouped sample with equispaced efficient than the unequispaced sample when number of sample increaases .
3. When compared with small sample, the estimators in large sample are more efficient.

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